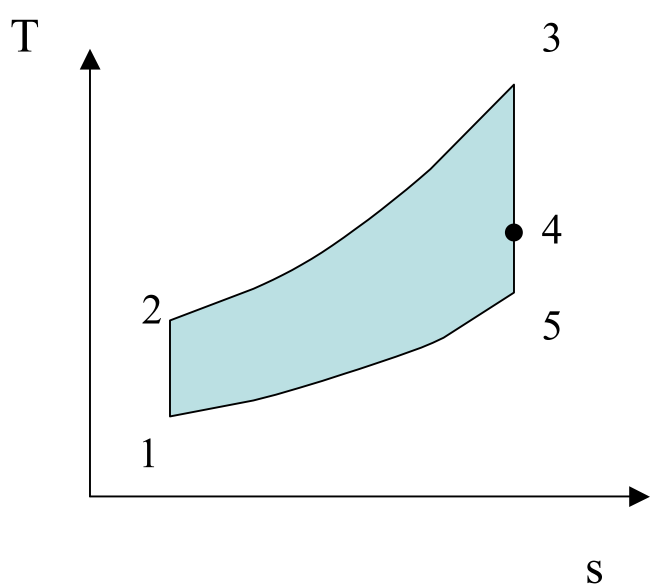
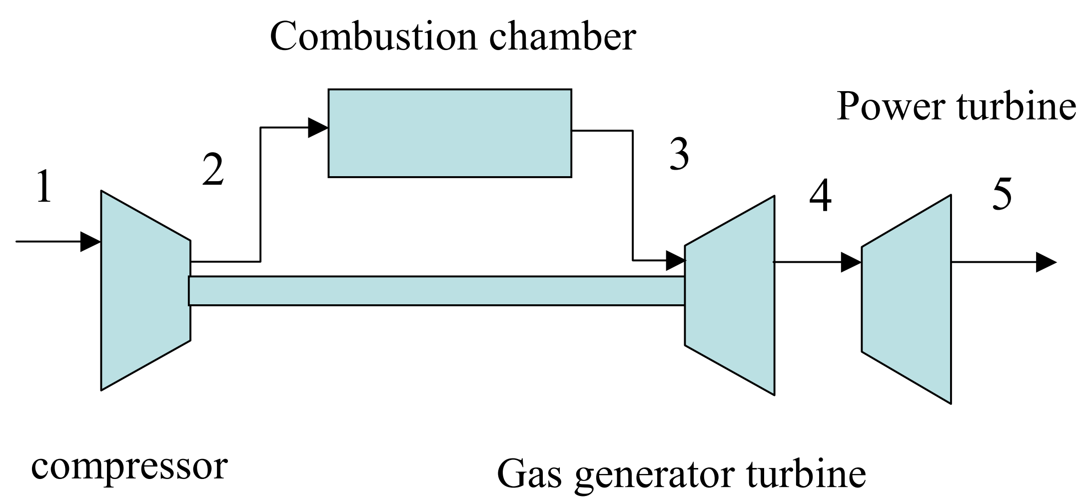
Ideal Brayton Cycle (Air Standard) Example 1

An ideal air standard gas turbine engine has a compressor inlet temperature of 288 K and a turbine temperature of 1400 K. Assume variable specific heats, calculate the compressor work, heat added, thermal efficiency, net work developed by the engine if the compressor pressure ratio is 12. (example 5.2b in the text)





Example (1-2)

- From table B.1, one can find the enthalpy and relative pressure for states 1 (entering compressor) and 3 (leaving combustion chamber).

	T(K)	h(kJ/kg)	Pr
1	288	-14.3	1.2093
2	580	284.1	14.51
3	1400	1212.7	450.9
5	751	465.7	37.58

$$\frac{\mathrm{Pr}_2}{\mathrm{Pr}_1} = \frac{p_2}{p_1} = 12, \qquad \mathrm{Pr}_2 = (12)(1.2093) = 14.51$$

From table B.1, $\mathrm{T}_2 \approx 580\mathrm{K}$, $\mathrm{h}_2 \approx 284.1$ (kJ/kg)

$$\mathrm{Wcomp{=}h_2{-}h_1{=}298.4(kJ/kg)}, \quad \left(\text{note: } \frac{p_3}{p_5} = \frac{p_2}{p_1} = 12\right)$$

$$\mathrm{Pr}_5 = \mathrm{Pr}_3\left(\frac{p_5}{p_3}\right) = 37.58, \quad \text{Table B.1} \Rightarrow \mathrm{T}_5 \approx 751 \text{ K}, \ \mathrm{h}_5 \approx 465.7 \text{(kJ/kg)}$$

