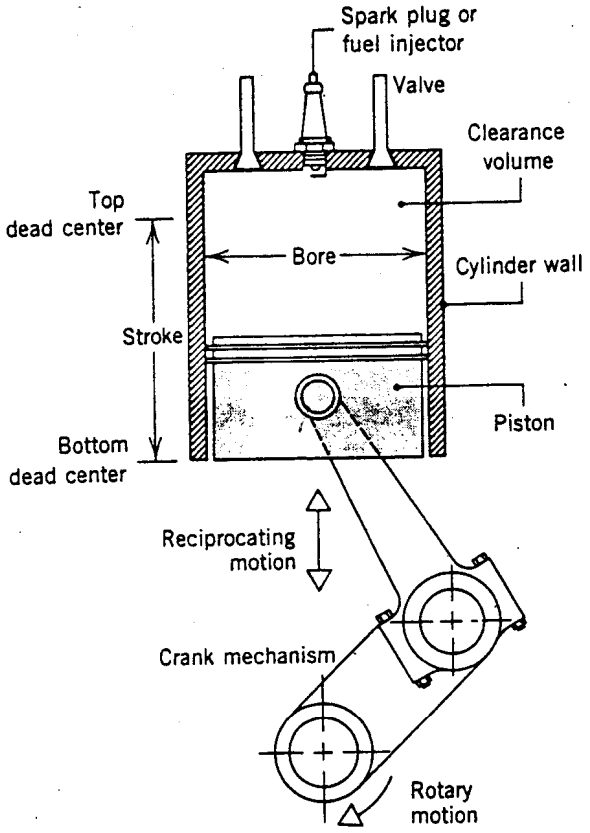
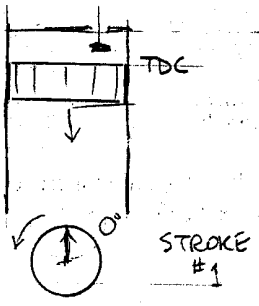
Otto Cycle Engine

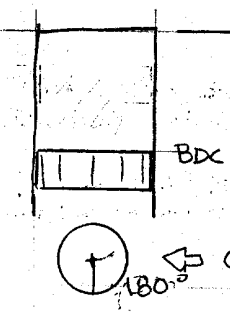
The Otto cycle engine is an internal combustion engine. Combustion occurs inside the piston-cylinder volume.

The following is a representation of the reciprocating piston cylinder engine.

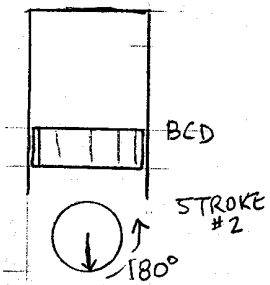


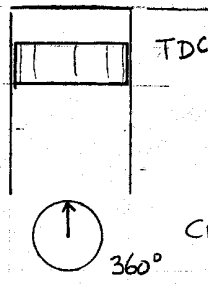
The cycle includes two rotations of the crank, and four strokes of the piston.



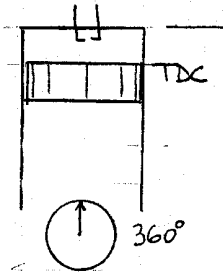


STROKE #1: Intake of air-fuel mixture though intake valve



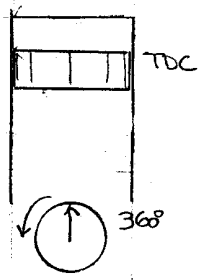


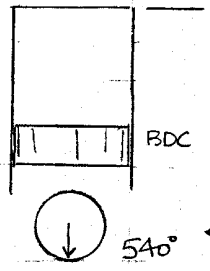
STROKE #2: Compression of air-fuel mixture



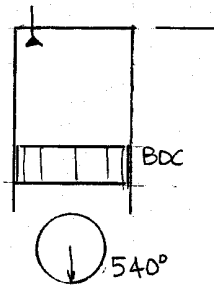
COMBUSTION Ignition of air-fuel mixture (heat addition at constant volume - piston in 360° position)

Air-fuel mixture reacts to form combustion products.

