Moment of Momentum

Linear momentum equation:

$$\sum \vec{F} = \frac{\partial}{\partial t}\int \rho \vec{V} d\forall + \int \rho \vec{V}(\vec{V} \bullet d\vec{A}) = \int \rho \vec{V}(\vec{V} \bullet d\vec{A}) \text{ (for a steady state flow)}$$

Momen of momentum equation:

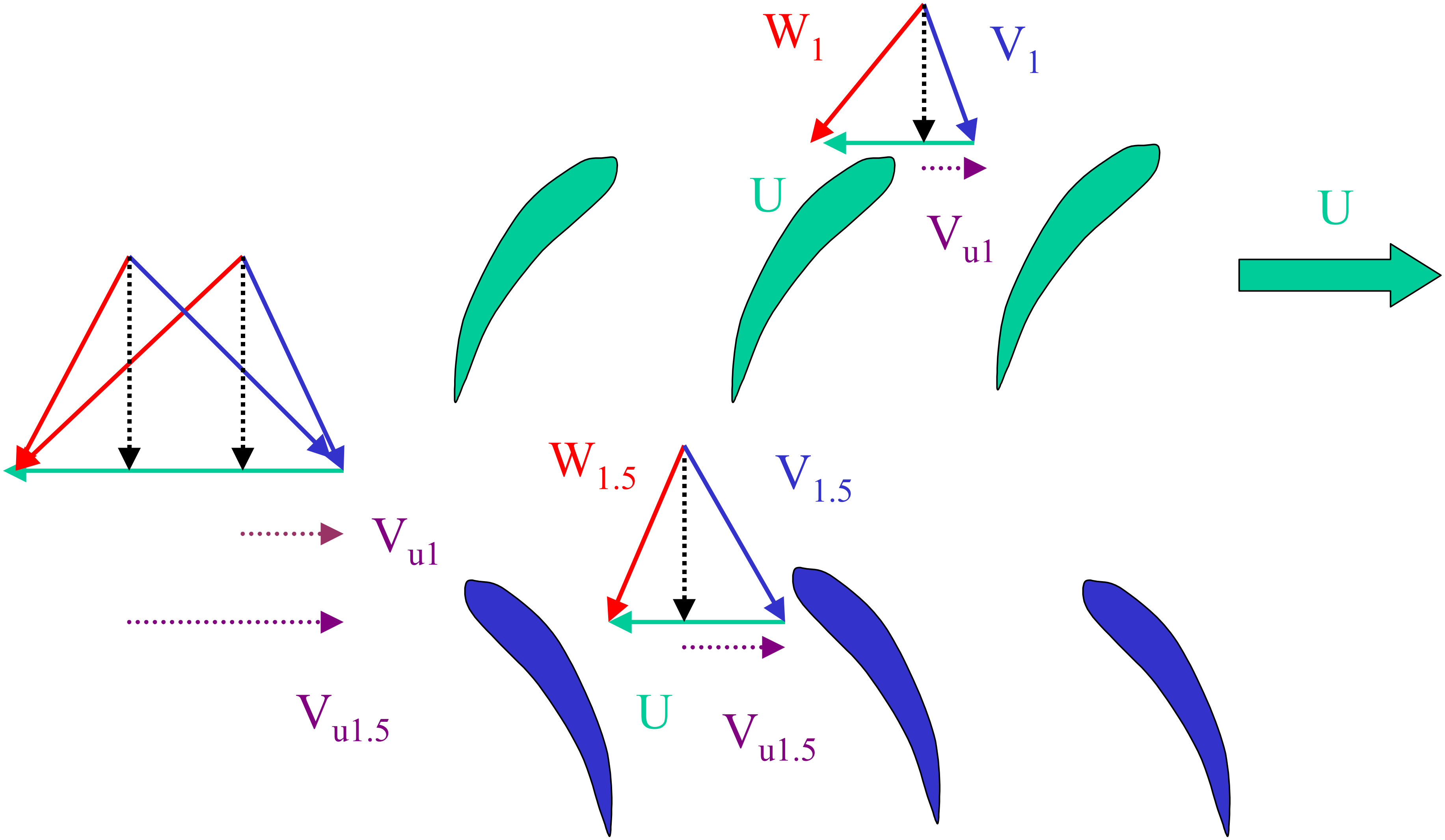
$$\vec{\tau} = \vec{r} \times \sum \vec{F} = \int \vec{r} \times \rho \vec{V}(\vec{V} \bullet d\vec{A}) = \int \left(\vec{r} \times \vec{V}\right)(\rho \vec{V} \bullet d\vec{A})$$

Evaluating this at the inlet and outlet control surfaces and recognize the fact that $\dot{m}_{IN} = \dot{m}_{OUT} = \dot{m}$:

$$\vec{\tau} = \dot{m}\left[\left(\vec{r} \times \vec{V}\right)_{OUT} - \left(\vec{r} \times \vec{V}\right)_{IN}\right]$$

This is the torque exerted on the fluid.

