Propulsion Efficiency

- Defined as the ratio of the thrust power to the jet power

$$\eta_P = \frac{\text{thrust power}}{\text{net jet power}} = \frac{\mathrm{T}V}{\dot{m}_a\left[(1+f)\left(V_e^2/2\right) - \left(V^2/2\right)\right]}$$

Assume $f \approx 0$ and pressure contribution is small

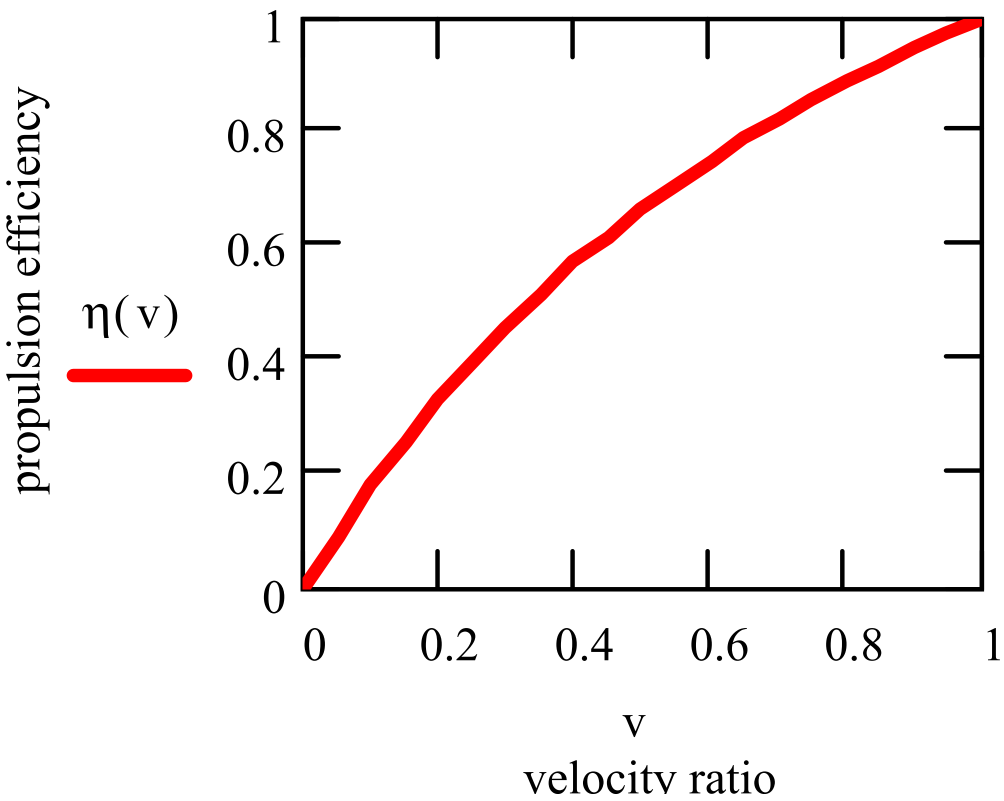
$$\mathrm{T} = \dot{m}_a\left[(1+f)V_e - V\right] \approx \dot{m}_a\left[V_e - V\right]$$

$$\eta_P = \frac{V\left(V_e - V\right)}{\left(V_e^2/2\right) - \left(V^2/2\right)} = \frac{2\left(V/V_e\right)}{1+\left(V/V_e\right)} = \frac{2\nu}{1+\nu}$$

where $\nu = \frac{\mathrm{V}}{\mathrm{V_e}}$ is the ratio between the flight velocity (V)

and the exhaust velocity ($\mathrm{V_e}$)

Propulsion Efficiency



- The propulsion efficiency increases as the velocity ratio is increased.
- It reaches a maximum at v=1 and η_P=1. No lost kinetic energy but also no thrust since $V=V_e$.
- It also suggests that in order to increase the propulsion efficiency one would like to operate at relatively low jet nozzle velocity.
- In order to avoid the loss of thrust, the mass flow rate has to be increased.

