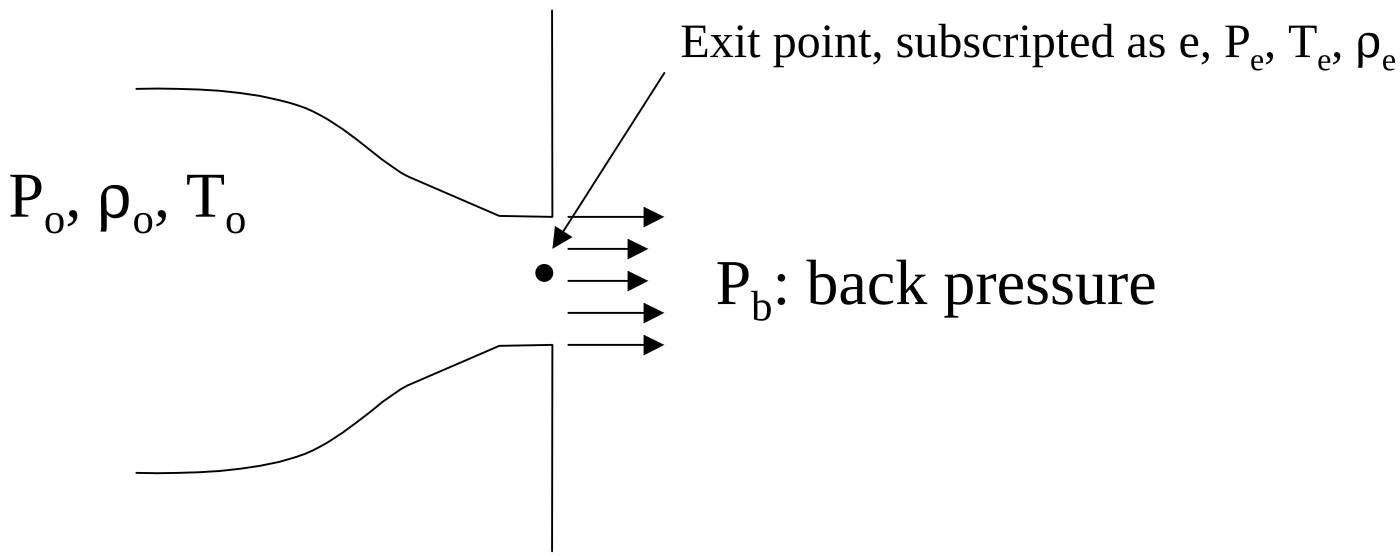
Choked Flow

Examine a convergent nozzle as show below. As the back pressure decreases, the mass flow exhausting out of the nozzle will increase until the nozzle reaches its choked condition. That is when the local flow speed reaches the local speed of the sound. Under this condition, no information can be transmitted upstream through the nozzle and any further increase of the back pressure produces no increase of the mass flow and the flow is said to be "choked".



Effect of Back Pressure

From the isentropic expansion relation: $\frac{P_O}{P} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\gamma/(\gamma-1)}$

If we decrease the back pressure, it can be seen that the Mach number will increase accordingly since there is higher differential pressure to drive the fluid particles. This situation will continue until the exit velocity reaches the local speed of sound, that M_e =1. It can be easily shown that the back pressure can be expressed as: $P_b = p^* = p_O \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$. Further decrease of the back pressure will produce no increase of mass flow and the nozzle is "choked". Note: one can still increase the mass flow rate if the stagnation pressure inside the channel is increased as will be seen later.

