Supplementary homework 1 for graduate student (due Jan. 25, Tuesday)

This is to follow up on the homework problem I. For graduate students, you have to integrate the pressure relation in problem 2 so that you can come up with the pressure variation as a function of crank angle (from –180° to 180°). Using thermodynamic relation and ideal gas assumption, you should be able to determine the temperature and work profiles under the same operating condition. In order to do this, you will need to know how heat is releasing into the cylinder and we can use a finite heat release model to simulate the heat addition process in an engine power cycle. First, define the heat release $Q(\theta)=Q_{in}x_b(\theta)$, where Q_{in} is the amount of heat addition and the cumulative heat release fraction $x_b(\theta)$ is defined as

$$x_b(\theta) = 1 - \exp\left[-a\left(\frac{\theta - \theta_S}{\theta_d}\right)^n\right],$$

where θ=crank angle, θ_S is the start of heat release, θ_d is duration of heat release, n is Weibe form factor, and a is the Weibe efficiency factor.

The values of a =5 and n=3 have been used to provide satisfactory results as compared to experimental data. Use this model to integrate the pressure relation in problem 2. You might want to use the following suggestions for this problem:

- Use a fourth order Runge-Kutta integration
- Proceed the integration degree by degree from the bottom dead center (θ=180°) to the top dead center (θ=0°) and back.
- Once the pressure is determined, calculate the work by $\delta W = PdV$
- Calculate the temperature using the ideal gas law $T = \frac{PV}{mR}$

Use the following parameters for your calculation: single cylinder with a bore of 0.1 m, stroke of 0.1 m, and connecting rod length of 0.15 m, a compression ratio of 10. The initial temperature, cylinder pressure at the bottom dead center are 300K and 1 bar (10^5 Pa). The heat addition (Q_{in}) is 1800 J. Assume ideal gas with a specific heat ratio of 1.4, and the gas molecular weight is 29. The combustion duration θ_d=40°, the Weibe parameters are a =5 and n=3.

Compare the effect of start of heat release at θ_S=-20° and θ_S=0°.

(a) Determine the net work, the thermal efficiency of the cycle, and the mean effective pressure.

(b) Plot the pressure, temperature, and work profiles as a function of crank angle.