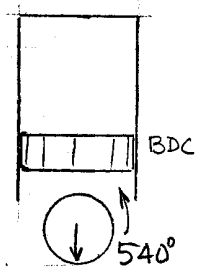


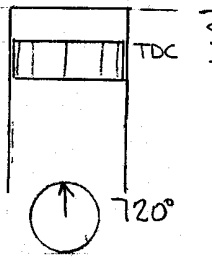


STROKE #3 Expansion of combustion products (power stroke)



EXHAUST Expulsion of combustion products (heat rejection at constant volume - piston in 540° position)

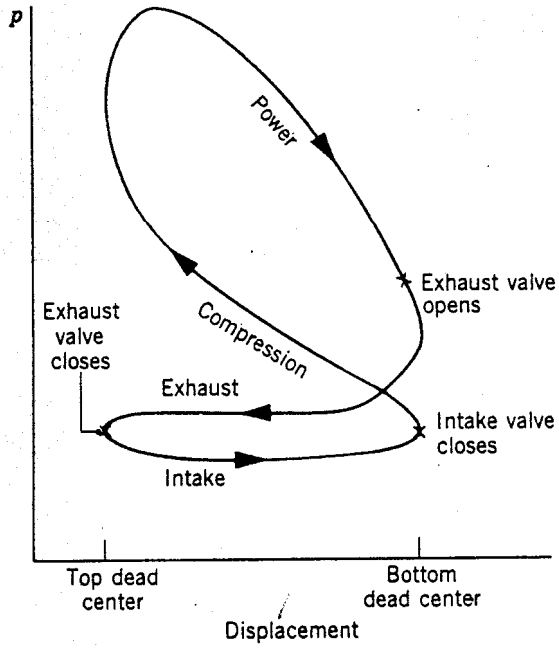




STROKE #4 Expulsion of remaining combustion products in cylinder

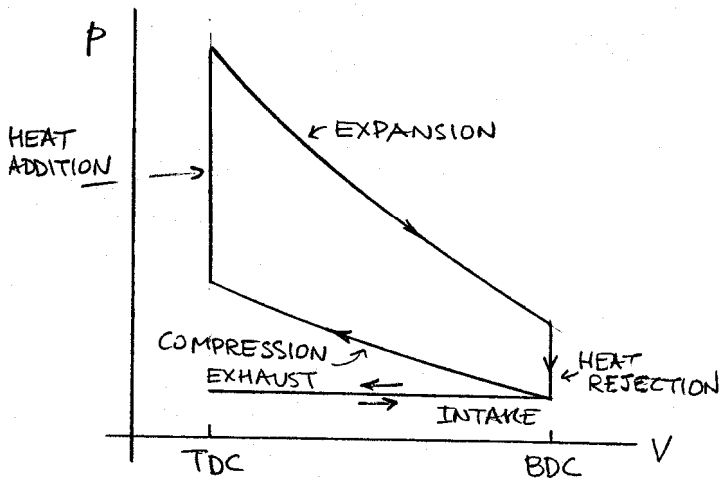
Return to original position in cycle. Completion of two rotations of the crank and 4 strokes of the piston

If we were to connect a pressure transducer (pressure gage) to the cylinder we would obtain the following pressure vs. position of piston diagram. (indicator diagram)



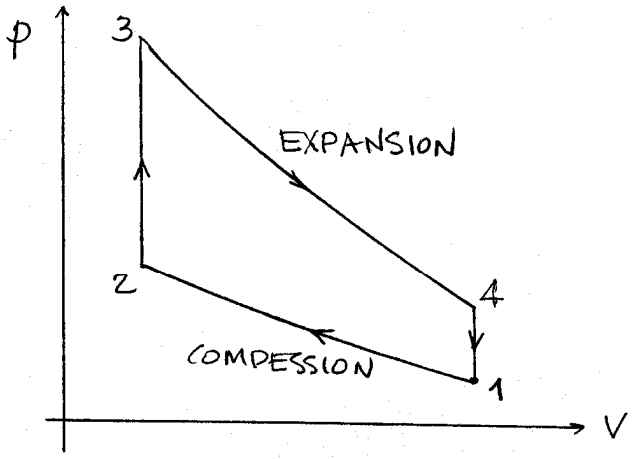
not to scale

We represent the above pressure displacement diagram by a simpler diagram for purposes of analysis



We further simplify the cycle representation by eliminating the exhaust and intake from the diagram.

