Ideal Gas Model

• For many gases, the ideal gas assumption is valid and the P-v-T relationship can be simplified by using the ideal gas equation of state: $Pv=RT$ (Compressibility factor $Z=1$)

• For an ideal gas, it can be shown that the specific internal energy u is a function of temperature only, $u=u(T)$

• Accordingly, specific enthalpy is a function of temperature only, since $h(T)=u(T)+pv=u(T)+RT$

• Physically, a gas can be considered ideal only when the intermolecular forces are weak. This usually happens in low pressure and high temperature ranges.

$$c_v = \frac{du}{dT}, \; c_P = \frac{dh}{dT} = \frac{d(u+Pv)}{dT} = \frac{d(u+RT)}{dT} = c_v + R$$

$$c_P = c_v + R$$

Ideal Gas Model(cont.)

- Specific ratio $k=c_p/c_v$, $c_p=c_v+R$

$$c_P(T)=\frac{kR}{k-1},\ \ c_v(T)=\frac{R}{k-1}$$

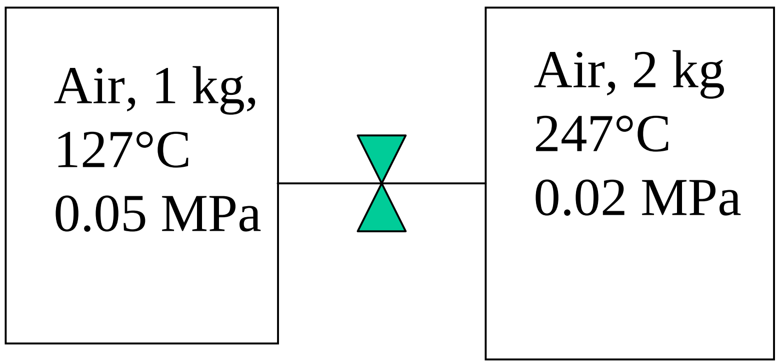
$$du=c_v(T)dT,\ \text{ integrate from 1 to 2}$$

$$\mathrm{u(T_2)}-u(T_1)=\int_1^2 c_v(T)dT,\ \ \text{similarly}$$

$$\mathrm{h(T_2)}-h(T_1)=\int_1^2 c_P(T)dT$$

- In some cases, the temperature dependence of the specific heat can be written in polynomial form, see Table A-2, p. 845
- Otherwise, ideal gas tables are also available. They are easier to use compared to the thermodynamic tables since temperature is the only parameter.

Example: Two tanks filled with air are connected by a valve as shown. If the valve is opened and the gases are allowed to mix while receiving heat from the surrounding. The final temperature is 227°C. Determine (a) the final pressure of the mixture, (b) the amount of heat transfer during the mixing process. Assume ideal gas model is valid.



Assume: little change of KE & PE
Both the initial and the final states are in equilibrium

From table A-1, T_c=132.5 K, P_c=3.77 Mpa
T_r=3, P_r=0.0133 for tank 1
From Figure A-13, compressibility factor Z=1, good ideal gas assumption

$$P_f = \frac{mRT_f}{V}, \text{ where } \mathrm{V} = \mathrm{V}_1 + \mathrm{V}_2 \text{ is the total volume of both tanks.}$$

$$\mathrm{m} = \mathrm{m} + \mathrm{m} \text{ is the total mass of both tanks.}$$

$$V_1 = \frac{m_1 R T_1}{P_1}, V_2 = \frac{m_2 R T_2}{P_2}$$

$$P_f = \frac{(m_1 + m_2) R T_f}{\frac{m_1 R T_1}{P_1} + \frac{m_2 R T_2}{P_2}} = \frac{(1+2)(273+227)}{\frac{(1)(400)}{0.05} + \frac{(2)(520)}{0.02}} = 0.025(MPa)$$

(b) The heat transfer can be found from the energy balance

$$\mathrm{E_f} - E_i = U_f - U_i = Q - W$$

$$Q = U_f - U_i = (m_1 + m_2) u(T_f) - \left[m_1 u(T_1) + m_2 u(T_2) \right]$$

$$Q = m_1[u(T_f) - u(T_1)] + m_2[u(T_f) - u(T_2)]$$

$$= m_1 C_{v,avg,f-1}(T_f - T_1) + m_2 C_{v,avg,f-2}(T_f - T_2) \text{ (see eq 3-46, p. 112)}$$

where $C_{v,avg,f-1} = 1/2(0.726 + 0.742) = 0.734(kJ/kgK)$

can be found using averaged C_v and table A-2 (p. 844)

$C_{v,avg,f-2} \approx 0.742$ (kJ/kg K) since $T_f \approx T_2$

$$Q = (1)(0.734)(500-400) + (2)(0.742)(500-520) = 43.72(\text{kJ})$$