7-6. A three-phase 60-Hz two-pole induction motor runs at a no-load speed of 3580 r/min and a full-load speed of 3440 r/min. Calculate the slip and the electrical frequency of the rotor at no-load and full-load conditions. What is the speed regulation of this motor [Equation (4-57)]?

SOLUTION The synchronous speed of this machine is 3600 r/min. The slip and electrical frequency at no-load conditions is

$$s_{\text{nl}} = \frac{n_{\text{sync}} - n_{\text{nl}}}{n_{\text{sync}}} \times 100\% = \frac{3600 - 3580}{3600} \times 100\% = 0.56\%$$

$$f_{r,\text{nl}} = sf_e = (0.0056)(60 \text{ Hz}) = 0.33 \text{ Hz}$$

The slip and electrical frequency at full load conditions is

$$s_{\text{fl}} = \frac{n_{\text{sync}} - n_{\text{nl}}}{n_{\text{sync}}} \times 100\% = \frac{3600 - 3440}{3600} \times 100\% = 4.44\%$$

$$f_{r,\text{fl}} = sf_e = (0.0444)(60 \text{ Hz}) = 2.67 \text{ Hz}$$

The speed regulation is

$$\text{SR} = \frac{n_{\text{nl}} - n_{\text{fl}}}{n_{\text{fl}}} \times 100\% = \frac{3580 - 3440}{3440} \times 100\% = 4.1\%$$

$$Z_{\mathrm{TH}} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

$$Z_{\mathrm{TH}} = \frac{jX_M(R_1 + jX_1)[R_1 - j(X_1 + X_M)]}{[R_1 + j(X_1 + X_M)][R_1 - j(X_1 + X_M)]}$$

$$Z_{\mathrm{TH}} = \frac{\left[-R_1X_1X_M + R_1X_1X_M + R_1X_M^{\,2}\right] + j\left[R_1^{\,2}X_M + X_1^{\,2}X_M + X_1X_M^{\,2}\right]}{R_1^{\,2} + (X_1 + X_M)^2}$$

$$Z_{\mathrm{TH}} = R_{\mathrm{TH}} + jX_{\mathrm{TH}} = \frac{R_1X_M^{\,2}}{R_1^{\,2} + (X_1 + X_M)^2} + j\frac{R_1^{\,2}X_M + X_1^{\,2}X_M + X_1X_M^{\,2}}{R_1^{\,2} + (X_1 + X_M)^2}$$

The Thevenin resistance is $R_{\mathrm{TH}} = \dfrac{R_1X_M^{\,2}}{R_1^{\,2} + (X_1 + X_M)^2}$. If $X_M >> R_1$, then

$R_1^{\,2} + (X_1 + X_M)^2 \approx (X_1 + X_M)^2$, so

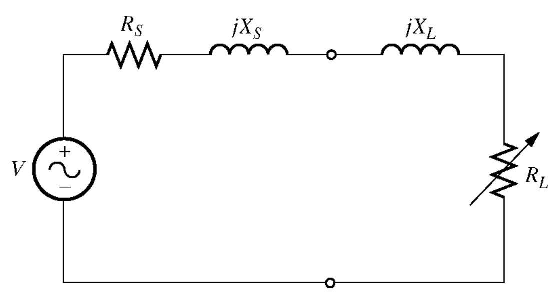
$$\boxed{R_{\mathrm{TH}} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2}$$

The Thevenin reactance is $X_{\mathrm{TH}} = \dfrac{R_1^{\,2}X_M + X_1^{\,2}X_M + X_1X_M^{\,2}}{R_1^{\,2} + (X_1 + X_M)^2}$.

If $X_M >> R_1$ and $X_M >> X_1$ then $X_1X_M^{\,2} >> R_1^{\,2}X_M + X_1^{\,2}X_M$ and $(X_1 + X_M)^2 \approx X_M^{\,2} >> R_1^{\,2}$, so

$$\boxed{X_{\mathrm{TH}} \approx \frac{X_1X_M^{\,2}}{X_M^{\,2}} = X_1}$$

7-13. Figure P7-1 shows a simple circuit consisting of a voltage source, two resistors, and two reactances in series with each other. If the resistor R_L is allowed to vary but all the other components are constant, at what value of R_L will the maximum possible power be supplied to it? *Prove* your answer. (*Hint:* Derive an expression for load power in terms of V, R_S, X_S, R_L and X_L and take the partial derivative of that expression with respect to R_L.) Use this result to derive the expression for the pullout torque [Equation (7-52)].



