Momentum Conservation

The linear momentum conservation equation is derived based on the Newton's second law: The sum of all external forces actin on a system is equal to the time rate change of the linear momentum of the system.

$\vec{F} = (\frac{d\vec{P}}{dt})_{system}$, where the linear momentum of the system is given by

$$\vec{P} = \int_{system} \rho \vec{V} d\forall.$$

Using Reynolds transport theorem, it is obtained that

$$(\frac{d\vec{P}}{dt})_{system} = [\frac{\partial}{\partial t}\int_{CV} \rho \vec{V} d\forall] + [\int_{CS} \rho \vec{V}(\vec{V} \cdot d\vec{A})]$$

$= [$Time change of the linear momentum within the control volume$]$

$+ [$net rate of linear momentum flux in & out through the control surface$]$

Linear Momentum

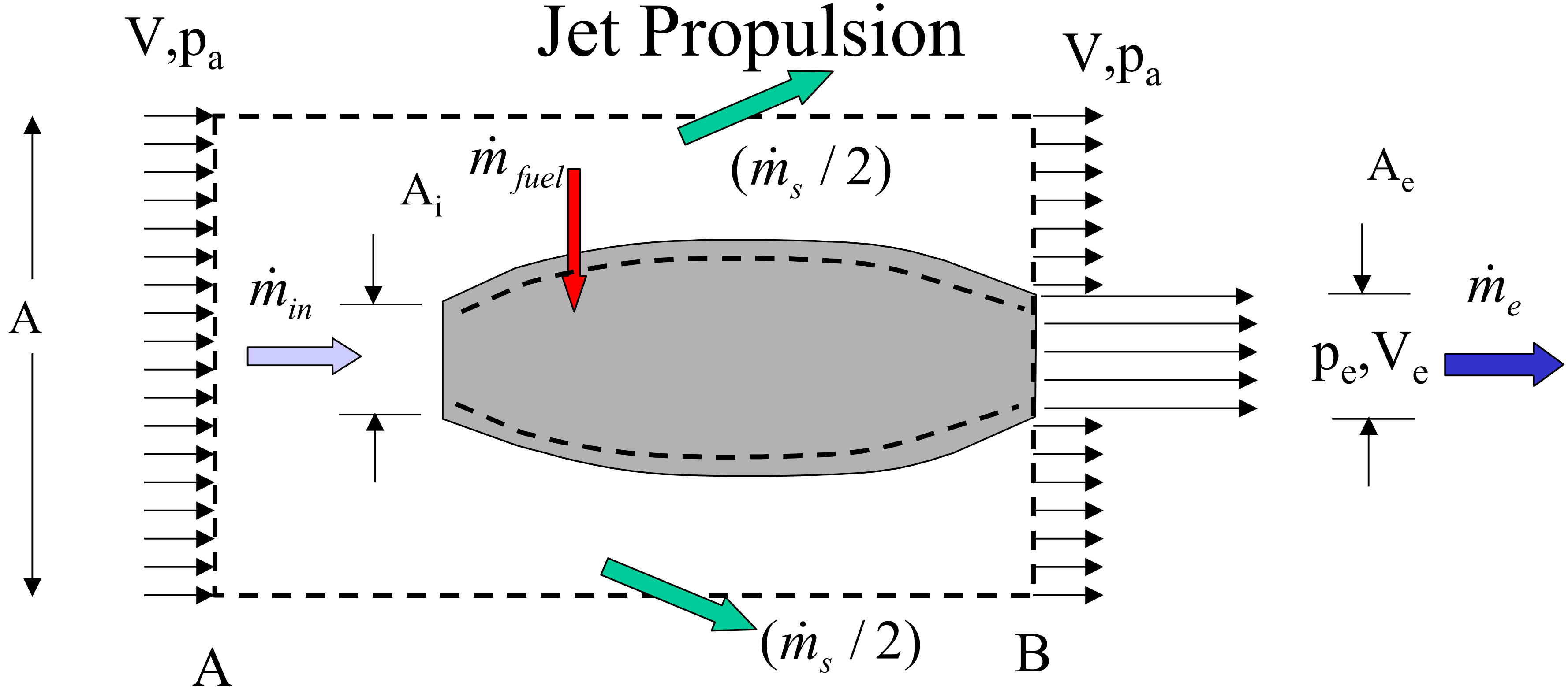
In general, there are two types of forces acting on the fluids: surface forces and body forces. Surface forces (F_s) are external forces acting on the surfaces between solids and the fluids or between different fluid layers. Examples: pressure forces, shear forces, surface tension, etc. Body forces (F_B)are acting on the entire body of the fluids, such as gravity and magnetohydrodynamic forces.