Our final pressure – displacement (volume) diagram consists of four processes.



Note: The compression and expansion processes occur so rapidly that there is insufficient time for significant loss of heat due to combustion. Hence these process are said to occur ADIABATICALLY

Adiabatic means no heat transfer to the surroundings. (This does not mean constant temperature)

The four processes are:

1-2 Adiabatic compression, $Q_{12} = 0$

2-3 Heat addition at constant volume, $V_2 = V_3$

3-4 Adiabatic expansion, $Q_{34} = 0$

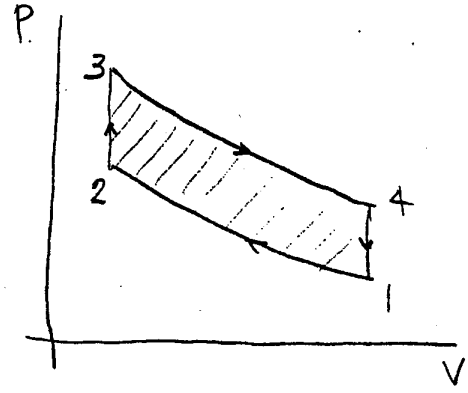
4-1 Heat rejection at constant volume, $V_4 = V_1$

NOTE: Engine speed of 2,000 RPM $\rightarrow \frac{\text{min}}{2000 \text{ Rev}} \frac{60 \text{ sec}}{\text{min}} = \frac{3 \text{ sec}}{100 \text{ Rev}} = 0.03 \text{ sec/rev} = 30 \frac{\text{ms}}{\text{Rev}}$

$= 15 \text{ ms/stroke.}$

We shall now analyse the cycle, one process at a time

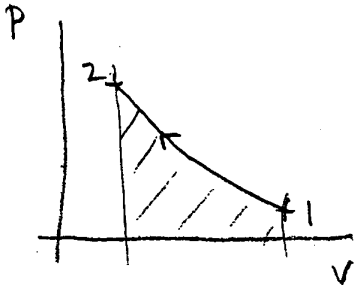
OTTO cycle.



The Otto cycle consists of four processes.

1-2 : compression
2-3 : heat addition
3-4 : expansion
4-1 : heat rejection

Process 1-2. Compression. Compression occurs ADIABATICALLY



ADIABATIC process → $Q_{12} = 0$ no heat transfer between the system and its surroundings

1st Law

$$\Delta U = \cancel{Q_{12}}^{0} - {}_1W_2 = -W_{12}$$

$$W_{12} = \int_1^2 p\,dV$$

Note that at this point we can not integrate for work since we do not know the relationship between p and V for an adiabatic process.

However we can evaluate work from the change in internal energy.

$$-W_{12} = \Delta_{12} U$$

Recall from the definition of specific heat at constant volume

$$c_v = \frac{du}{dT} \rightarrow du = m c_v dT,$$

Hence $$W_{12} = -\Delta U = -\int_1^2 m c_v dT = -m c_v \int_1^2 dT = -m c_v (T_2 - T_1)$$

$$W_{12} = -m c_v (T_2 - T_1)$$

↑ NOTE: We assumed c_v = constant
COLD AIR-STANDARD ANALYSIS

NOTE:

