Exhaust Nozzle Example

- The exhaust gases (R=0.288 kJ/(kg K), γ=1.32) of a gas turbine are expanded through a converging nozzle from stagnation conditions of P_o=200 kPa and T_o=1400K to ambient. The mass flow rate is 100 kg/s. Assume isentropic expansion inside the nozzle. (a) Determine the exit area. (b) What is the exit speed of the sonic jet? (c) Determine the nozzle exit pressure. (d) Determine the nozzle inlet pressure if the inlet Mach number is 0.5.

Note: review lecture notes on choked flow and compressible flow

$$(a)\quad \dot{m} = \sqrt{\gamma \rho_O P_O}\, A_e \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}, \text{ where } \rho_O = \frac{P_O}{RT_O}$$

$$A_e = \frac{\dot{m}}{\sqrt{\gamma \rho_O P_O}} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 0.316(m^2)$$

(b) The exit speed is the local speed of sound a^* and $M = 1$ (choked):

$$a^* = \sqrt{\gamma R T^*} = 677(m/s), \quad \frac{T^*}{T_O} = \left[1 + \frac{\gamma-1}{2} M^2\right]^{-1} = \frac{2}{\gamma+1} = 0.862$$

Exhaust Nozzle Example (2)

(c) Nozzle exit pressure can also be determined by isentropic relation:

$$\frac{P_O}{P^*} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\gamma/(\gamma-1)} = \left[\frac{\gamma + 1}{2}\right]^{\gamma/(\gamma-1)}$$

$$P^* = 108.4\, kPa$$

(d) Nozzle area A_e

$$\frac{A_e}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{\gamma - 1}{2} M^2}{1 + \frac{\gamma - 1}{2}}\right]^{(\gamma+1)/2(\gamma-1)} \quad \text{, for M=0.5}$$

$$A_e = 0.427\, m^2$$