## T-F Lecture 10. Reversibility and the Carnot Cycle

As we compare efficiencies of various cycles, we observe that some cycles are more efficient than others in converting thermal to mechanical energy. One might ask the question: what is the most efficient cycle and what are its attributes?

Clearly, a given cycle (system) becomes more efficient as one reduces the various energy dissipating modes. For example, one would expect the efficiency to increase as the amount of friction is reduced. In particular, the efficiency should be maximum in the absence of friction.

Our analysis of processes and cycles always excluded friction and any other dissipative effects. Such a process is considered **reversible**. Since friction is generally internal to the system, we will call such a process **internally reversible**.

A process is reversible between states 1 and 2 if it can be retraced identically from 2 to 1. Such processes are shown as solid lines on p-v diagrams.

A reversible process is an idealization that cannot be realized in any real process.

Reversible expansion will thus produce the maximum amount of work, while reversible compression will require the minimum amount of work.



The above reversibility refers to mechanical interactions such as work and accompanying friction. What about heat interaction? Can heat interaction be reversible?

Note that heat flows from a high to low temperatures over some finite temperature difference. Such heat flow however is not reversible, i.e. heat does not, of its own accord, flow from a low to a high temperature. **Reversible heat flow** is defined as heat flow over an infinitesimal, or differential temperature difference  $dT \rightarrow 0$ .

As  $dT \rightarrow 0$  it takes an infinite amount of time to transfer a finite amount of heat. Since time is not a factor here, this is not a problem conceptually. In a real heat transfer, however we need a finite temperature difference that renders such heat transfer irreversible.

A process wherein heat transfer occurs over a finite temperature difference  $\Delta T$  is called an **externally irreversible** process, and one in which heat transfer occurs over a differential (infinitesimal) temperature difference dT is called **externally reversible**.

A cycle that is **both internally and externally reversible** therefore should have the **highest efficiency** possible.

The cycles we have been analyzing have all been internally reversible but not externally reversible. How might an externally reversible cycle be configured?

Since heat must be added to the system from some high temperature reservoir and rejected to some low temperature reservoir (typically the surroundings), there should be two such processes. Furthermore for theses processes to be externally reversible, the temperature difference between the system and the high temperature reservoir, and the system and its low temperature reservoir cannot be larger that dT.

An isothermal heat addition process at the temperature of the high temperature reservoir, and an isothermal heat rejection process at the temperature of the low temperature reservoir can meet such requirements. Processes that link these two isothermal processes to complete a cycle must not have any heat transfer, they must be adiabatic. Thus a simple internally and externally reversible cycle will have two isothermal and two adiabatic processes. Such a cycle is called a **Carnot cycle**.

### A Carnot cycle configured in a piston-cylinder arrangement

The cylinder has insulated sidewalls and a thermally conducting endwall. The endwall can be in contact with either an insulated endwall or a thermal reservoir for heating or cooling the gas inside the cylinder. The piston is frictionless.



### Isothermal Heat Addition Process.

The cylinder is at its minimum volume and the endwall is in contact with the high temperature thermal reservoir at the temperature  $T_H$ . As the gas inside the cylinder is heated the gas begins to expand and the piston moves out. The expansion is such that the gas in the piston remains at the hot reservoir temperature  $T_H$ . The process is an **isothermal expansion** process. High temperature endwall at  $T_H$ 



### Adiabatic Expansion Process.

After the heat addition, the hot reservoir is removed form the conducting endwall and replaced with an insulating endwall. The gas is allowed to expand and cool to some low temperature  $T_C$ . The process is an **adiabatic expansion** process.

### Isothermal Heat Rejection Process.

The cylinder is now at its minimum volume, the insulating endwall is replaced with the low temperature thermal reservoir at the temperature  $T_C$ . As the gas inside the cylinder is cooled the gas begins to compress as the piston moves inward. The compression is such that the gas in the piston remains at the low reservoir temperature  $T_C$ . The process is an **isothermal compression** process.

### Adiabatic Compression Process.

After the heat rejection, the low temperature reservoir is removed form the conducting endwall and replaced with an insulating endwall. The gas is compressed and heats up to the high temperature  $T_H$ . The process is an **adiabatic compression** process.