SOLUTION The current flowing in this circuit is given by the equation

$$\mathbf{I}_L = \frac{\mathbf{V}}{R_S + jX_S + R_L + jX_L}$$

$$I_L = \frac{V}{\sqrt{(R_S + R_L)^2 + (X_S + X_L)^2}}$$

The power supplied to the load is

$$P = I_L^{\,2} R_L = \frac{V^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{\left[(R_S + R_L)^2 + (X_S + X_L)^2\right] V^2 - V^2 R_L \left[2(R_S + R_L)\right]}{\left[(R_S + R_L)^2 + (X_S + X_L)^2\right]^2}$$

To find the point of maximum power supplied to the load, set $\partial P / \partial R_L = 0$ and solve for R_L.

$$\left[(R_S + R_L)^2 + (X_S + X_L)^2\right] V^2 - V^2 R_L \left[2(R_S + R_L)\right] = 0$$

$$\left[(R_S + R_L)^2 + (X_S + X_L)^2\right] = 2R_L (R_S + R_L)$$

$$R_S^{\,2} + 2R_S R_L + R_L^{\,2} + (X_S + X_L)^2 = 2R_S R_L + 2R_L^{\,2}$$

$$R_S^{\,2} + R_L^{\,2} + (X_S + X_L)^2 = 2R_L^{\,2}$$

$$R_S^{\,2} + (X_S + X_L)^2 = R_L^{\,2}$$

Therefore, for maximum power transfer, the load resistor should be

$$\boxed{R_L = \sqrt{R_S^{\,2} + (X_S + X_L)^2}}$$

7-14. A 440-V 50-Hz six-pole Y-connected induction motor is rated at 75 kW. The equivalent circuit parameters are

$R_1 = 0.082\ \Omega$	$R_2 = 0.070\ \Omega$	$X_M = 7.2\ \Omega$
$X_1 = 0.19\ \Omega$	$X_2 = 0.18\ \Omega$	
$P_{\text{F\&W}} = 1.3\text{ kW}$	$P_{\text{misc}} = 150\text{ W}$	$P_{\text{core}} = 1.4\text{ kW}$

For a slip of 0.04, find

(a) The line current I_L

(b) The stator power factor

(c) The rotor power factor

(d) The stator copper losses P_{SCL}

(e) The air-gap power P_{AG}

(f) The power converted from electrical to mechanical form P_{conv}

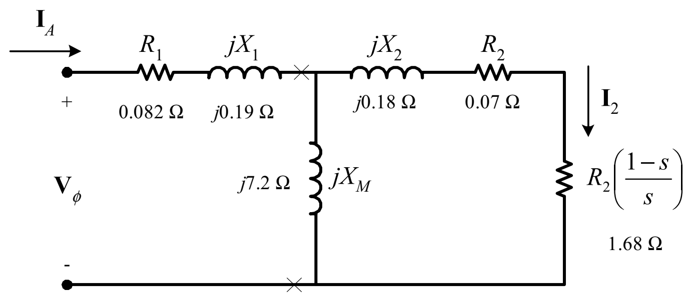
(g) The induced torque τ_{ind}

(h) The load torque τ_{load}

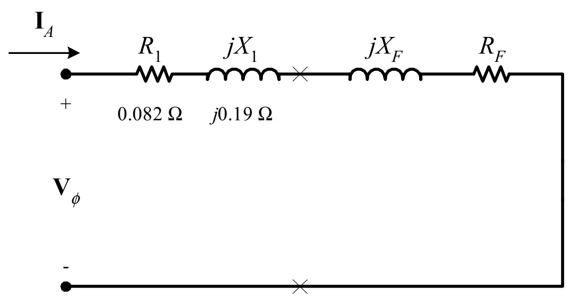
(i) The overall machine efficiency η

(j) The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j7.2\,\Omega} + \frac{1}{1.75 + j0.18}} = 1.557 + j0.550 = 1.67\angle 19.2°\ \Omega$$

The phase voltage is 440/$\sqrt{3}$ = 254 V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{254\angle 0°\ \text{V}}{0.082\,\Omega + j0.19\,\Omega + 1.557\,\Omega + j0.550\,\Omega}$$

$$I_L = I_A = 141\angle -24.3°\ \text{A}$$

(b) The stator power factor is

$$\text{PF} = \cos 24.3° = 0.911\ \text{lagging}$$

(c) To find the rotor power factor, we must find the impedance angle of the rotor

$$\theta_R = \tan^{-1}\frac{X_2}{R_2/s} = \tan^{-1}\frac{0.18}{1.75} = 5.87°$$

Therefore the rotor power factor is

$$\text{PF}_R = \cos 5.87° = 0.995\ \text{lagging}$$

(d) The stator copper losses are

$$P_{\text{SCL}} = 3I_A{}^2 R_1 = 3(141\ \text{A})^2(0.082\ \Omega) = 4890\ \text{W}$$

