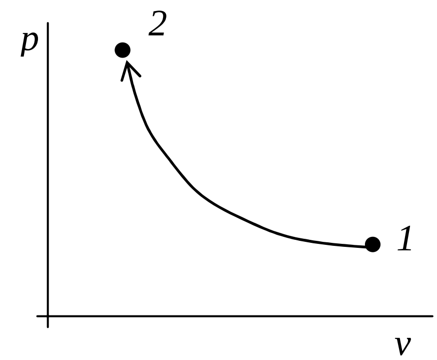
T-F Lecture 6B – The First Law and the Stirling Cycle.

Recall the expression for the First Law for a process of a closed system in terms of intensive variables:

$$\Delta u_{12} = q_{12} - w_{12}$$



We have shown previously that the work for a process is given by

$$w_{12} = \int_1^2 p dv$$

where p must be expressed as a function of v.

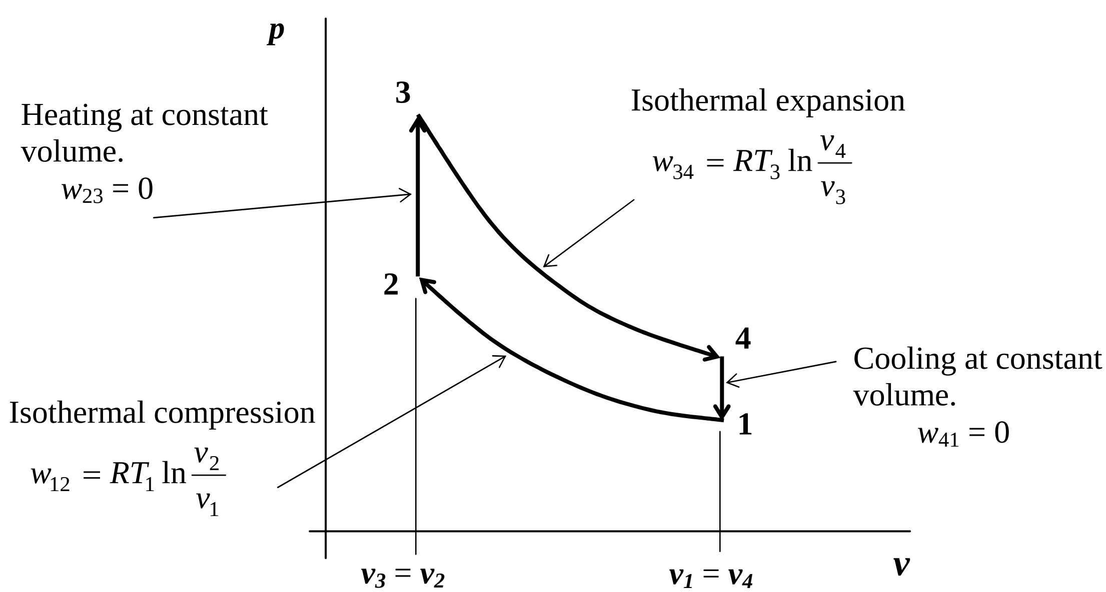
We have just shown that the change in internal energy is given by:

$$\Delta u_{12} = \int_1^2 du = \int_1^2 c_v dT = c_v (T_2 - T_1) = c_v \Delta T$$

We can now evaluate the heat transfer for a process. From the First Law we write

$$q_{12} = \Delta u_{12} + w_{12} = \int_1^2 c_v dT + \int_1^2 p dv$$

Recall the Stirling cycle and the work calculations for each process:



Calculate the heat transfer for each process and the cycle as a whole. Assume a constant specific heat.

