Convective Heat Transfer

- Accurate prediction of the convective heat transfer usually involves the understanding of very complicated flow and heat transfer phenomena such as turbulent flow, boundary layer behavior, transient heat transfer, among others. In general, no analytical solution can be found in such problems such that numerical or empirical methods are usually required.

- The heat convection can be modeled using Newton's cooling law: $q=hA(T_S-T_\infty)$. Where h is the convective coefficient and it can be related to a dimensionless parameter, Nu, the Nusselt number as: $Nu=hD/k_f$, where D is a characteristic dimension and k_f is the thermal conductivity of the fluid.

Convective Heat Transfer

- Based on dimensional analysis and empirical relation, the Nusselt number can be found as a function of other dimensionless parameters: Nu=f(Re, Pr).

$\mathrm{Re} = \dfrac{\rho U D}{\mu}$, where ρ is the fluid density, μ is the dynamic viscosity, and it characterizes the flow behaviors (such as laminar-turbulent transition, convection speed, etc.)

$\mathrm{Pr} = \dfrac{\nu}{\alpha}$, where ν is the kinematic viscosity and α is the thermal diffusivity, and it measures the relative importance between the momentum diffusion (viscous effect) and the thermal diffusion (heat conduction) processes.

Convective Heat Transfer

In general, the empirical relation is in the following form:

$Nu = f(\mathrm{Re},\ \mathrm{Pr}) = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$, where C, m, and n are all empirical constants dictated by the operating configurations.

Stanton number, St, is sometimes used as a modified Nusselt number:

$$\mathrm{St} = \frac{\mathrm{h}}{\rho \mathrm{U} \mathrm{c}_\mathrm{P}} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}}$$

For a boundary layer flow on a flat plate (no pressure gradient, Pr=1), Stantaon number can be related to the skin friction coefficient $\mathrm{C_f}$ as:

$\frac{\mathrm{C_f}}{2} = St$ and this is called the Reynolds analogy, relating the thermal and velocity boundary layer (can be applied to flow with pressure gradient if the flow is fully turbulent).

This can be modified to include the variation of Pr for $\mathrm{Pr} \neq 1$ by

$\frac{\mathrm{C_f}}{2} = St\,\mathrm{Pr}^{2/3}$, and this is called Colburn analogy.

Engine Convection Correlation Model

It has been shown by Woschni* that the averaged convection coefficient for a spark ignition engine can be expressed as

$$\bar{h}(W/m^2 \cdot K) = 3.26\, b(m)^{-0.2}\, p(kPa)^{0.8}\, T(K)^{-0.55}\, U(m/s)^{0.8}$$

where b is the bore size, p is the cylinder pressure, T is the mean cylinder gas temperature, U is the averaged gas velocity inside the cylinder. Please note that the equation is an empirical one and therefore no real physical meaning. Proper units (as shown) for all terms have to be used to obtain a correct correlation. The convection coefficient so obtained is a spatially averaged value inside the cylinder.

* Woschni, G., SAE Paper 670931, SAE Transaction Vol. 76, 1967.

Engine Convection Correlation Model

The averaged gas velocity, U (m/s), can be expressed as follows:

$$U = C_1 \bar{U}_P + C_2 \frac{V_d T_O}{p_O V_O}(p - p_m)$$ **(without swirl)**

where C_1 and C_2 are two constants, $\bar{U}_P$ is the mean piston speed (m/s), V_d is the displaced volume (m^3). T_O, V_O, and p_O are temperature (K), cylinder volume (m^3), and pressure (kPa) at intake valve closing. p is the instantaneous cylinder pressure (kPa), and p_m is the motored cylinder pressure (kPa) without combustion.

For the gas exchange period: C_1=6.18, C_2=0

For the compression period: C_1=2.28, C_2=0

For the combustion and expansion periods: C_1=2.28, C_2=0.00324

Engine Convection Correlation Model

- It has been clearly seen that, during gas exchange and and compression periods, the gas speed is proportional to the mean piston speed and $C_2=0$. During combustion and expansion, gas velocities can be changed drastically by the results from combustion and a term due to combustion ($p-p_m$) is introduced $C_2>0$ and the gas speed is increased due to combustion process.
- For engines with **swirl**, the heat transfer is higher as a result of a higher gas speed. The previous constants can be modified as

For the gas exchnage period: $C_1=6.18+0.417\frac{v_S}{\bar{U}_P}$

For the rest of cycle: $C_1=2.28+0.308\frac{v_S}{\bar{U}_P}$

where $v_s=\frac{b\omega_P}{2}$ and ω_P is the rotation speed of the paddle wheel used to measure the swirl velocity.

Finite Heat Release Model (with heat loss)

The convection heat exchange between the engine inside wall and the cylinder gas can be modeled into the finite heat release model discussed earlier. Therefore, the pressure variation can be expressed as

$$\frac{\mathrm{dP}}{\mathrm{d}\theta} = \frac{\gamma - 1}{V} Q_{in} \frac{dx_b}{d\theta} - \gamma \frac{P}{\mathrm{V}} \frac{dV}{d\theta} - \left[\frac{dQ_{wall}}{d\theta} \right]$$

Since $\mathrm{q}_{\mathrm{wall}} = \frac{dQ_{wall}}{dt} = \frac{dQ_{wall}}{d\theta} \frac{d\theta}{dt} = h(\theta) A_w(\theta)[T_g(\theta) - T_w(\theta)]$

$$\frac{dQ_{wall}}{d\theta} = h(\theta) A_w(\theta)[T_g(\theta) - T_w(\theta)] / N$$

where $\mathrm{A_w}(\theta) = \mathrm{A_{wall}} + \mathrm{A_{head}} + \mathrm{A_{piston}} = \pi \mathrm{by} + 2\frac{\pi}{4} b^2 = \pi \mathrm{by} + \frac{\pi}{2} b^2$, y is the exposed cylinder wall at a particular instant.