Compressor Configuration



W: relative to blades, V: absolute velocity, V_u: tangential compenent

Compressor Pressure Ratio

$$\vec{\tau} = \dot{m}\left[\left(\vec{r}\times\vec{V}\right)_{OUT} - \left(\vec{r}\times\vec{V}\right)_{IN}\right]:$$

$$T_{shaft} = \dot{m}\left(r_{1.5}V_{u1.5} - r_1 V_{u1}\right)$$

Power delivered: $\mathrm{P} = \dfrac{dW}{dt} = \dfrac{d}{dt}\left(Fx\right) = F\,\dfrac{dx}{dt}$

$$\mathrm{P} = FV_u = Fr\omega = T_{shaft}\omega = \omega\left[\dot{m}\left(r_{1.5}V_{u1.5} - r_1 V_{u1}\right)\right]$$

Work per unit mass: ${}_1w_{1.5} = \dfrac{\mathrm{P}}{\dot{m}} = \omega\left(r_{1.5}V_{u1.5} - r_1 V_{u1}\right)$

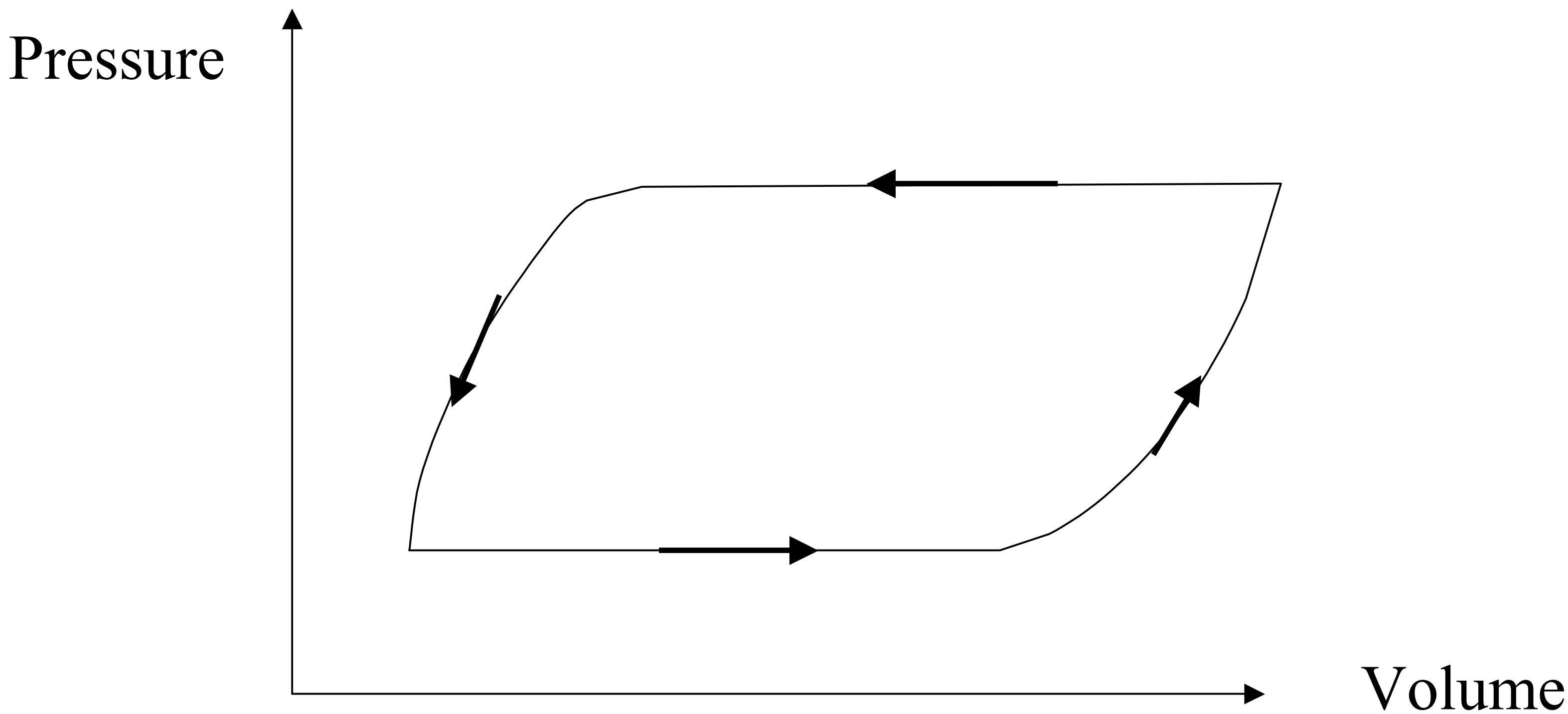
Energy balance: $h_{1.5} = h_1 + {}_1w_{1.5}, \quad c_P T_{O,1.5} = c_P T_{O,1} + {}_1w_{1.5}$

$$T_{O,1.5} = T_{O,1} + \frac{{}_1w_{1.5}}{c_P} = T_{O,1} + \frac{\omega\left(r_{1.5}V_{u1.5} - r_1 V_{u1}\right)}{c_P}$$

Assume isentropic compression: $\dfrac{P_{O,1.5}}{P_{O,1}} = \left(\dfrac{T_{O,1.5}}{T_{O,1}}\right)^{\frac{\gamma}{\gamma-1}}$

Surge

- Surge: Strong flow instability induced as a result of a massive flow reversal in one or several compressor stage(s)$\Rightarrow$flow reversal $\rightarrow$reduce volume flow $\rightarrow$adverse pressure drop $\rightarrow$ flow reattaches$\rightarrow$volume increase $\rightarrow$adverse pressure re-establishes $\rightarrow$flow reversal and cycle repeats itself



Rotating Stall

• One or several blades experience massive flow separation, i.e., "stalled". This package of stalled fluid then propagates around the blade row and very large structural vibrations can be induced as the blades periodically enter and depart from stalled state.

➢ Mass flow is redirected to other neighboring blades

➢ Effective angle attack increases and new cell stalls