- Consequently, turbofan engine is a more efficiency engine as compared to the turbojet engine since the fan can induce large amount of the propellant into the engine and can operate at a relatively low jet exhaust speed.

Thermal and overall Efficiency

- Defined as the ratio of the change in kinetic energy of the gases due to combustion to the rate of total thermal energy consumption

$$\eta_{th} = \frac{\dot{m}_a\left[(1+f)\left(V_e^2/2\right) - V^2/2\right]}{\dot{m}_f(\Delta H_C)} = \frac{\left[(1+f)\left(V_e^2/2\right) - V^2/2\right]}{f(\Delta H_C)}$$

where ΔH_C is the lower heating value of the fuel.

(A complete combustion is assumed here)

- Overall efficiency is the combination of these two efficiencies:

$$\eta_O = \eta_{th}\eta_P$$

Takeoff Thrust (Static Thrust)

- One of the most important design considerations for jet engine is its capability to provide low-speed thrust to lift the aircraft off from the runway. That is, when the flight velocity is almost zero:

Statis thrust: $\mathrm{T}=\dot{m}_a\left[(1+f)V_e - V\right] \approx \dot{m}_a (1+f) V_e$

$$\frac{\mathrm{T}_{static}}{\dot{m}_a} = (1+f)V_e$$

Static thrust is directly proportional to the exhaust jet velocity.

This implicitly implies that we should have higher jet exhaust velocity in order to take off effectively.

However, another very important issue is for a given fuel flow rate, how the static thrust varies with the jet exhaust velocity?

Takeoff Thrust (2)

- Take into consideration of fuel consumption by using the thermal efficiency defined earlier:

$$\eta_{th} = \frac{\dot{m}_a \left[(1+f)\left(V_e^2 / 2 \right) - V^2 / 2 \right]}{\dot{m}_f (\Delta H_C)} \approx \frac{\dot{m}_a \left[(1+f)\left(V_e^2 / 2 \right) \right]}{\dot{m}_f (\Delta H_C)}$$

$$\dot{m}_a (1+f) V_e^2 = 2\eta_{th} \dot{m}_f (\Delta H_C), \text{ and}$$

$$\mathrm{T}_{static} = \dot{m}_a (1+f) V_e = \frac{2\eta_{th} \dot{m}_f (\Delta H_C)}{V_e}$$

For a given fuel flow rate and thermal efficiency (engine design), the takeoff thrust is actually inversely proportional to exhaust velocity. In other words, one can increase the takeoff thrust by pushing through a large amount of airflow through the engine, while keeping the exhaust jet velocity low.

Takeoff Thrust (3)

- The takeoff thrust is important since **it determines approximately the required size of the engine** in relating to the thrust requirement. For engine with the same combustion temperature, the fuel-air ratio is roughly about the same (recall the calculation of adiabatic flame temperature and the excess air in steady flow combustion). For a modern engine (either a turbojet or a turbofan), this temperature is dictated by the material limitations and turbine design and does not vary much. Therefore, for a fixed fuel flow rate, the air flow rate passing the combustion chamber is the same for most engines. Therefore, the engine size should be similar for both designs since all components (chamber, turbine, etc.) are basically handling the same amount of hot flow for fixed fuel flow rate. However, **the turbofan can accelerate a much larger amount of air mass through the engine due to its bypass fan design**, therefore, should provide better takeoff performance in consideration of the thrust requirement.

TSFC (Thrust-Specific Fuel Consumption)

- Defined as the fuel mass flow rate divided by the thrust developed by the engine

$$TSFC = \frac{\dot{m}_f}{\mathrm{T}}$$

For a turbojet (assuming $P_e = P_a$)

$$TSFC = \frac{\dot{m}_f}{\dot{m}_a\left[(1+f)V_e - V\right]} = \frac{f}{(1+f)V_e - V}$$

One of the engine design criteria is to minimmize the TSFC for economical consideration.