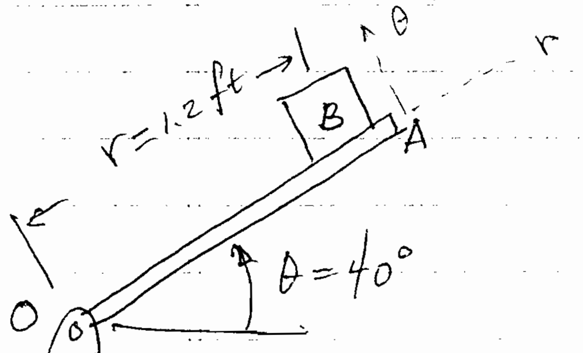
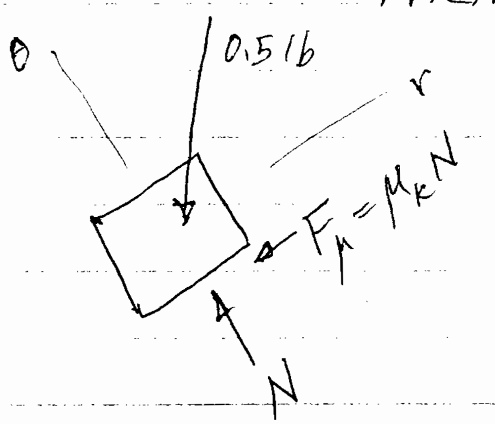
Ex. The 0.5 lb block B slides along the rotating bar OA. The coefficient of kinetic friction between B and OA is $\mu_K = 0.2$. In the position shown, $\dot{r} = 2$ ft/s, $\dot{\theta} = 5$ rad/s and $\ddot{\theta} = 3$ rad/s². For this position, determine $\ddot{r}$, the acceleration of B relative to bar OA.



Sol) Free body diagram

Note $\dot{r} = 2$ ft/s > 0 means the block B is moving away from O

$\Rightarrow$ Frictional force F_μ is opposite to $\dot{r}$



$$\Sigma F_\theta = m a_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$N - (0.5)\cos 40^\circ = \left(\frac{0.5}{32.2}\right)\left[(1.2)(3) + 2(2)(5)\right]$$

$$N = 0.7495 \text{ lb} \quad *$$

$$\Sigma F_r = m a_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$-0.5\sin 40^\circ - 0.2N = \left(\frac{0.5}{32.2}\right)\left[\ddot{r} - (1.2)(5)^2\right]$$

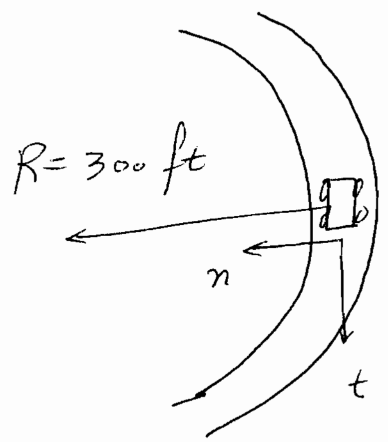
$$\ddot{r} = -0.351 \text{ ft/s}^2 \quad *$$

Ex.

Show that the exit velocity (max) can be estimated for a freeway off-ramp in circular motion. Assume the radius of the circle is 300 ft and the coefficient of static friction between tires and road surface is $\mu_s = 0.4$.

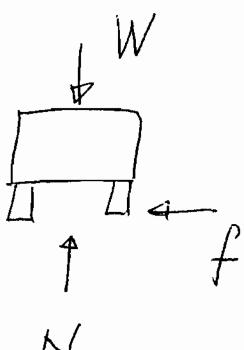
$$\Sigma F_n = m a_n$$

$$f = m a_n = m \frac{v^2}{R}$$



$$f_{max} \geq f = m \frac{v^2}{R}$$

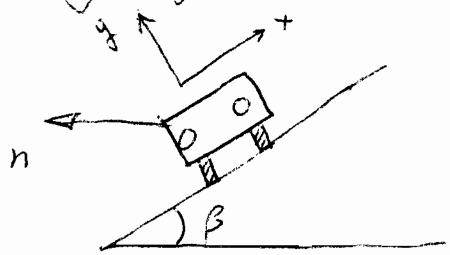
$$\mu_s N = \mu_s m g = m \frac{V_{max}^2}{R}$$