From $_1\Delta_2 U = -W_{12}$ we obtain

$$m\int_1^2 c_v dT = -\int_1^2 p\,dV = -m\int_1^2 p\,dv$$

or $\boxed{c_v dT = -p\,dv}$ Adiabatic

substitute for p from the ideal gas equation of state

$pv = RT$ (R is the particular gas constant)

$$p = \frac{RT}{v}$$

$$c_v dT = -RT\frac{dv}{v}$$

$$\int_1^2 \frac{dT}{T} = -\int_1^2 \frac{R}{c_v}\frac{dv}{v} = -\frac{R}{c_v}\int\frac{dv}{v}$$

$$\int_1^2 d\ln T = -\frac{R}{c_v}\int_1^2 d\ln v$$

$$\ln T_2 - \ln T_1 = -\frac{R}{c_v}(\ln v_2 - \ln v_1)$$

$$\ln\frac{T_2}{T_1} = -\frac{R}{c_v}\ln\frac{v_2}{v_1} = \ln\left(\frac{v_2}{v_1}\right)^{-R/c_v}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^{-R/c_v}}$$

Substitute for T from the ideal gas equation of state

$T = \frac{pv}{R}$

$$\frac{p_2 v_2}{R}\frac{R}{p_1 v_1} = \left(\frac{v_2}{v_1}\right)^{-R/c_v}$$

$$\frac{p_2}{p_1} = \left(\frac{v_2}{v_1}\right)^{-R/c_v}\frac{v_1}{v_2} = \left(\frac{v_1}{v_2}\right)^{R/c_v}\left(\frac{v_1}{v_2}\right)^{1}$$

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{1+\frac{R}{c_v}} = \left(\frac{v_1}{v_2}\right)^{\frac{c_v+R}{c_v}}$$

Note the relationship between c_p and c_v.

From the definition of enthalpy: $h = u + pv$
and the specific heats at constant pressure and volume:

$$c_p = \frac{dh}{dT} \quad \text{and} \quad c_v = \frac{du}{dT}$$

For an ideal gas $pv = RT$, hence

$$h = u + RT$$

Take the differential of the above equation

$$dh = du + R\,dT$$

substitute for dh and du:

$$c_p\,dT = c_v\,dT + R\,dT$$

Hence:

$$\boxed{c_p = c_v + R}$$

We can thus write for an ADIABATIC and REVERSIBLE process:

$$\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^{-\frac{R}{c_v}} \qquad \text{where } -\frac{R}{c_v} = -\frac{c_p - c_v}{c_v} = -(k-1) \text{ where } k = \frac{c_p}{c_v}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^{1-k}}$$

Similarly $\boxed{\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{k}}$ and $\boxed{\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}}$

Note: $p_2 v_2^k = p_1 v_1^k$ implies $pv^k = \text{constant}_1$

similarly $Tv^{k-1} = \text{constant}_2$

and $Tp^{(1-k)/k} = \text{constant}_3$

Now that we have demonstrated that

$$pv^k = \text{constant}$$

for an ADIABATIC process we can evaluate work directly from

$$W_{12} = \int_1^2 p\,dv$$

let $pV^k = \text{constant} = p_1V_1^k = p_2V_2^k$

or $$p = \frac{p_1V_1^k}{V^k}$$

$$W_{12} = \int_1^2 p_1V_1^k \frac{dV}{V^k} = p_1V_1^k \int_1^2 \frac{dV}{V^k} = p_1V_1^k \int_1^2 V^{-k}\,dV$$

$$W_{12} = p_1V_1^k \left[\frac{1}{-k+1} V^{-k+1}\right]_1^2 = \frac{p_1V_1^k}{1-k}\left[V_2^{1-k} - V_1^{1-k}\right]$$

$$= \frac{p_1V_1^kV_2^{1-k} - p_1V_1^kV_1^{1-k}}{1-k} = \frac{p_2V_2^kV_2^{1-k} - p_1V_1^kV_1^{1-k}}{1-k}$$

$$\underline{W_{12} = \frac{p_2V_2 - p_1V_1}{1-k}} = \frac{m(p_2v_2 - p_1v_1)}{1-k}$$