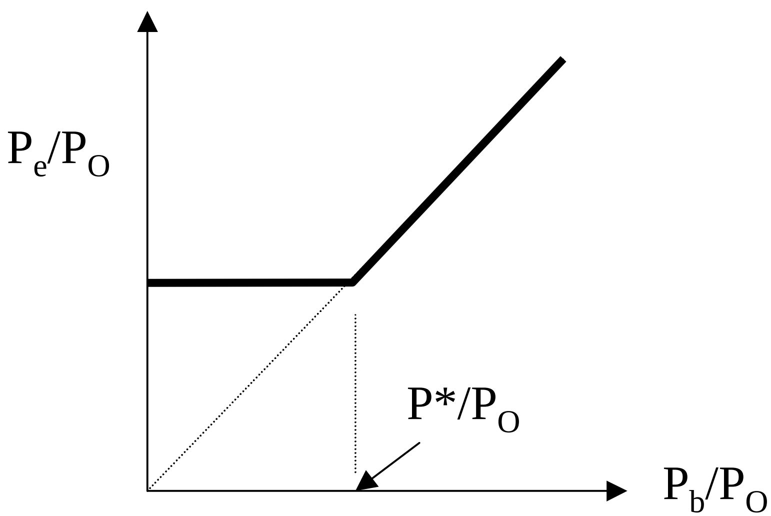
Effect of Back Pressure

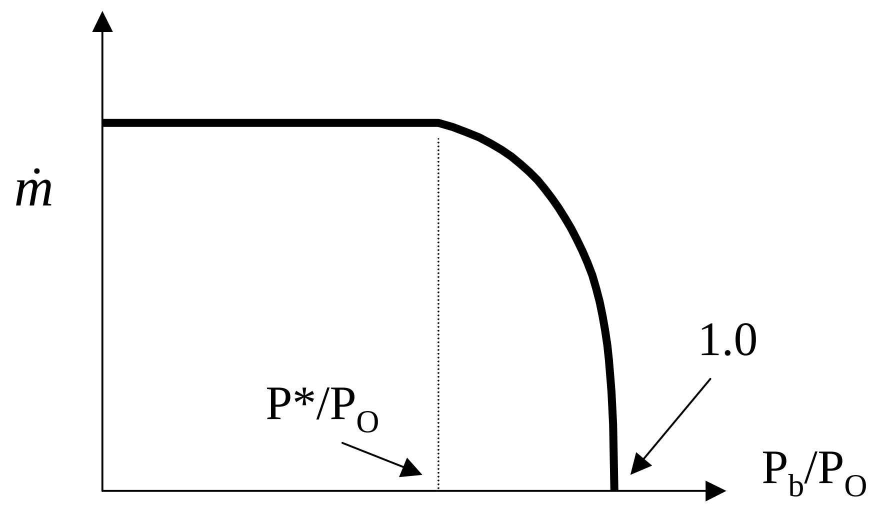
Two conditions exit: when $p_b > p^*$, not choked $\Rightarrow p_e = p_b$

$$\dot{m} = \rho_o A_e \left(\frac{P_b}{P_O}\right)^{1/\gamma} \left\{ \left(\frac{2\gamma}{\gamma - 1}\right)\left(\frac{P_O}{\rho_O}\right)\left[1 - \left(\frac{P_b}{P_O}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2}$$

when $p_b \le p^*$, choked $\Rightarrow \dfrac{P_e}{P_O} = \left(\dfrac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$, P_e is a constant

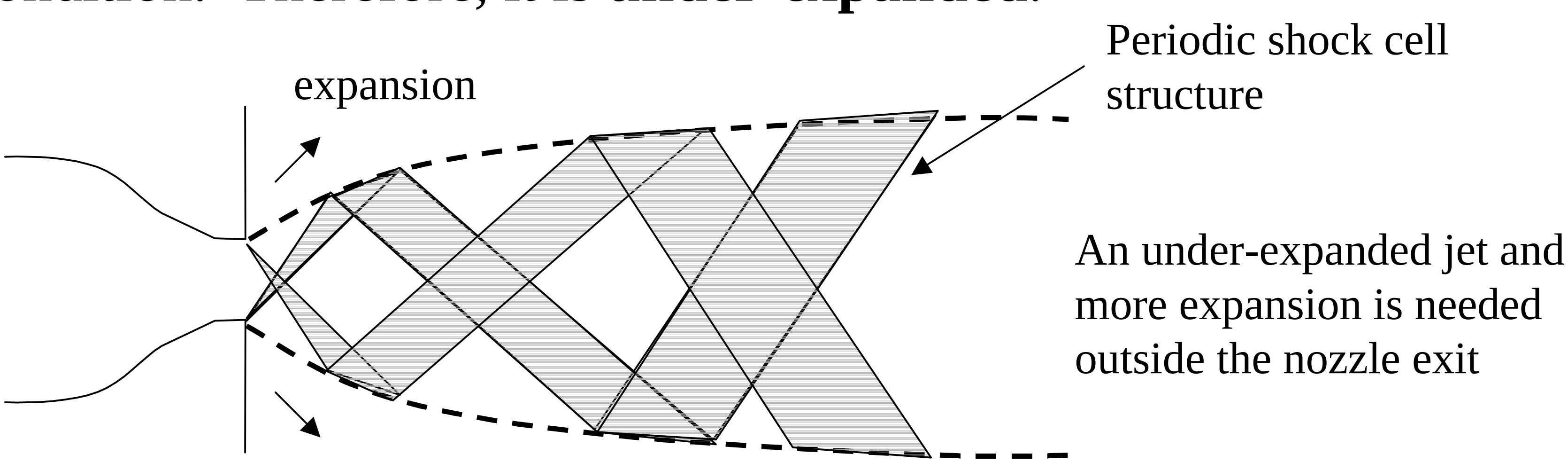
$$\dot{m} = \sqrt{\gamma \rho_O P_o} A_e \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}, \text{ a constant}$$





