EML 4421 Homework #3 due Feb 1

1. Show that by using energy equation $c_p T_O = c_p T + \frac{U^2}{2}$, one can obtain the following relationships for an isentropic 1-D acceleration process:

$$\frac{T_O}{T} = \left[1 + \frac{\gamma - 1}{2} M^2\right], \frac{\rho_O}{\rho} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{1/(\gamma-1)}, \text{ and } \frac{P_O}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\gamma/(\gamma-1)}.$$

2. A pitot-static tube is usually being used to measure the air speed for an incompressible flow (see figure 6.4(b), p 250 in IFM book). If the compressibility effect is neglected, the following formula can be used to relate the velocity to the pressure difference between the stagnation and the local static pressures: $U = \sqrt{\frac{2(p_O - p)}{\rho}}$. Determine the error incurred in the estimation of the air speed by neglecting the compressibility effect as a function of the flow Mach number. Define the error as: $\varepsilon = \left| \frac{(U_{actual} - U_{incompressible})}{U_{actual}} \right|$. Plot ε as a function of the Mach number (for γ=1.4). Discuss your observation.

3. **For graduate students**: Derive an equation about the use of pitot tube to measure flow Mach number in supersonic flow, that is an equation relating the Mach number to stagnation and static pressures $p_{o2}/p_1 = f(\gamma, M_1)$. Discuss all relevant phenomena associated with this application and describe how can a pitot tube be used to determine the local Mach number. To be consistent, use the following parameters for your derivation: p_{o1}: stagnation pressure upstream of the shock, p_{o2}: stagnation pressure downstream of the shock, p_1: static pressure upstream of the shock, M_1: Mach number upstream of the shock, γ: specific heat ratio.