Note since $pV = mRT$, we can also write

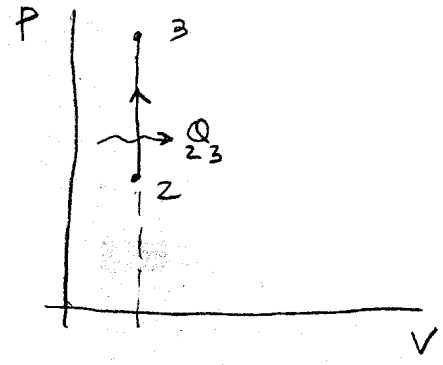
$$W_{12} = \frac{mR(T_2 - T_1)}{1-k}$$

Note also, $R = c_p - c_v$ and $k = c_p/c_v$, hence

$$\frac{R}{1-k} = \frac{c_p - c_v}{1 - c_p/c_v} = \frac{c_p - c_v}{\frac{1}{c_v}(c_v - c_p)} = -c_v$$

Now $\underline{W_{12} = -mc_v(T_2 - T_1)}$ which is the same result we obtained earlier from $W_{12} = -\int_1^2 du = -mc_v(T_2 - T_1)$

Process 2–3 Constant volume heat addition

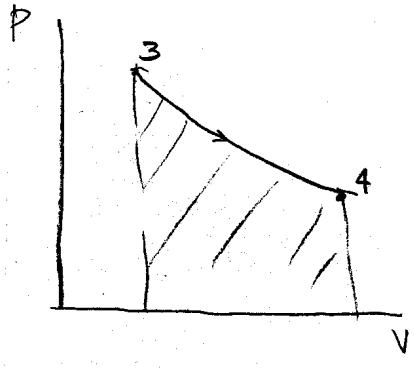


1st Law: $\Delta_{23}U = Q_{23} - W_{23}$

$W_{23} = \int_1^2 p\,dV = 0$ since $dV = 0$

$Q_{23} = \Delta_{23}U = \int_2^3 m c_v\,dT = m c_v (T_3 - T_2)$

Process 3–4 Expansion. Expansion occurs ADIABATICALLY (No heat exchange between the system and its surroundings)

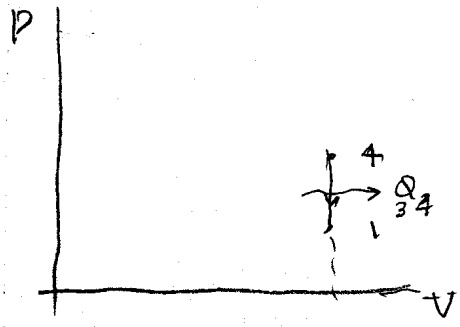


1st Law $\Delta_{34}U = \cancel{Q_{34}}^{\,0} - W_{34}$

$W_{34} = -\Delta_{34}U = -mc_v \int_3^4 dT$

$W_{34} = -mc_v (T_4 - T_3)$

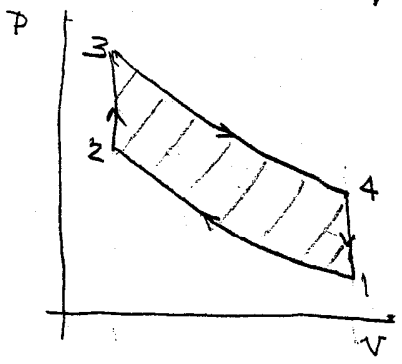
Process 4–1 Heat rejection at constant volume



1st Law $\Delta_{41}U = Q_{41} - \cancel{W_{41}}^{\,0}$

${}_4Q_1 = \Delta_{41}U = mc_v (T_1 - T_4)$

Summerize the cycle:



Net Work $= \sum W = W_{12} + W_{23} + W_{34} + W_{41}$ (W_{23} and W_{41} $\to 0$)

$$W_{net} = W_{12} + W_{34}$$
$$= -mc_v(T_2 - T_1) - mc_v(T_4 - T_3)$$
$$W_{net} = mc_v(T_1 - T_2 + T_3 - T_4)$$