Under-Expanded Nozzle

As stated before, the exit fluid particles can not sense the flow condition outside the nozzle if the exit speed exceeds the local speed of sound. However, they will be SURPRISED once coming out of the nozzle since they will suddenly experience a lower than expected pressure. This condition will cause the flow to expand immediately outside the nozzle. This kind of the nozzle flow is called an under-expanded jet since it should have expanded more after the throat to reach supersonic speed. However, it did not get the chance due to the choked condition. Therefore, it is **under-expanded**.



An under-expanding jet visualized by shadowgraph

Periodic shock structure seen trailing behind the launching missiles

Ideally-Expanded Nozzle

A supersonic nozzle is said to be an ideally-expanded if its nozzle contour is designed so that the exit flow is expanding isentropically (without the presence of shock or expansion waves). Therefore, the ideally-expanded supersonic jet can only be generated using a convergent-divergent (CD) nozzle (why?). The contour for such a nozzle can easily be designed using the isentropic relation (see p621, chapter 12.3 IFM):

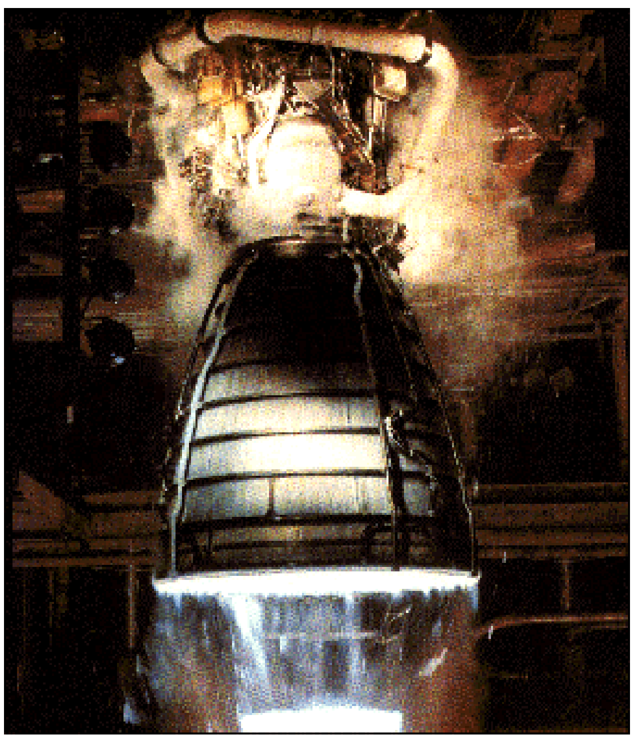
Using mass conservation $\rho AU = \rho^* A^* U^*$ and critical throat conditions derived earlier, we can determine

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}}\right]^{(\gamma+1)/2(\gamma-1)} ,$$

where A^* is the throat area and M is the local Mach number.

Ideally-Expanded C-D Nozzle

A table of the isentropic flow functions has been included in the Appendix E of IFM (p. 724). It can be seen that in order to produce a Mach two jet (M=2), the nozzle exit area should be 68.8% greater than the throat area (since $A/A^*=1.688$). For higher Mach number, this ratio can be much greater (for example, $A/A^*=25$ for M=5).