Example (1-3)

$$W_{turb} = h_3 - h_5 = 747 (kJ/kg)$$

$$W_{net} = W_{turb} - W_{comp} = 448.6 (kJ/kg)$$

$$q_{in} = h_3 - h_2 = 928.6 (kJ/kg), \qquad \eta_{th} = \frac{W_{net}}{q_{in}} = 48.3\%$$

However, all mechanical devices operate with losses and can not be exactly isentropic. This imperfection can be taken into consideration by using isentropic efficiency, defined as:

Compressor isentropic efficiency: $\eta_{\text{comp}} = \dfrac{w_{1-2,i}}{w_{1-2,a}} = \dfrac{h_{2,i} - h_1}{h_{2,a} - h_1}$

Only a percentage of work can be delivered to the working fluid through a compressor. Actual work required is higher.

Turbine isentropic efficiency: $\eta_{\text{turb}} = \dfrac{w_{3-4,a}}{w_{3-4,i}} = \dfrac{h_3 - h_{4,a}}{h_3 - h_{4,i}}$

Actual work delivered by a turbine is lower than a ideal turbine.

Isentropic Efficiency Example 2

Following the previous example, but consider the effect of isentropic efficiency for all devices as follows: compressor efficiency 87%, gas generator turbine efficiency 89%, and power turbine efficiency 89%. (a) Determine power delivered by the turbine, (b) Determine the overall thermal efficiency.

	T(K)	h(kJ/kg)	Pr	p(kPa)
1	288	-14.3	1.2093	101.3
2i	580	284.1	14.51	1215.6
2a	623	328.7		
3	1400	1212.7	450.9	1215.6
4i	1072	827.3	151.4	408.2
4a	1109	869.7	173.36	408.2
5i	777	495.5	43.02	101.3
5a	815	536.7		101.3

$\eta_C = 0.87,\ \eta_{GT} = 0.89,\ \eta_{PT} = 0.89$

Ideal compressor work: $w_{C,i} = h_{2,i} - h_1 = 298.4(kJ/kg)$

$w_{C,a} = \dfrac{w_{C,i}}{\eta_C} = 343(kJ/kg)$: More work is required to provide a 12:1 compression ratio.

Isentropic Efficiency Example (2-2)

$h_{2,a} = h_1 + w_{C,a} = 328.7(kJ/kg)$

From table B.1 $\Rightarrow \mathrm{T}_{2,a} \approx 623\mathrm{K}$

(actual temperature is higher when compared to an ideal compressor)

3-4 (Gas generator turbine stage):

$w_{GT,a} = w_{C,a}$=343(kJ/kg) (Assume no shaft loss from GT to Compressor)

$w_{GT,i} = \dfrac{w_{GT,a}}{\eta_{GT}} = \dfrac{343}{0.89} = 385.4(kJ/kg)$: Actual GT receives less power

$h_{4,i} = h_3 - W_{GT,i} = 827.3$(kJ/kg)

$\mathrm{Pr}_{4,\mathrm{i}}$=151.4, $\mathrm{T}_{4,\mathrm{i}} \approx 1072$ K from table B.1

$$p_4 = p_3\left(\frac{\mathrm{Pr}_{4,\mathrm{i}}}{\mathrm{Pr}_3}\right) = (1215.6)\left(\frac{151.4}{450.9}\right) = 408.2(kPa)$$

Isentropic Efficiency Example (2-3)

Power turbine (4 to 5)

$h_{4,a} = h_3 - W_{GT,a} = 869.7\text{(kJ/kg)}$

$T_{4,a} \approx 1109\text{ K} > T_{4,i}, \quad \mathrm{Pr}_{4,a} = 173.36$

$\mathrm{Pr}_{5,i} = \mathrm{Pr}_{4,a}\left(\dfrac{p_5}{p_4}\right) = 43.02, \;\; h_{5,i} = 495.5(kJ/kg)$ from table B.1

$(a) W_{PT,a} = \eta_{PT}(h_{4,a} - h_{5,i}) = (0.89)(869.7 - 495.5) = 333.0(kJ/kg)$

$h_{5,a} = h_{4,a} - W_{PT,a} = 536.7(kJ/kg)$

$T_{5,a} \approx 815K$ from table B.1

$q_{in} = h_3 - h_{2,a} = 884(kJ/kg)$

$(b) \eta_{th} = \dfrac{W_{PT,a}}{q_{in}} = 37.7\%$