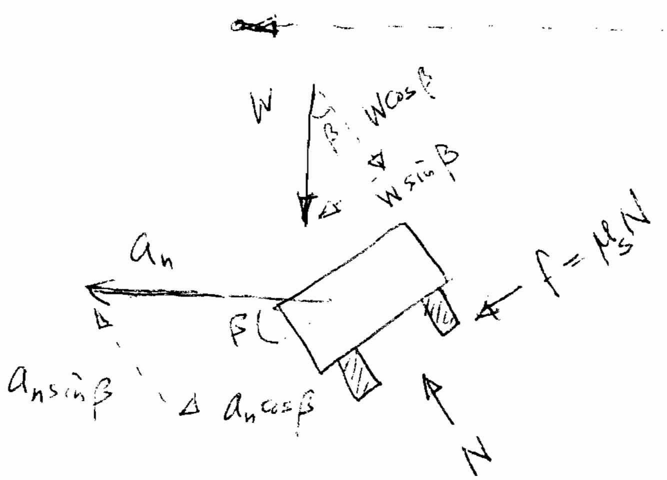


$$V_{max} = \sqrt{\mu_s g R}$$

$$= \sqrt{(0.4)(32.2)(300)} = 62.2 \text{ ft/s}$$

$\underline{\underline{Ex}}$ ~~Turning~~ Turning with a banking angle of β $\Rightarrow$ find v.





$$\begin{cases} \Sigma F_x = m a_x \\ \Sigma F_y = m a_y \end{cases}$$

$$\Sigma F_x = -W\sin\beta - \mu_s N = -m a_n \cos\beta \qquad -(1)$$

$$\Sigma F_y = -W\cos\beta + N = m a_n \sin\beta$$

$$N = m a_n \sin\beta + W\cos\beta$$

Substitute into (1)

$$\Rightarrow \quad W(\sin\beta + \mu_s \cos\beta) = m a_n (\cos\beta - \mu_s \sin\beta)$$

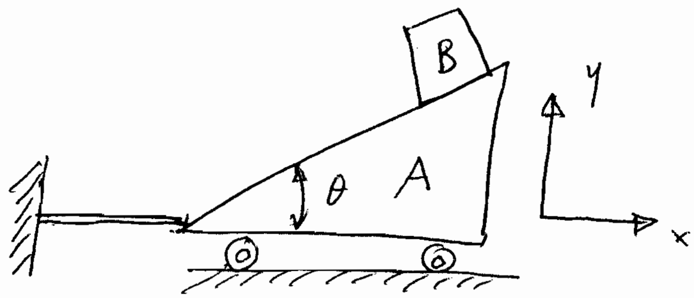
$$= m \frac{v^2}{R} (\cos\beta - \mu_s \sin\beta)$$

$$v = \sqrt{gR\left(\frac{\sin\beta + \mu_s \cos\beta}{\cos\beta - \mu_s \sin\beta}\right)} \qquad *$$

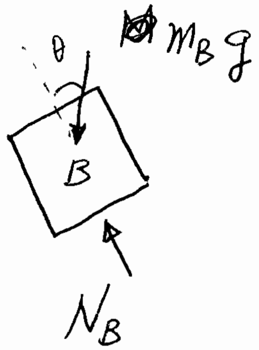
It can be shown that $(\ \ ) > \mu_s$ as long as $\frac{1}{\mu} > \tan\beta$.

since $\mu < 1$, $\Rightarrow$ This is advantageous for β as high as 45°

13-45 $m_A = 30\ kg$, $m_B = 10\ kg$, $\theta = 30^\circ$. Determine the tension in cord to hold the cart from sliding. (Neglect friction)



← logical selection of coordinate system

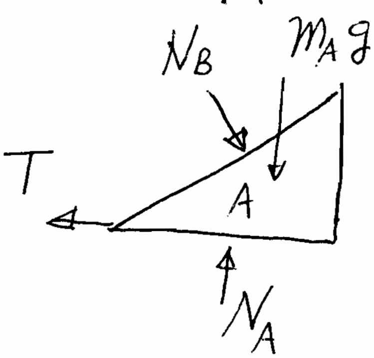


$$N_B - m_B g \cos\theta = 0$$

$$N_B = (10)(9.8)\cos(30^\circ) = 84.9\ (N).$$

since block A is the relevant object to consider (do not have to deal with components from T and N_A if the coordinate is so chosen)

Cart A:



$$\Sigma F_{A_x} = 0$$

$$\Rightarrow -T + N_B \sin\theta = 0$$

$$T = N_B \sin\theta = (m_B g \cos\theta)\sin\theta = \left(\frac{m_B g}{2}\right)\sin(2\theta) = 42.5\ (N).$$

13-45. (cont.)

