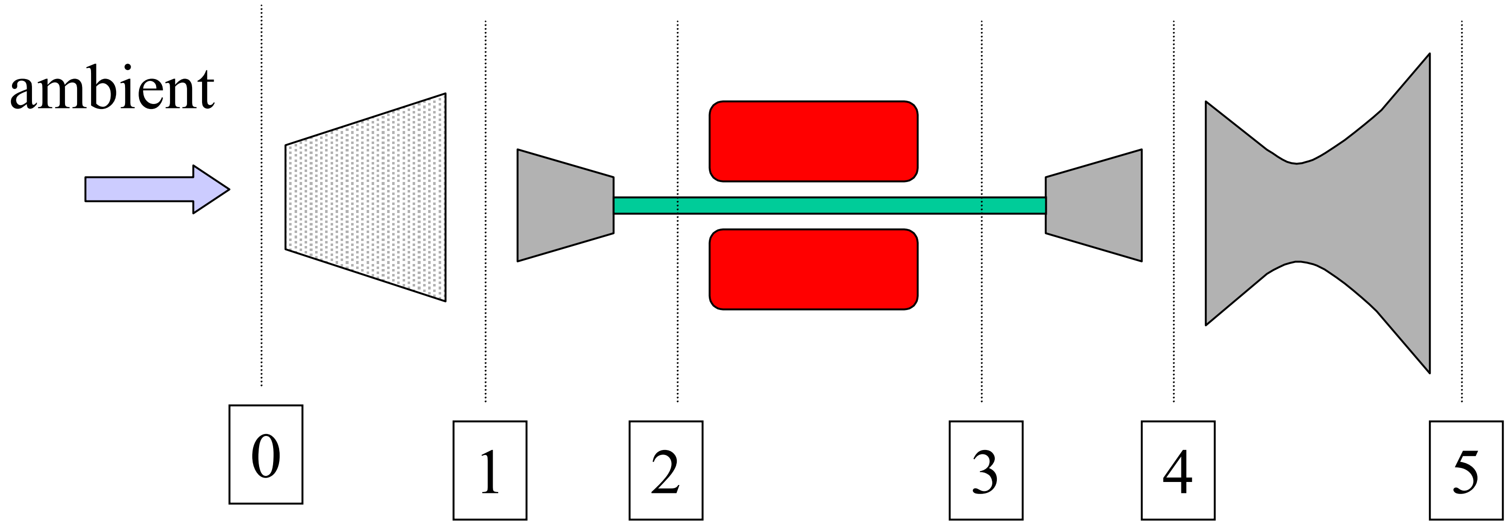
Turbojet Example

- A turbojet is operating under the following conditions:
  - Altitude 25,000 ft
  - Mach number 0.85
  - Air flow 1 lb/s
  - Compressor pressure ratio 17
  - Efficiencies:
    - Diffuser (η_D) 100%
    - Compressor (η_C) 87%
    - Turbine (η_T) 89%
    - Jet nozzle (η_N) 97%
  - Maximum stagnation temperature 2520°R

Assume an air standard cycle but take into account the mass of fuel (liquid n-octane, C_8H_{18}) added. Determine the thrust developed by the engine and the thrust-specific fuel consumption if (a) the nozzle is converging-diverging; (b) if the nozzle is converging only.

This is actually problem 6.6 in the textbook

Example (2)



- From table C.1, the ambient temperature and pressure can be found: T_{amb}=430°R, P_{amb}=5.454 psia

Speed of sound $a=\sqrt{\gamma RT}=\sqrt{(1.4)(32.2)(53.35)(430)}=1017(ft/s)$

Flight speed $V=Ma=(0.85)(1017)=864(ft/s)$

From table B.2 h_{amb}=-27.4(Btu/lb), Pr_{amb}=0.6262

$$h_{O,1}=h_{amb}+\frac{V^2}{2}=-27.4+\frac{(864)^2}{2(32.2)(778)}=-12.5\ \text{(Btu/lb)}$$

(Note: 1 Btu=778 $ft\text{-}lbf$)

Example (3)