Recall again

$$q_{12} = c_v\left(T_2 - T_1\right) + \int_1^2 p dv$$

Process 1-2. Isothermal compression. $T_2 = T_1$. Recall also that $p_1 v_1 = p_2 v_2 = constant$.

$$q_{12} = c_v\left(T_2 - T_1\right) + RT_1 \ln\frac{v_2}{v_1} = RT_1 \ln\frac{v_2}{v_1}$$

Process 2-3. Heat addition at constant volume. $v_2 = v_1 = constant$.

$$q_{23} = c_v\left(T_3 - T_2\right) + \int_2^3 p dv = c_v\left(T_3 - T_2\right)$$

Process 3-4. Isothermal expansion. $T_4 = T_3$. Recall also that $p_4 v_4 = p_3 v_3 = constant$.

$$q_{34} = c_v\left(T_4 - T_3\right) + RT_3 \ln\frac{v_4}{v_3} = RT_3 \ln\frac{v_4}{v_3}$$

Process 4-1. Cooling at constant volume. $v_2 = v_1 = constant$.

$$q_{41} = c_v\left(T_1 - T_4\right) + \int_4^1 p dv = c_v\left(T_1 - T_4\right)$$

Cycle Calculation.

There are three quantities of primary interest.

- Net work of the cycle
- Total heat input to the cycle
- Thermal efficiency of the cycle

Net work of the cycle:

$$w_{net} = \sum w_{ij} = w_{12} + w_{23} + w_{34} + w_{41}$$

$$w_{net} = RT_1 \ln \frac{v_2}{v_1} + 0 + RT_3 \ln \frac{v_4}{v_3}$$

Recall that $v_3 = v_2$ and $v_4 = v_1$, also note that $v_1 > v_2$, thus $\ln \frac{v_2}{v_1} = -\ln \frac{v_1}{v_2}$, and

$$w_{net} = RT_1 \ln \frac{v_2}{v_1} + RT_3 \ln \frac{v_1}{v_2} = -RT_1 \ln \frac{v_1}{v_2} + RT_3 \ln \frac{v_1}{v_2}$$

$$w_{net} = R(T_3 - T_1) \ln \frac{v_1}{v_2}$$

Since $v_1 > v_2$ and $T_3 > T_1$, $w_{net} > 0$: the work is positive indicating a net work output. Recall our sign convention that positive work represents work output and negative work represents work input.

Heat input to the cycle:

According to our sign convention, positive values of heat represent heat input and negative values represent heat output. We therefore must examine the sign of the heat transfer in each of the processes.

Process 1-2. $q_{12} = RT_1 \ln \frac{v_2}{v_1} = -RT_1 \ln \frac{v_1}{v_2} < 0 \Rightarrow$ heat output

Process 2-3. $q_{23} = c_V \left(T_3 - T_1\right) > 0 \Rightarrow$ heat input

Process 3-4. $q_{34} = RT_3 \ln \frac{v_4}{v_3} = RT_3 \ln \frac{v_1}{v_2} < 0 \Rightarrow$ heat input

Process 4-1. $q_{41} = c_V \left(T_1 - T_4\right) = -c_V \left(T_3 - T_1\right) > 0 \Rightarrow$ heat output

Total heat input , $q_{in} = q_{23} + q_{34}$:

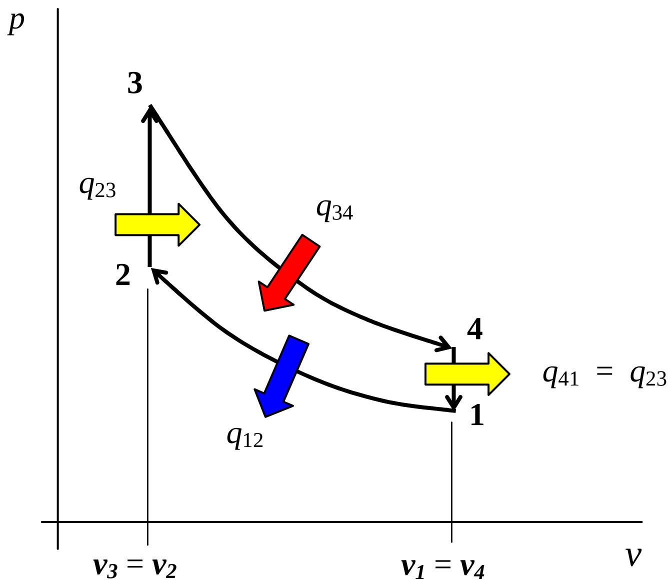
$$q_{in} = c_V \left(T_3 - T_1\right) + RT_3 \ln \frac{v_1}{v_2}$$

