T-F Lecture 7. Stirling Cycle Numerical Example

To solve the Stirling cycle, we must solve for all the state variables and the processes that make up the cycle.

A summary of the energy equations developed in the last lecture is represented on the p-v diagram below.



Before beginning with an actual numerical calculation example let us review the variables and available equations for the cycle.

There are twelve (12) p, v, T variables and twelve (12) energy variables, a total of 24 variables:

$$p_1, v_1, T_1; p_2, v_2, T_2; p_3, v_3, T_3; p_4, v_4, T_4;$$

 $\Delta u_{12}, q_{12}, w_{12}; \Delta u_{23}, q_{23}, w_{23}; \Delta u_{34}, q_{34}, w_{34}; \Delta u_{41}, q_{41}, w_{41};$

There are four (4) state equations, and four (4) process equations among the state variables:

$$p_1v_1 = RT_1, p_2v_2 = RT_2, p_3v_3 = RT_3, p_4v_4 = RT_4,$$

 $T_1 = T_2, v_2 = v_3, T_3 = T_4, \text{ and } v_1 = v_4.$

Additionally there are twelve equations for the energy variables, three equations for each process. These equations are summarized in the p-v diagram above.

In summary we have 24 variables and 20 equations. To solve the system we must specify four independent variables. In our example we shall specify an initial state, say p_1 and T_1 , and any two additional independent variables.

Consider the following given four pieces of information:

Initial pressure,	<i>p</i> ₁ =100 kPa
Initial temperature,	$T_1 = 25^{\circ} \text{C},$
Volume compression ratio,	$v_1 = 6 v_2$, and
Maximum cycle temperature,	$T_3 = 1100^{\circ}$ C.

The working fluid is air with the following properties:

 $R = 0.287 \text{ kJ/kg} \cdot \text{K}$ and $cv = 0.718 \text{ kJ/kg} \cdot \text{K}$

The cycle is solved with the aid of the Mathcad mathematics software below.

Stirling Cycle Calculation Example

Consider an ideal Stirling cycle with the following given information:



Start the cycle calculation from the initial ambient state 1.

State 1: Using the ideal gas equation of state, pv = RT, calculate the initial volume, v1:

Let $kJ := 10^3 \cdot J$ The gas constant R for air is $R := 0.287 \frac{kJ}{kg \cdot K}$ $v_1 := \frac{R \cdot T_1}{p_1}$ $v_1 = 0.855 \frac{m^3}{kg}$

Process 1 - 2: Isothermal compression process. $T_2 := T_1$

State 2: From the given compression ratio we have: $v_2 := \frac{v_1}{6}$ $v_2 = 0.143 \frac{m^3}{kg}$

The isothermal process, $T_2 := T_1$, can alternatively be expressed as $p_1 \cdot v_1 = p_2 \cdot v_2$ for an ideal gas.

The pressure at state 2 is thus
$$p_2 := p_1 \cdot \left(\frac{v_1}{v_2}\right)$$
 $p_2 = 600 \text{kPa}$
The expression for *work* is $w_{12} := \int_1^2 p \, dv$ with $p := \frac{R \cdot T_1}{v}$

$$w_{12} := \int_{v_1}^{v_2} \left(\frac{R \cdot T_1}{v}\right) dv$$
 $w_{12} = -153.242 \frac{kJ}{kg}$

Note that alternatively, one can calculate the work from the evaluated integral, which is:

$$w_{12} := R \cdot T_1 \cdot ln\left(\frac{v_2}{v_1}\right)$$
 $w_{12} = -153.242 \frac{kJ}{kg}$

Calculate the change in *internal energy*, assuming constant specific heat $c_v := 0.718 \frac{kJ}{kg \cdot K}$

$$\Delta u_{12} \coloneqq c_v \cdot \left(T_2 - T_1\right) \qquad \qquad \Delta u_{12} = 0 \frac{kJ}{kg}$$

Heat transfer is calculated from the First Law expression $\Delta u := q - w$

Hence
$$q_{12} := \Delta u_{12} + w_{12}$$
 $q_{12} = -153.242 \frac{kJ}{kg}$

Process 2 - 3: Constant volume process $v_3 := v_2$

State 3: From the ideal gas equaton of state and a constant volume process

$$p_3 := p_2 \cdot \frac{T_3}{T_2} \qquad p_3 = 2.764 \times 10^3 \, \text{kPa}$$

The expression for *work* is $w_{23} := \int_2^3 p \, \text{dv}$ with $p := \frac{R \cdot T}{v_1}$
Since $dv = 0$ $w_{23} := 0 \cdot \frac{kJ}{kg}$

Change in *internal energy*:
$$\Delta u_{23} := c_v \cdot (T_3 - T_2)$$
 $\Delta u_{23} = 771.85 \frac{kJ}{kg}$

