

Exhaust Nozzle Example

- The exhaust gases ($R=0.288 \text{ kJ/(kg K)}$, $\gamma=1.32$) of a gas turbine are expanded through a converging nozzle from stagnation conditions of $P_o=200 \text{ kPa}$ and $T_o=1400\text{K}$ to ambient. The mass flow rate is 100 kg/s . Assume isentropic expansion inside the nozzle. (a) Determine the exit area. (b) What is the exit speed of the sonic jet? (c) Determine the nozzle exit pressure. (d) Determine the nozzle inlet area if the inlet Mach number is 0.5.

Note: review lecture notes on [choked flow](#) and [compressible flow](#)

$$(a) \quad \dot{m} = \sqrt{\gamma \rho_o P_o} A_e \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}}, \text{ where } \rho_o = \frac{P_o}{RT_o}$$

$$A_e = \frac{\dot{m}}{\sqrt{\gamma \rho_o P_o}} \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} = 0.316(m^2)$$

(b) The exit speed is the local speed of sound a^* and $M = 1$ (choked):

$$a^* = \sqrt{\gamma R T^*} = 677(m/s), \quad \frac{T^*}{T_o} = \left[1 + \frac{\gamma-1}{2} M^2 \right]^{-1} = \frac{2}{\gamma+1} = 0.862$$

Exhaust Nozzle Example (2)

(c) Nozzle exit pressure can also be determined by isentropic relation:

$$\frac{P_o}{P^*} = \left[1 + \frac{\gamma - 1}{2} M^2 \right]^{\gamma/(\gamma-1)} = \left[\frac{\gamma + 1}{2} \right]^{\gamma/(\gamma-1)}$$

$$P^* = 108.4 \text{ kPa}$$

(d) Nozzle inlet area A_{inlet}

$$\frac{A_{\text{inlet}}}{A^*} = \frac{1}{M} \left[\frac{1 + \frac{\gamma - 1}{2} M^2}{1 + \frac{\gamma - 1}{2}} \right]^{(\gamma+1)/2(\gamma-1)}, \text{ for } M=0.5$$

$$A_{\text{inlet}} = 0.427 \text{ m}^2$$