Polytropic Efficiency

- It is sometime more convenient to define a stage efficiency for compressor efficiency, usually referred to as the polytropic efficiency

- Tds=dh-vdP=0 for an isentropic process across a single compressor stage

- The polytropic efficiency is defined as

$$\eta_p = \frac{dT_i}{dT_{act}}, \quad dT_i = \eta_p dT_{act}, \quad dh_i = c_p dT_i = \eta_p c_p dT_{act} = vdP = \frac{RT}{P} dP$$

$$\eta_p c_p \left(\frac{dT_{act}}{T} \right) = R \left(\frac{dP}{P} \right),$$

integrate across one compressor stage a to b : $\eta_p c_p \ln\left(\frac{T_b}{T_a}\right) = R \ln\left(\frac{P_b}{P_a}\right)$

Polytropic Efficiency

$$\frac{P_b}{P_a} = \left(\frac{T_b}{T_a}\right)^{\eta_P\left(\frac{\gamma}{\gamma-1}\right)}, \; or \;\; \frac{T_b}{T_a} = \left(\frac{P_b}{P_a}\right)^{\left(\frac{\gamma-1}{\gamma\eta_P}\right)}$$

The overall isentropic efficiency of the compressor is defined as

$$\eta_\mathrm{i} = \frac{w_{ideal}}{w_{actual}} = \frac{h_{O,2i} - h_{O,1}}{h_{O,2a} - h_{O,1}} = \frac{\left(\frac{T_{o,2i}}{T_{o,1}}\right) - 1}{\left(\frac{T_{o,2a}}{T_{o,1}}\right) - 1} = \frac{\left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1}{\left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma\eta_P}\right)} - 1}$$

Example

An axial compressor with a total pressure ratio of 20 and each stage compressor efficiency of 0.95. Determine the isentropic efficiency of the compressor by assuming a constant specific heat and γ=1.4.

$$\eta_i = \frac{(20)^{\frac{1.4-1}{1.4}} - 1}{(20)^{\frac{1.4-1}{1.4(0.95)}} - 1} = 0.93$$