Heat transfer is calculated from the First Law expression

$$q_{23} := \Delta u_{23} + w_{23}$$
 $q_{23} = 771.85 \frac{kJ}{kg}$

Process 3 - 4: Isothermal expansion process, $T_4 := T_3$ $T_4 = 1.373 \times 10^3 \text{ K}$

State 4: Expansion occurs from state 3 to state 4. Since the last process is at constant vole the expansion is from v3 to v4, and

 $v_4 := v_1$

From the ideal gas equation of state:
$$p_4 := p_3 \cdot \left(\frac{v_3}{v_4}\right)$$
 $p_4 = 460.738 \text{kPa}$

The expression for *work* is $w_{34} := \int_{3}^{4} p \, dv$ with $p := \frac{R \cdot T_3}{v}$

$$\mathbf{w}_{34} \coloneqq \int_{\mathbf{V}_3} \left(\frac{\mathbf{R} \cdot \mathbf{\Gamma}_3}{\mathbf{v}}\right) d\mathbf{v} \qquad \mathbf{w}_{34} = 706.045 \frac{\mathbf{kJ}}{\mathbf{kg}}$$

Calculate the change in *internal energy*, assuming constant specific heat $c_v := 0.718 \frac{kJ}{kg \cdot K}$

$$\Delta u_{34} \coloneqq c_v \cdot \left(T_4 - T_3\right) \qquad \qquad \Delta u_{34} = 0 \frac{kJ}{kg}$$

Heat transfer is calculated from the First Law expression $\Delta u := q - w$

Hence
$$q_{34} := \Delta u_{34} + w_{34}$$
 $q_{34} = 706.045 \frac{kJ}{kg}$

Process 4 - 1: Constant volume process $v_4 := v_1$

State 4 is known since $v_4 := v_1$ and $T_4 := T_3$ and the pressure at state 4 is

$$p_4 := p_1 \cdot \left(\frac{T_4}{T_1}\right)$$
 $p_4 = 460.738 kPa$

The expression for *work* is
$$w_{41} := \int_{4}^{1} p \, dv$$
 with $p := \frac{R \cdot T}{v_4}$

Since dv = 0 $w_{41} := 0 \cdot \frac{kJ}{kg}$

Change in *internal energy*:
$$\Delta u_{41} := c_v \cdot (T_1 - T_4)$$
 $\Delta u_{41} = -771.85 \frac{kJ}{kg}$

Heat transfer is calculated from the First Law expression

$$q_{41} := \Delta u_{41} + w_{41}$$
 $q_{41} = -771.85 \frac{kJ}{kg}$

 $\label{eq:cycle summation: q_net} \textbf{Cycle summation:} \quad q_{net} \coloneqq q_{12} + q_{23} + q_{34} + q_{41} \quad w_{net} \coloneqq w_{12} + w_{23} + w_{34} + w_{41}$

 $q_{net} = 552.803 \frac{kJ}{kg}$ $w_{net} = 552.803 \frac{kJ}{kg}$ Note $q_{net} = w_{net}$, as expected for a cycle

$$\Delta u_{net} := \Delta u_{12} + \Delta u_{23} + \Delta u_{34} + \Delta u_{41} \qquad \Delta u_{net} = 0 \frac{kJ}{kg} \qquad \text{As expected for a cycle}$$

Cycle efficiency: $\eta := \frac{w_{net}}{q_{in}}$

The net work has been calculated above. Consider next the heat input to the cycle. Examine the heat terms for each processand and remember that our sign convention for heat is: heat into the system is positive, and heat out of the system is negative. List the heat transfer for each process:

$$q_{12} = -153.242 \frac{kJ}{kg}$$
 $q_{23} = 771.85 \frac{kJ}{kg}$ $q_{34} = 706.045 \frac{kJ}{kg}$ $q_{41} = -771.85 \frac{kJ}{kg}$

From the above listing it is clear that the heat input to the system is represented by:

$$q_{in} := q_{23} + q_{34}$$
 $q_{in} = 1.478 \times 10^3 \frac{kJ}{kg}$

Hence the thermal efficiency becomes: $\eta := \frac{w_{net}}{q_{in}}$ $\eta = 0.374$

Notice in the above heat transfer values, the heat rejected by process 4-1 is equal to the heat input to process 2-3. If we take the heat rejected from process 4-1 and use it to supply process 2-3, then the only heat input to the cycle become just the heat input to process 3-4. This is called **regeneration**.

Thus, with regeneration the cycle efficiency becomes

$$q_{in} := q_{34} \qquad \qquad \eta := \frac{w_{net}}{q_{in}} \qquad \qquad \eta = 0.783$$