Heat in put $Q_{23} = mc_v(T_3 - T_2)$

Thermal efficiency

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{mc_v(T_1 - T_2 + T_3 - T_4)}{mc_v(T_3 - T_2)} = \frac{(T_3 - T_2) - (T_4 - T_1)}{(T_3 - T_2)}$$

$$\boxed{\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}}$$ ← OTTO CYCLE

We can also write

$$\eta_{th} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

Now we know $pv^k = \text{constant}$

$p_1 v_1^k = p_2 v_2^k$ and $p_3 v_3^k = p_4 v_4^k$

Also $v_2 = v_3$ and $v_1 = v_4$

Hence $p_1 v_1^k = p_2 v_3^k$ and $p_3 v_3^k = p_4 v_1^k$

Divide one by the other

$$\frac{p_1 v_1^k}{p_4 v_1^k} = \frac{p_2 v_3^k}{p_3 v_3^k} \Rightarrow \boxed{\frac{p_1}{p_4} = \frac{p_2}{p_3}}$$ use $p = \frac{RT}{v}$ and substitute for p

$$\left(\frac{RT_1}{v_1}\right)\left(\frac{v_4}{RT_4}\right) = \left(\frac{RT_2}{v_2}\right)\left(\frac{v_3}{RT_3}\right)$$

$$\frac{T_1}{T_4} = \frac{T_2}{T_3} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$ ⇒ Return to η_{th} expression.

We now write

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_1\left(T_4/T_1 - 1\right)}{T_2\left(T_3/T_2 - 1\right)} = 1 - \frac{T_1\left(T_4/T_1 - 1\right)}{T_2\left(T_4/T_1 - 1\right)}$$

and

$$\eta_{th} = 1 - \frac{T_1}{T_2}$$

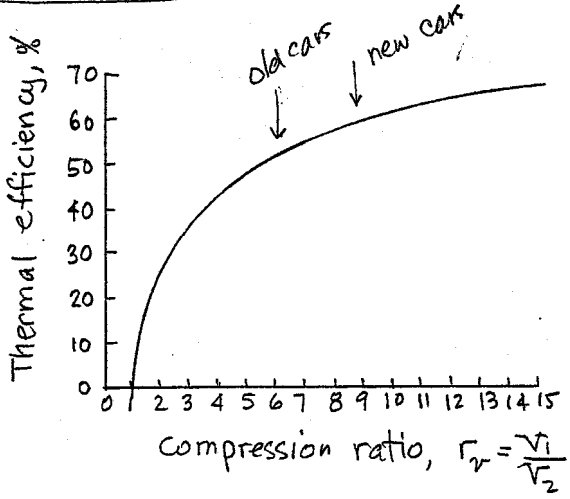
Note: $\frac{T_1}{T_2} = \frac{P_1 V_1}{P_2 V_2}\frac{P_2 V_2^{k}}{P_1 V_1^{k}} = \frac{V_1^{1-k}}{V_2^{1-k}} = \left(\frac{V_1}{V_2}\right)^{1-k}$

$$\eta_{th} = 1 - \left(\frac{V_1}{V_2}\right)^{1-k}$$

Note: $\frac{V_1}{V_2} \Rightarrow r_v \equiv$ compression ratio (volume)

$$\eta_{th} = 1 - (r_v)^{1-k}$$

cold air-standard (c_v = constant)



Note:
Upper limit on r_v due to tendancy for the fuel to detonate.
"Knock" → fuels developed with better antiknock characteristics

Deviations from cold air-standard analysis:

* $c_v = f(T)$
* incomplete combustion
* pressure drops across valves (more work required)
* heat transfer between gas and cylinder walls
* irreversibilities associated with temperature and pressure gradients.

A cycle can also be referred to in terms of the so-called "MEAN EFFECTIVE PRESSURE"

$$W_{net} = m\, p_{mep} (v_2 - v_1) \qquad \rightarrow 1 \rightarrow 2 \Rightarrow \text{compression}$$

$$p_{mep} = \frac{W_{net}}{m(v_2 - v_1)}$$