$m_B = 10$ kg, $g = 9.81$ m/s^2, $m_A = 30$ kg

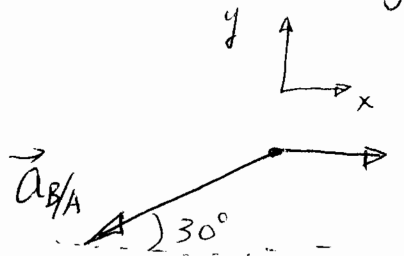
$\theta = 30°$

$$T = \left(\frac{m_B g}{2}\right)\sin(2\theta) = 42.5 \text{ (N)}$$

If the cable is cut $\Rightarrow$ determine the acceleration $\vec{a}_A$ and $\vec{a}_B$

Block A is free to move to the right

Block B is ~~sligh~~ sliding down the slope

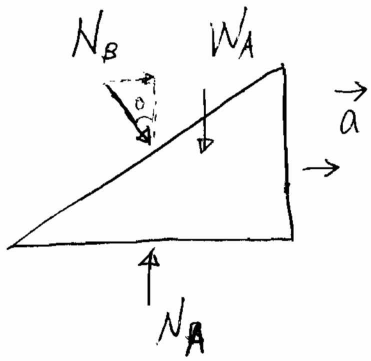


$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$= a_A\vec{i} + \left(-a_{B/A}\cos 30°\,\vec{i} - a_{B/A}\sin 30°\,\vec{j}\right)$$

$$= (a_A - 0.866\,a_{B/A})\vec{i} - a_{B/A}(0.5)\vec{j}$$

$\Sigma F_{A_x} = m_A a_{A_x}$ $\qquad$ $N_B \sin\theta = m_A a_A$ — (1)

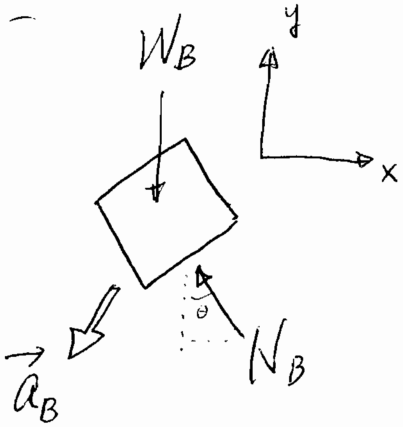


$\Sigma F_{A_y} = 0$, ~~$W_A + N_B\cos\theta = N_A$~~

$$N_A - (W_A + N_B\cos\theta) = 0$$

$$N_A = W_A + N_B\cos\theta = 294.3 + (0.866)N_B$$

$$\Sigma F_{B_x} = m_B a_{B_x} , \quad -N_B \sin\theta + 0 = m_B (a_A - 0.866\, a_{B/A})$$



from (1)

$$-m_A a_A = m_B a_A - m_B (0.866) a_{B/A}$$

$$a_{B/A} = \frac{m_A + m_B}{(0.866)\, m_B} a_A = 4.62\, a_A$$

$$\Sigma F_{B_y} = m_B a_{B_y}, \quad N_B \cos\theta - W_B = -m_B (0.5) a_{B/A}$$

$$(0.866) N_B - (10)(9.81) = -5\, a_{B/A} = -23.1\, a_A \quad -(2)$$

(1) (2) $\Rightarrow$

$$a_A = 1.31 \ (m/s^2)$$

$$a_{B/A} = 6.04 \ (m/s^2)$$

$$N_B = 78.6 \ (N)$$

$$\vec{a}_B = -3.92\,\vec{i} - 3.02\,\vec{j} \ (m/s^2)$$