Therefore, the linear momentum equation can be written as

$$\vec{F} = \vec{F}_S + \vec{F}_B = [\frac{\partial}{\partial t}\int_{CV} \rho \vec{V} d\forall] + [\int_{\mathrm{CS}} \rho \vec{\mathrm{V}}(\vec{\mathrm{V}} \cdot \mathrm{d}\vec{\mathrm{A}})]$$

Note: These forces are forces acting on the fluids in order to produce the necessary changes of the linear momentum of the fluids. In order to evaluate the forces acting on the external devices, one needs to apply the Newton's third law: reaction force is equal in magnitude and opposite in direction with respect to the action force. That is, R = - F

Jet Propulsion



- Assume steady state flow, neglect body force and only x-momentum is considered
- Assume fuel is added vertically and does not contribute to the x-momentum
- Control volume is large compared to the engine

Propulsion Equation (1)

- Mass conservation: mass in + fuel in = mass out

$$\dot{m}_A + \dot{m}_{fuel} = \dot{m}_s + \dot{m}_B$$

$$\rho VA + \dot{m}_{fuel} = \dot{m}_s + \rho_e V_e A_e + \rho V\left(A - A_e\right)$$

$$\dot{m}_{fuel} = \dot{m}_s + A_e(\rho_e V_e - \rho V)$$

For the engine alone, $\dot{m}_a + \dot{m}_{fuel} = \dot{m}_e$, therefore,

$$\dot{m}_s = \rho V\left(A_e - A_i\right)$$

Calculate the x-momentum flux on all external control surfaces:

$$\int_{\mathrm{CS}} \rho V_x(\vec{\mathrm{V}} \cdot \mathrm{d}\vec{\mathrm{A}})$$

$$= [\text{surface B}] + (\text{surfaces above and below}) + \{\text{surface A}\}$$

$$= [\dot{m}_e V_e + \rho V\left(A - A_e\right)V] + (\dot{m}_s V) - \{\dot{m}_a V + \rho V\left(A - A_i\right)V\}$$

$$= \dot{m}_e V_e - \dot{m}_a V$$

Propulsion Equation (2)

The surface force terms can be obtained by integrating surface stresses on all control surfaces:

surface A+ surface B + engine surfaces

$$\sum F_{S,x} = p_a A - p_a (A - A_e) - p_e A_e + \mathrm{T}$$

where the thrust: $\mathrm{T} = \int_{engine\,surfaces} (\tau_x + p_x) dA$

is the integrated force over all engine surfaces, including inlet, compressor, combustion chambers, turbine, etc.

$\sum F_{S,x} = (p_a - p_e) A_e + \mathrm{T},$ where the exit pressure might be different from the inlet pressure because the exhaust flow can be supersonic (under- or over-expansion can occur).

Propulsion Equation (3)

Combining the previous relations, and the thrust can be written as:

$$\mathrm{T}=(p_e - p_a)A_e + \dot{m}_e V_e - \dot{m}_a V$$

$$= (p_e - p_a)A_e + \dot{m}_a[(1+f)V_e - V]$$

where $\dot{m}_e = \dot{m}_a + \dot{m}_f = (1+f)\dot{m}_a$, f is the fuel-air ratio

This equation is good for a turbojet engine where one stream passing through the entire engine. For a turbofan engine, there might be two streams (one going through the combustion chamber, specified as hot (H), while the other stream bypass the chamber, specified as cold (C).):

$$\mathrm{T}_{two\ streams}=(\dot{m}_{\mathrm{a,H}} + \dot{m}_f)V_{e,H} - \dot{m}_{\mathrm{a,H}}V + \dot{m}_{\mathrm{a,C}}(V_{e,C} - V) + (p_{e,H} - p_a)A_{e,H}$$

$$+(p_{e,C} - p_a)A_{e,C}$$