Low temperature endwall at  $T_C$ 





Representation of the Carnot cycle on a *p*-*v* diagram:



# Isothermal Heat Addition, Process 1-2. Isothermal heat addition, р $q_{12} = q_H$ $T_2 = T_1$ and $p_2 v_2 = p_1 v_1$ (process) $p = \frac{RT}{v}$ (Ideal gas law) $du = \delta q - \delta w$ (First Law) $du = c_v dT = 0$ (constant specific heat) $\delta q = \delta w = p dv$ (reversible work) V $q_{12} = w_{12} = \int_{1}^{2} p dv = RT_1 \ln \frac{v_2}{v_1}$ р Adiabatic Expansion, Process 2-3. Isothermal heat addition, $q_{12} = q_H$ $\delta q = 0$ (process) Adiabatic 2 $du = \delta q - \delta w = - \delta w$ expansion, $q_{23} = 0$ $c_v dT = -p dv = -RT \frac{dv}{v} = -RT d \ln v$ 3 $\int_{2}^{3} d\ln v = -\frac{c_{v}}{R} \int_{2}^{3} d\ln T$ V $\ln \frac{v_3}{v_2} = -\frac{c_v}{R} \ln \frac{T_3}{T_2} = \ln \left(\frac{T_3}{T_2}\right)^{-\frac{c_v}{R}} = \ln \left(\frac{T_2}{T_3}\right)^{\frac{c_v}{R}}$ $\frac{v_3}{v_2} = \left(\frac{T_2}{T_2}\right)^{c_v / R}$

Using the ideal gas law we obtain the following relationship between p and v

$$\frac{p_3}{p_2} = \left(\frac{v_2}{v_3}\right)^{\frac{R}{c_v} + 1} = \left(\frac{v_2}{v_3}\right)^k \text{ Note that } c_p = c_v + R \text{ and } \mathbf{k} = \mathbf{c}_p/\mathbf{c}_v$$

and  $p_3(v_3)^k = p_2(v_2)^k = constant$ . This is the same result we obtained earlier for a reversible adiabatic process. Since we have a relation ship between T and v, and p and v, we can evaluate both the change in internal energy and the work for this process.

$$\Delta u_{23} = c_v (T_3 - T_2)$$
 and from the work integral we obtain an alternative expression:  
 $w_{23} = \frac{p_3 v_3 - p_2 v_2}{1 - k}$ 

Note that on the p-v diagram we have drawn the adiabatic process (process 2-3) steeper than the isothermal process (process 1-2). Can you show why it is steeper?

Solving for the slope at a given point on the p-v diagram for each of the processes can demonstrate this.

The *p*-*v* relationship for the isothermal process passing through the point at  $p_0$  and  $v_0$  is:

$$pv = p_0 v_0 = constant$$
$$p = \frac{p_0 v_0}{v}$$
$$\frac{dp}{dv} = -\frac{p_0 v_0}{v^2} = -\frac{p}{v}$$

The *p*-*v* relationship for the adiabatic process passing through the point at  $p_0$  and  $v_0$  is:

$$pv^{k} = p_{0}(v_{0})^{k} = constant$$
$$p = \frac{p_{0}v_{0}^{k}}{v^{k}}$$
$$\frac{dp}{dv} = -k\frac{p_{0}v_{0}^{k}}{v^{k}} = -k\frac{p}{v}$$

The ratio of the adiabatic to isothermal slopes is

$$\frac{\left(\frac{dp}{dv}\right)_{ADIABATIC}}{\left(\frac{dp}{dv}\right)_{ISOTHERMAL}} = k$$

since k > 1, and each of the slopes is negative, the adiabatic process slope is greater than the isothermal process slope.



### Isothermal Heat Rejection, Process 3-4.

This process is similar to the heat addition process.

$$T_{4} = T_{3} \text{ and } p_{4}v_{4} = p_{3}v_{3} \text{ (process)}$$

$$p = \frac{RT}{v} \text{ (ideal gas law)}$$

$$du = \delta q - \delta w \text{ (First Law)}$$

$$du = c_{v}dT = 0 \text{ (constant specific heat)}$$

$$\delta q = \delta w = pdv \text{ (reversible work)}$$

$$q_{34} = w_{34} = \int_{3}^{4} pdv = RT_{3} \ln \frac{v_{4}}{v_{3}}$$



### Adiabatic Compression, Process 4-1.

This process is similar to the adiabatic expansion process.

$$\delta q = 0$$
 and  $pv^k = constant$  (process)

$$\mathrm{d} u = \delta q - \delta w = - \delta w$$

 $du = c_v dT = -\delta w = -p dv$ 

 $\Delta u_{41} = c_{\rm v} \left( T_1 - T_4 \right)$ 

and from the work integral we obtain an alternative expression:

$$w_{41} = \frac{p_1 v_1 - p_4 v_4}{1 - k}$$



#### **Summary of Process Energy Calculations**

Isothermal heat addition, process 1-2:  

$$q_{12} = w_{12} = RT_1 \ln \frac{v_2}{v_1}$$
Adiabatic expansion, process 2-3:  

$$\Delta u_{23} = c_v (T_3 - T_2) = -w_{23} = -\frac{p_3 v_3 - p_2 v_2}{1 - k}$$
Isothermal heat rejection, process 3-4:  

$$q_{34} = w_{34} = RT_3 \ln \frac{v_4}{v_3}$$
Adiabatic compression, process 4-1:  

$$\Delta u_{41} = c_v (T_1 - T_4) = -w_{41} = -\frac{p_1 v_1 - p_4 v_4}{1 - k}$$

#### **Thermal Efficiency**

$$\eta = \frac{w_{NET}}{q_{IN}}$$

Net work =  $w_{NET} = w_{12} + w_{23} + w_{34} + w_{41}$ 

$$w_{NET} = RT_1 \ln \frac{v_2}{v_1} + c_v (T_3 - T_2) + RT_3 \ln \frac{v_4}{v_3} + c_v (T_1 - T_4)$$