Thermal efficiency of the Stirling Cycle, η.

$$\eta = \textit{thermal efficiency} = \frac{\textit{net work output}}{\textit{heat input}} = \frac{w_{net}}{q_{in}}$$

$$\eta = \frac{R(T_3 - T_1)\ln\frac{v_1}{v_2}}{c_V(T_3 - T_1) + RT_3 \ln\frac{v_1}{v_2}}$$

Look at the Stirling cycle processes 2-3 and 4-1 heat transfer in terms of their magnitudes.



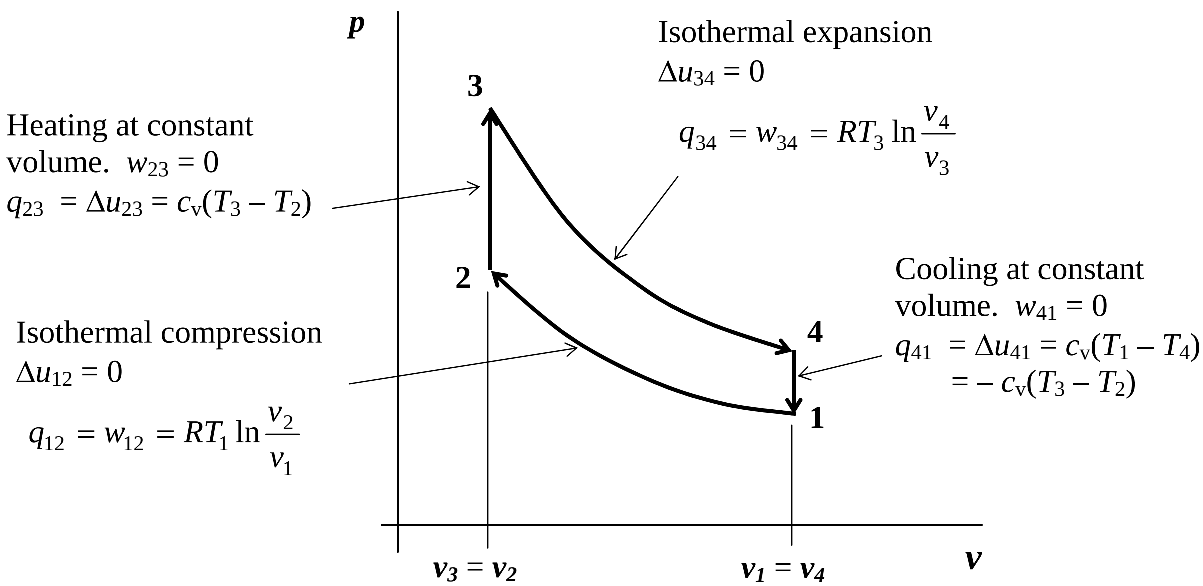
Note that the magnitude of the heat output q_{41} is equal to the magnitude of the heat input q_{23}. Is it possible to use the heat output q_{41} to supply the heat input q_{23}? Yes, in principle, it can. If this is done then the only heat input required will be q_{34}.

The use of heat output to supply heat input is called **regeneration**. Since the only external heat supplied is now q_{34}, the thermal efficiency with regeneration becomes:

$$\eta_{reg} = \frac{R(T_3 - T_1)\ln\frac{v_1}{v_2}}{RT_3 \ln\frac{v_1}{v_2}} = \frac{T_3 - T_1}{T_3} = 1 - \frac{T_1}{T_3}$$

Summary of the energy calculations.

A summary of the energy calculation is represented on the *p*-*v* diagram below.



sdf
Sdv
sdf