T-F Lecture 6A – Heat Capacity, Internal Energy, and Heat Energy.

DEFINITION:

Heat Capacity – Amount of heat energy required to raise the temperature of a substance per unit degree.

Heat Energy

$$C = \frac{Q}{\Delta T}$$

Temperature Difference

Heat Capacity

Specific Heat Capacity – Heat Capacity per unit mass

$$c = \frac{C}{m}$$

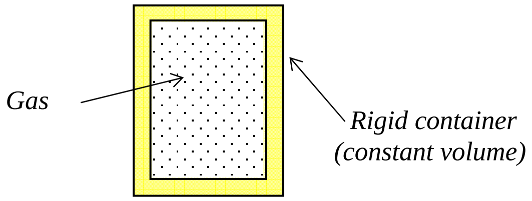
In differential form the specific heat is defined as:

$$c = \frac{1}{m}\frac{\delta Q}{dT}$$

Note that for gases the amount of heat energy required to raise the temperature by a degree will depend on the particular process. We therefore must specify the process. Consider the constant volume and constant pressure processes.

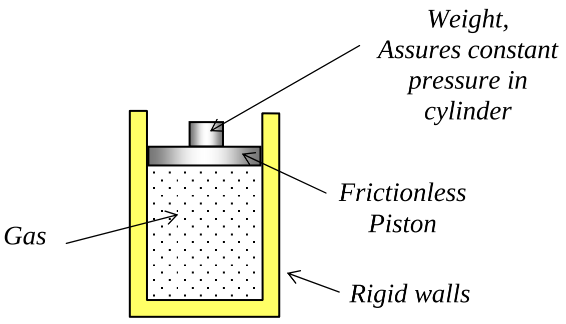
Specific heat at constant volume:

$$c_v = \frac{1}{m}\frac{\delta Q_v}{dT}$$



Specific heat at constant pressure:

$$c_p = \frac{1}{m}\frac{\delta Q_p}{dT}$$



From the differential form of First Law of thermodynamics,

$$dU = \delta Q - \delta W$$

we obtain an expression for heat energy,

$$\delta Q = dU + \delta W.$$

Compression / expansion work in a closed system is given by:

$$\delta W = pdV$$

Hence,

$$\boxed{\delta Q = dU + pdV}$$

Constant Volume Process

For a constant volume process, V = constant, and $dV = 0$. Hence, the heat transfer for a constant volume process is:

$$\delta Q_v = dU$$

From the definition of constant volume specific heat capacity, we obtain

$$c_v \equiv \frac{1}{m}\frac{\delta Q_v}{dT} = \frac{1}{m}\frac{dU}{dT} = \frac{du}{dT}$$

Note that the expression for the constant volume heat capacity,

$$\boxed{c_v = \frac{du}{dT}}$$

is an expression in terms of state variables u and T, hence c_v is also a property of the system. This expression also allows us to calculate the change in the specific internal energy du. We write:

$$du = c_v dT$$

This expression can be integrated for any process between any two state variables since this is an expression in terms of state variables and system properties that are independent of any particular processes.

$$\boxed{\Delta u_{12} = \int_1^2 du = \int_1^2 c_v dT}$$

In general c_v is a function of temperature. For constant c_v, we have

$$\Delta u = c_v \Delta T$$

Constant Pressure Process

For a constant pressure process we again obtain an expression for the heat transfer δQ. We start with the earlier expression:

$$\delta Q = dU + pdV$$

Additionally we define a new state variable, called *Enthalpy* and denoted by the symbol H.

$$H = U + pV$$

where U is the internal energy, p the pressure, and V the volume. In differential form we write:

$$dH = dU + pdV + Vdp$$

Solving for dU and substituting into the δQ equation above we obtain

$$\delta Q = (dH - pdV - Vdp) + Vdp = dH - Vdp$$

Thus for a constant pressure process, $p = constant$ and $dp = 0$.

$$\delta Q_p = dH$$

From the earlier definition of specific heat

$$c_p = \frac{1}{m}\frac{\delta Q_p}{dT}$$

we obtain

$$c_p = \frac{1}{m}\frac{dH}{dT} = \frac{dh}{dT}$$

Note that the expression for the constant pressure heat capacity,

$$\boxed{c_p = \frac{dh}{dT}}$$

is an expression in terms of state variables h and T, hence c_p is also a property of the system. This expression also allows us to calculate the change in the specific internal enthalpy dh. We write:

$$dh = c_p dT$$

This expression can be integrated for any process between any two state variables since this is an expression in terms of state variables and system properties that are independent of any particular processes.

$$\boxed{\Delta h = \int_1^2 dh = \int_1^2 c_p dT}$$

In general c_p is a function of temperature. For constant c_p, we have

$$\Delta h = c_p \Delta T$$

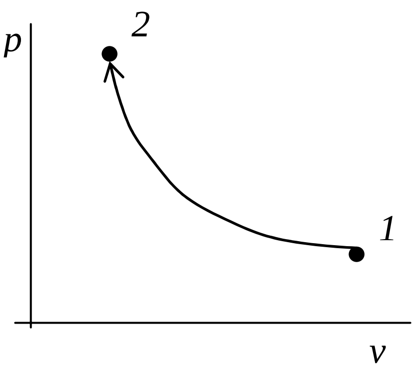
Expression for δQ, the heat energy transfer, in a general process:

We have shown previously that specific work for a closed system is given by

$$w_{12} = \int_1^2 p dv$$

and we have just shown that the specific internal energy is given by

$$\Delta u = \int_1^2 c_v dT$$



We can now write a general expression for Q_{12} based on the first law of thermodynamics:

$$Q_{12} = \Delta U_{12} + W_{12}$$

$$Q_{12} = m\int_1^2 c_v dT + m\int_1^2 p dv$$

Note that $\delta Q = m\delta q$, $V = mv$, *and* $U = mu$.

For constant specific heat we obtain

$$Q_{12} = mc_v \Delta T + m\int_1^2 p dv$$

Note that to obtain Q_{12} we must still specify the process in order to evaluate the work integral. To do so, we must have an expression for p as a function of v, i.e. we must have

$$p = f(v).$$