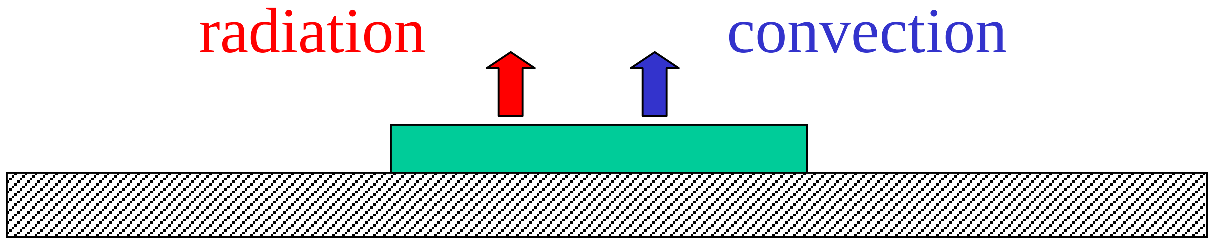
Example: Convection? Radiation? Or Both?

Heat transfer takes place between objects with different temperatures and all three modes of heat transfer exist simultaneously. However, there are many situations when one mode dominates over the others. Question: how do we know which modes to neglect in order to simplify the calculation? The following is an example to provide a general guideline to determine the relative importance between convection and radiation. By neglecting the less important one can usually lead to a simpler solution without significant error.

Example: An electronic chip can dissipation between 0.1 W to 200 W of power depending on its operating configuration. Determine the operating temperature of the chip under three different power settings: (a) 0.1 W, (b) 200 W, (c) 5 W. Assume the convection heat transfer coefficient (h) over the chip is 10 W/(m K), the ambient temperature surrounding the chip is 27°C, the surface emissivity (ε) of the chip is 0.3 and the chip has a surface area of 0.0004 m^2.



Question: what happens to conduction?

$$q = q_{conv} + q_{radiation} = hA(T_S - T_\infty) + e s A(T_S^4 - T_{surr}^4)$$

$$q = (10)(0.0004)(T_S - 300) + (0.3)(5.67\times 10^{-8})(0.0004)(T_S^4 - 300^4)$$

$$q = 0.004(T_S - 300) + (6.8\times 10^{-12})(T_S^4 - 300^4)$$

A fourth order equation that can not be solved easily without a programmable calculator.

Maybe we can neglect one of the modes to simplify the expression.

Question : which one?

Rule of thumb 1 : neglect the one that contributes the less.

Question : how can one tell?

Rule of thumb 2 : neglect convection when the temperature is low.

However, neglect radiation when the temperature is high.

Question: what do you mean by saying high or low temperature?

Unfortunately, there is no easy answer to this question. It will depend on your engineering judgement. Hope the following calculation can help to answer this question.

$$(a)\quad 0.1\quad W\quad Power:\quad q = 0.1 = 0.004(T_S - 300) + (6.8\times 10^{-12})(T_S^4 - 300^4)$$

It might be reasonable that this will give a low temperature (?)

neglect the radiation term

$0.1 = 0.004(T_S - 300),\quad T_S = 325\quad K,\quad$ not really that low but let's check.

Based on this temperature, the radiation contribution will be

$$\mathrm{q}_{\mathrm{radiation}} = (6.8\times 10^{-12})(T_S^4 - 300^4) = 0.021(W)$$

and it is small compared to 0.1 W so it is reasonable to neglect the radiation since it does not contribute significantly to the heat loss. However, we can still consider this contribution as shown in the following example.

$(a)\, 200W\ Power: q = 200 = 0.004(T_S - 300) + (6.8\times10^{-12})(T_S^4 - 300^4)$

It might be reasonable this time to assume a high temperature (?)

neglect the convection term

$200 = (6.8\times10^{-12})(T_S^4 - 300^4),\ T_S = 2329K,$ very high indeed.

Based on this temperature, the convection contribution will be

$\mathrm{q}_{\mathrm{conv}} = 0.004(T_S - 300) = 8.1(W)$

and it is also small as compared to the radiation term.

What is the radiation term? We do not know since the calculated temperature is not exact. However, can we estimate it?

$\mathrm{q} = 200 = \mathrm{q}_{\mathrm{conv}} + q_{rad} = 8.1 + q_{rad}$

By neglecting convection, we ask radiation to do too much work and now we can relief some of its load and recalculate the new temperature.

$$\text{q} - \text{q}_{\text{conv}} = 200 - 8.1 = (6.8\times10^{-12})(T_S^4 - 300^4),$$
$$T_S = 2305K$$

The surface temperature does decrease as it should since convection also contributes now. Check convection

$$\text{q}_{\text{conv}} = 0.004(2305 - 300) = 8.02(W)$$

That should be good enough!!

However, what happens if I am a perfectionist and want to get a even better answer? No problem. Just substitute this number and recalculate the surface temperature until those numbers match!!

$(a)\, 5\,W\ Power:\ q = 5 = 0.004(T_S - 300) + (6.8\times10^{-12})(T_S^4 - 300^4)$

From our previous experience this might give us trouble since the temperature might be neither too high nor too low.

Do we have to keep both terms??

Not really. One should still dominate over the other.

Check : radiation only

$5 = (6.8\times10^{-12})(T_S^4 - 300^4),\ \ T_S = 930K$

convection only

$5 = 0.004(T_S - 300),\ T_S = 1550K$

Therefore, the radiation is probably the more dominant one. Why?

Neglect convection and determine T_S

$$5 = (6.8\times10^{-12})(T_S^4 - 300^4),\ \ (T_S)_1 = 930K$$

Determine the contribution by the convection if the temperature is at this level

$$(q_{conv})_1 = 0.004((T_S)_1 - 300) = 0.004(930 - 300) = 2.52$$

Take this into consideration and recalculate the temperature :

$$5 - (q_{conv})_1 = (6.8\times10^{-12})(T_S^4 - 300^4),\ \ (T_S)_2 = 782K$$

Repeat the procedure :

$$(q_{conv})_2 = 0.004((T_S)_2 - 300) = 0.004(782 - 300) = 1.93$$

$$5 - (q_{conv})_2 = (6.8\times10^{-12})(T_S^4 - 300^4),\ \ (T_S)_3 = 823K$$

$$(q_{conv})_3 = 0.004((T_S)_3 - 300) = 0.004(823 - 300) = 2.09$$

Repeat this correction process until it converges....

$$5\text{-}\ (\mathrm{q}_{\mathrm{conv}})_3 = (6.8\times10^{-12})(T_S^4 - 300^4),\ \ (T_S)_4 = 811K$$

$$(\mathrm{q}_{\mathrm{conv}})_4 = 0.004((T_S)_4 - 300) = 0.004(811 - 300) = 2.04$$

Should be good enough!!

Exact solution is 814K