Interpolate from table B.2, $T_{O,1}$=492°R, $Pr_{O,1}$=1.0032

$$P_{O,1} = \frac{\Pr_{O,1}}{\Pr_{amb}} P_{amb} = \frac{1.0032}{0.6262}(5.454) = 8.738(psia)$$

$1 \rightarrow 2$: Compressor: $P_{O,2} = (17)(8.738) = 148.5(psia)$

$$\Pr_{O,2i} = \frac{P_{O,2}}{P_{O,1}} \Pr_{O,1} = (17)(1.0032) = 17.054 \Rightarrow h_{O,2i} = 134.4$$

$$w_{C,a} = \frac{w_{C,i}}{\eta_C} = \frac{h_{O,2i} - h_{O,1}}{\eta_C} = \frac{14.4 - (-12.5)}{0.87} = 168.9(Btu/lb)$$

$T_{O,2a} \approx 1179°R$ from table B.2,

Example (4)

$2 \rightarrow 3$: Combustion chamber: Determine the % excess air and fuel air ratio

$$\mathrm{h}_{\mathrm{air,T_2}} = 156.4(Btu/lb) = 363.0(kJ/kg)$$

Convert into SI unit in order to use equations 4.18 and 4.19

$$X = \frac{4{,}697{,}815}{35118.8\text{-}28.965(\mathrm{kg/kmole})\mathrm{h}_{\mathrm{air,T_2}}} = \frac{4{,}697{,}815}{35118.8\text{-}(28.965)(363.0)}$$

$X = 190.3$ (Note: temperature 2520°R $\approx$ 1400K as in eqn 4.18 & 4.19)

$$f' = \frac{114.2336}{(X)(28.965)} = 0.0207 \text{ (Total mass } \dot{m}_{air}(1+f')\text{=}1.0207(lb/s))$$

$$\text{\% excess air=}\left(\frac{190.3\text{-}59.67}{59.67}\right)\times 100 = 218.9$$

See example 4.2 for more detailed derivation

Example (5)

$3 \rightarrow 4$ Turbine: $w_{T,a} = \dfrac{\dot{m}_{air} w_{C,a}}{\dot{m}_{fuel+air}} = 165.5(Btu/lb)$

Turbine inlet temperature is equal to the maximum temperature of 2520°R

$T_{O,3}$=2525°R, $h_{O,3}$=521.7, $Pr_{O,3}$=450.9 from Table B.2

$$h_{O,4i} = h_{O,3} - \frac{w_{t,a}}{\eta_T} = 335.7 \Rightarrow \mathrm{Pr}_{O,4i} \approx 129.6$$

$$P_{O,4} = \frac{\mathrm{Pr}_{O,4i}}{\mathrm{Pr}_{O,3}} P_{O,3} = \frac{129.6}{450.9} P_{O,2} = \frac{129.6}{450.9}(148.5) = 42.68\,psia$$

$$h_{O4,a} = h_{O,3} - w_{T,a} = 521.7 - 165.5 = 356.2 \Rightarrow \mathrm{Pr}_{O4,a} \approx 151.7$$

Example (6)
Nozzle configurations - CD nozzle

(a) If the exhaust nozzle is a Converging-Diverging nozzle, it can be assumed that the expansion inside the nozzle is isentropic and the exit pressure reaches the ambient pressure outside the nozzle. $P_5=P_{amb}=5.454$ psia. (Note: the nozzle might operate under non-ideally expanded conditions (either under- or over-expanded mode))

$$\Pr_{5,i} = \Pr_{O4,a}\frac{P_5}{P_{4,a}} = 151.7\frac{5.454}{42.68} = 19.385 \Rightarrow h_{5,i} \approx 144.1$$

$$\eta_N = \frac{h_{O,4} - h_{5,a}}{h_{O,4} - h_{5,i}} = \frac{V_{5a}^2/2}{V_{5i}^2/2}$$

$$V_{5a} = \sqrt{2\eta_N\left(h_{O,4} - h_{5,i}\right)} = \sqrt{2(0.97)(32.2)(778)(356.2 - 144.1)}$$

$$= 3,211(ft/s)$$

Example (7)
Nozzle configurations - CD nozzle

Thrust of the engine can be determined using equation (6.14):

$$\mathrm{T} = \left(P_e - P_{amb}\right) A_e + \dot{m}_a \left[\left(1+f\right)V_e - V\right]$$

