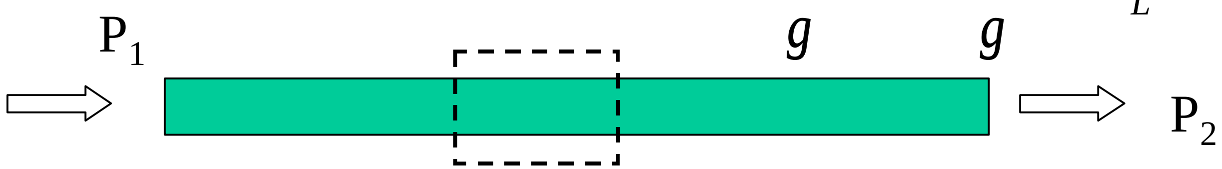
Frictional losses in piping system

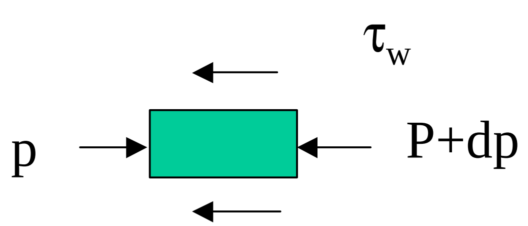
Extended Bernoulli's equation, $\frac{p_1}{g} + \frac{V_1^2}{2g} + z_1 + h_A - h_E - h_L = \frac{p_2}{g} + \frac{V_2^2}{2g} + z_2$

$$\frac{p_1 - p_2}{g} = \frac{\Delta p}{g} = h_L = \text{frictional head loss}$$



Consider a laminar, fully developed circular pipe flow

R: radius, D: diameter
L: pipe length
τ_w: wall shear stress



$$[p - (p + dp)](\pi R^2) = \tau_w (2\pi R)dx,$$

Pressure force balances frictional force

$$-dp = \frac{2\tau_w}{R} dx,$$ integrate from 1 to 2

Darcy's Equation: $\frac{\Delta p}{g} = \frac{p_1 - p_2}{g} = h_L = \frac{4\tau_w}{\rho g}\left(\frac{L}{D}\right) = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$

$$\tau_w = \left(\frac{f}{4}\right)\left(\frac{\rho V^2}{2}\right)$$

where f is defined as frictional factor characterizing pressure loss due to pipe wall shear stress

When the pipe flow is laminar, it can be shown (not here) that

$f = \frac{64m}{VDr}$, by recognizing that $\mathrm{Re} = \frac{rVD}{m}$, as Reynolds number

Therefore, $f = \frac{64}{\mathrm{Re}}$, frictional factor is a function of the Reynolds number

Similarly, for a turbulent flow, f = function of Reynolds number also

$f = F(\mathrm{Re})$. Another parameter that influences the friction is the surface

roughness as relativeto the pipe diameter $\frac{e}{\mathrm{D}}$.

Such that $f = F\left(\mathrm{Re}, \frac{e}{\mathrm{D}}\right)$: Pipe frictional factor is a function of pipe Reynolds

number and the relative roughness of pipe.

This relation is sketched in the Moody diagram as shown in the following page.

The diagram shows f as a function of the Reynolds number (Re), with a series of

parametric curves related to the relative roughness $\left(\frac{e}{\mathrm{D}}\right)$.