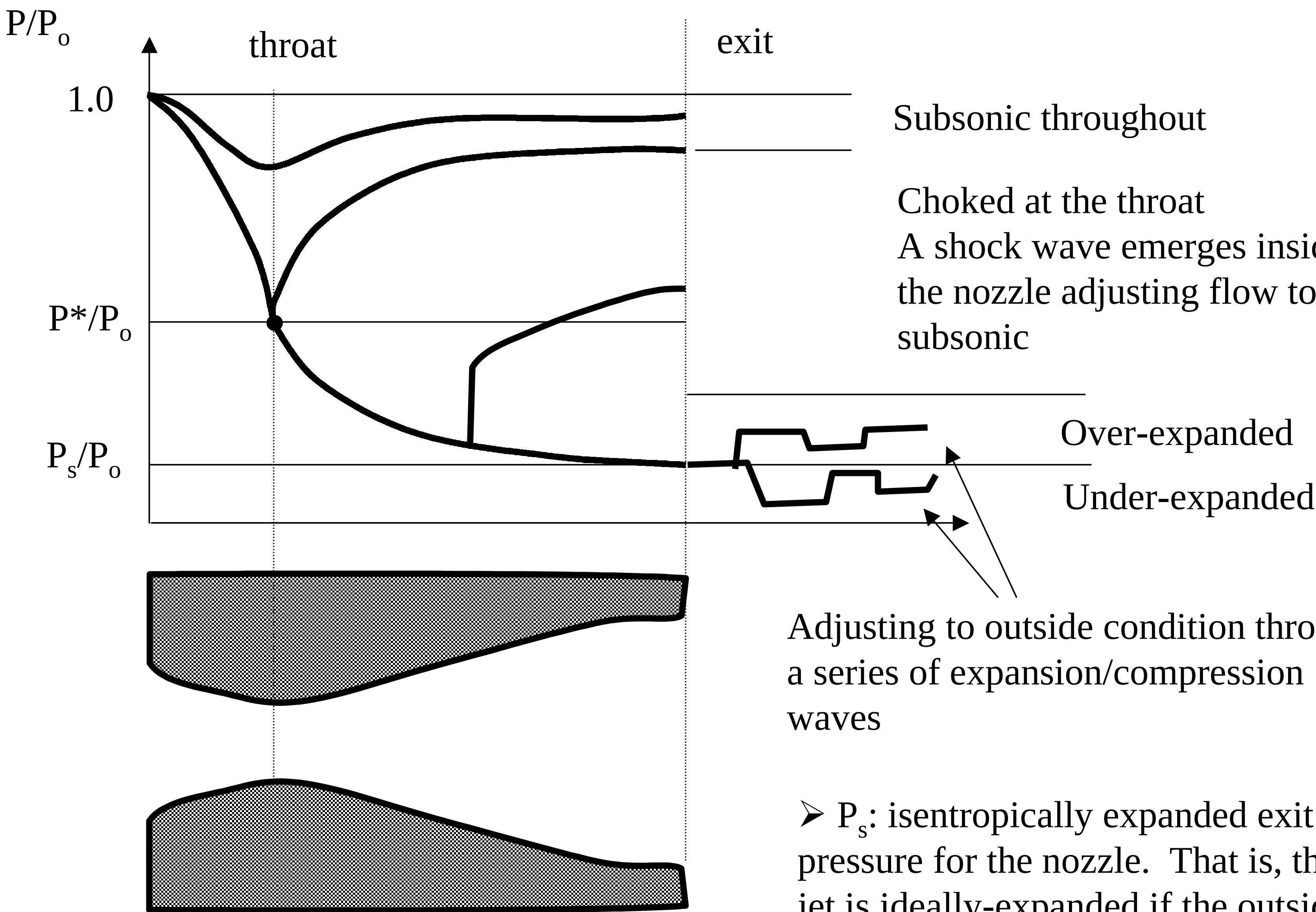


Show to the left is one of the shuttle's main engines and nozzle. Note the larger size of the nozzle as relative to the throat.

Under- and Over-expanded Supersonic C-D Nozzle

If the operating pressure ratio between the back pressure and the stagnation pressure does not match the ideally-expanded relationship. There is an immediate mismatch between the pressure when the gas molecules exhausting out from the nozzle since no upstream communication is possible for a supersonic flow. This type of operating condition is called non-ideally-expanded nozzle. It has two possibilities: (1) under-expanded situation when the back pressure is lower than the ideally expanded pressure and more expansion is needed in order to match this low pressure (as described before). (2) over-expanded when the back pressure is higher than the ideally expanded pressure and a compression is needed to increase the jet exhausting pressure to match the outside condition. Shocks/compression waves are required to achieve this.

Pressure distribution in a C-D nozzle



Subsonic throughout

Choked at the throat
A shock wave emerges inside the nozzle adjusting flow to subsonic

Adjusting to outside condition through a series of expansion/compression waves

- P_s: isentropically expanded exit pressure for the nozzle. That is, the jet is ideally-expanded if the outside pressure equals to P_s