Heat addition =  $q_{IN} = RT_1 \ln \frac{v_2}{v_1}$ 

$$\eta = \frac{RT_1 \ln \frac{v_2}{v_1} + c_v (T_3 - T_2) + RT_3 \ln \frac{v_4}{v_3} + c_v (T_1 - T_4)}{RT_1 \ln \frac{v_2}{v_1}}$$

Recall that  $T_1 = T_2 = T_H$  and  $T_4 = T_3 = T_C$ , hence  $c_v (T_1 - T_4) = c_v (T_2 - T_3) = -c_v (T_3 - T_2)$ . Thus the third and fourth terms in the numerator of the efficiency expression cancel. Recall also that for a reversible and adiabatic process

$$\frac{v_3}{v_2} = \left(\frac{T_2}{T_3}\right)^{c_v/R} \text{ and } \frac{v_4}{v_1} = \left(\frac{T_1}{T_4}\right)^{c_v/R} = \left(\frac{T_2}{T_3}\right)^{c_v/R}$$

Hence  $\frac{v_3}{v_2} = \frac{v_4}{v_1}$  or  $\frac{v_1}{v_2} = \frac{v_4}{v_3}$  and  $\ln \frac{v_4}{v_3} = \ln \frac{v_1}{v_2} = -\ln \frac{v_2}{v_1}$ . The ln term in the efficiency equation will cancel and the final expression for the thermal efficiency becomes:

$$\eta = 1 - \frac{T_3}{T_1} = 1 - \frac{T_C}{T_H}$$

The Carnot thermal efficiency is usually expressed in terms of the hot or high temperature thermal reservoir temperature  $T_H$ , and the cold or low temperature thermal reservoir temperature  $T_C$ .

$$\eta_{CARNOT} = 1 - \frac{T_C}{T_H}$$

Recall the conditions under which the above thermal efficiency is valid.

- The system is both internally and externally reversible
- Heat addition to the system is from a single thermal reservoir at the temperature  $T_H$  and the temperature difference between the system and the thermal reservoir is no greater than dT.
- Heat rejection from the system is to a single thermal reservoir at the temperature  $T_C$  and the temperature difference between the system and the thermal reservoir is no greater than dT.

### **Important conclusions**:

- The above thermal efficiency does not depend on the nature of the fluid medium or any details of the heat engine.
- This efficiency is a general result for heat engines operating between two single thermal reservoirs.
- This is the highest efficiency that can be attained for a heat engine operating between a single hot and a single cold thermal reservoir.

From the general definition of thermal efficiency we observe the following:

$$\eta = \frac{w_{NET}}{q_{IN}} = \frac{q_{IN} - q_{OUT}}{q_{IN}} = 1 - \frac{q_{OUT}}{q_{IN}} = 1 - \frac{q_C}{q_H} = 1 - \frac{T_C}{T_H}$$

Hence we observe that

$$\frac{q_C}{q_H} = \frac{T_C}{T_H}$$

The ratio of the heat rejected from the system to the heat added to the system is in the ratio of the corresponding temperatures of the thermal reservoirs. Note also that these temperatures are absolute temperatures, in units of K or <sup>o</sup>R and **not** <sup>o</sup>C or <sup>o</sup>F.





Operate the Otto cycle wetween the TH and Tc temperatures of the Carnot cycle; TH being the hottest, and Tc the coldest available temperatures.



 $\begin{aligned} & \operatorname{Recurs} = 1 - \frac{L_{H}}{L_{H}} \\ & \operatorname{Assume that} \ \mathcal{N}_{c} > \mathcal{N}_{0}, \text{ hence we must have } (\underline{L_{H}} - \underline{L}) \\ & (1 - \underline{L_{H}}) \\ & (1 - \underline{L_{H}) \\ & (1 - \underline{L_{H}) \\ & (1 - \underline{L_{H}}) \\ & (1 - \underline{L_{H}) \\ & (1 -$ 

Numerical Compartison I In Chample ) 10000  $T_3 = T_H = 2874 K$  and  $T_1 = T_C = 293 \rightarrow 2000 T_1 = \frac{293}{2874} = \frac{89.8\%}{2874}$