Since $P_e = P_{amb}$ for a C-D nozzle as assumed

$$\mathrm{T} = \dot{m}_a \left[\left(1+f\right)V_e - V\right] = \frac{(1)\left[(1+0.0207)(3211)-864\right]}{32.2} = 74.95(lbf)$$

Thrust Specific Fuel Consumption (TSFC) is defined as

$$\mathrm{TSFC} = \frac{\dot{m}_f}{\mathrm{T}} = \frac{(0.0207)(3600(\sec/hr))(1(lb/s))}{74.95(lbf)} = 0.994(lb/(lbf \bullet hr))$$

Example (8)
Nozzle configurations - Converging nozzle

(b) If the exhaust nozzle is a Converging nozzle, it can be assumed that the flow is isentropically accelerating to sonic speed at the exit and the exit pressure might not match the ambient pressure outside the nozzle. If the pressure is unknown, one should try to use the temperature instead:

$$T_{O,4a} \approx 1{,}932°R, \text{ since } h_{O,4a} = 356.2, \text{ from table B.2}$$

From isentropic compressible relation:

$$\frac{\mathrm{T_O}}{\mathrm{T}} = 1 + \frac{\gamma - 1}{2} M^2, \frac{T_5}{T_{O,4a}} = \frac{2}{\gamma + 1} \text{ where } \gamma \approx 1.33 \text{ from table B.3}$$

$$T_5 \approx 1932 \frac{2}{2.33} = 1658°R, \; h_5 \approx 281.6 \text{ (table B.2)}$$

$$\mathrm{V}_5 \approx \sqrt{2\left(h_{O,4} - h_5\right)}$$

$$= \sqrt{2(32.2)(778)(356.2 - 281.6)} = 1933(ft/s)$$

Example (9)
Nozzle configurations - Converging nozzle

This is not the end of the calculation. We will need to check to make sure that the exit Mach number actually reaches 1. Check with the speed of sound by

$$a = \sqrt{\gamma_{exit} R T_5} = \sqrt{(1.342)(32.2)(53.35)T_5} = 1{,}955(ft/s)$$

Note: $\gamma_{\text{exit}} \approx 1.342$ from table B.3 and $\text{T}_5 \approx 1658°\text{R}$

$V_5 = 1933 \neq a = 1955 \Rightarrow$ re-estimate the temperature until $V_5 = a$

$T_5 = 1653°R \Rightarrow h_5 = 280.2,\ V_5 = 1951,\ a = 1952$ (close enough)

$\text{Pr}_{5,\text{i}}$=81.44 from table B.2

$$P_5 = \frac{\text{Pr}_{5,i}}{\text{Pr}_{4,a}} P_{O,4} = \frac{81.44}{151.7} 42.68 = 22.91(psia)$$ significantly different

from the ambient pressure of 5.454 *psia*.

Example (10)
Nozzle configurations - Converging nozzle

Pressure term can not be neglected from the thrust equation since $P_e \neq P_{amb}$. Therefore, the value of A_e is needed also.

$$\mathrm{T} = \left(P_e - P_{amb}\right) A_e + \dot{m}_a \left[\left(1+f\right)V_e - V\right]$$

$$\dot{m}_{fuel+air} = \rho_e A_e V_5 = \left(\frac{P_e}{RT_e}\right) A_e V_5$$

$$A_e = \frac{\dot{m}_{fuel+air} RT_e}{P_e V_5} = \frac{(1.0207)(53.35)(1653)}{(22.91)(144)(1951)} = 0.014(ft^2)$$

$$\mathrm{T} = \left(22.91 - 5.454\right)(0.014)(144) + \frac{(1)\left[\left(1+0.0207\right)(1951) - 864\right]}{32.2}$$

$= 35.2 + 35 = 70.2(lbf)$ slightly lower than a C-D nozzle condition

$$TSFC = \frac{(0.0207)(3600)(1)}{70.2} = 1.065(lb/(lbf \bullet hr))$$