(e) The air gap power is $P_{\text{AG}} = 3I_2^{\,2}\dfrac{R_2}{s} = 3I_A^{\,2}R_F$

(Note that $3I_A^{\,2}R_F$ is equal to $3I_2^{\,2}\dfrac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{\text{AG}} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F = 3(141\text{ A})^2(1.557\ \Omega) = 92.6\text{ kW}$$

(f) The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = (1-0.04)(92.6\text{ kW}) = 88.9\text{ kW}$$

(g) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{6} = 1000\text{ r/min}$$

$$\omega_{\text{sync}} = (1000\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 104.7\text{ rad/s}$$

Therefore the induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{92.6\text{ kW}}{(1000\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 884\text{ N}\cdot\text{m}$$

(h) The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 88.9\text{ kW} - 1.3\text{ kW} - 1.4\text{ kW} - 300\text{ W} = 85.9\text{ kW}$$

The output speed is

$$n_m = (1-s)\,n_{\text{sync}} = (1-0.04)(1000\text{ r/min}) = 960\text{ r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{85.9\text{ kW}}{(960\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 854\text{ N}\cdot\text{m}$$

(i) The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}}\times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A\cos\theta}\times 100\%$$

$$\eta = \frac{85.9\text{ kW}}{3(254\text{ V})(141\text{ A})\cos 24.3^\circ}\times 100\% = 87.7\%$$

(j) The motor speed in revolutions per minute is 960 r/min. The motor speed in radians per second is

$$\omega_m = (960\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 100.5\text{ rad/s}$$

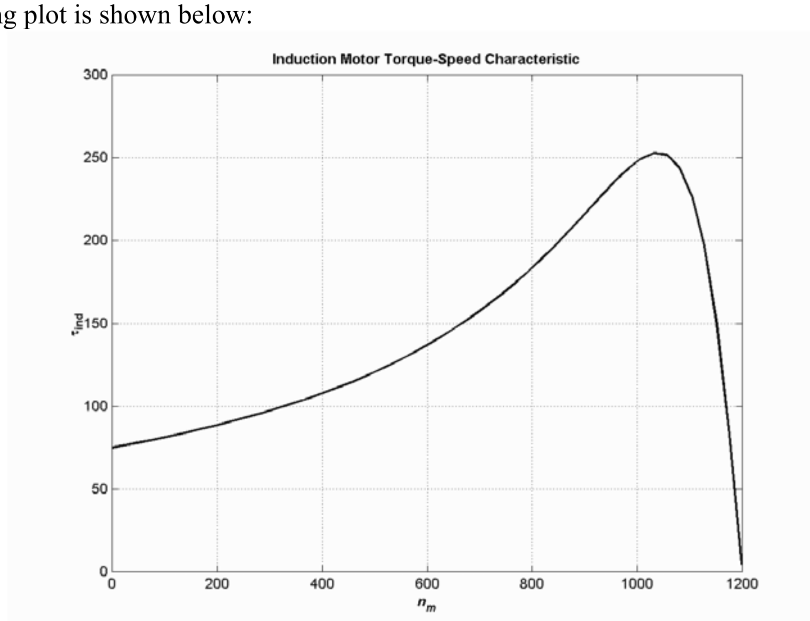
```
v_th = v_phase * ( xm / sqrt(r1^2 + (x1 + xm)^2) );
z_th = ((j*xm) * (r1 + j*x1)) / (r1 + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);

% Now calculate the torque-speed characteristic for many
% slips between 0 and 1.  Note that the first slip value
% is set to 0.001 instead of exactly 0 to avoid divide-
% by-zero problems.
s = (0:1:50) / 50;          % Slip
s(1) = 0.001;
nm = (1 - s) * n_sync;      % Mechanical speed

% Calculate torque versus speed
for ii = 1:51
   t_ind(ii) = (3 * v_th^2 * r2 / s(ii)) / ...
            (w_sync * ((r_th + r2/s(ii))^2 + (x_th + x2)^2) );
end

% Plot the torque-speed curve
figure(1);
plot(nm,t_ind,'b-','LineWidth',2.0);
xlabel('\bf\itn_{m}');
ylabel('\bf\tau_{ind}');
title ('\bfInduction Motor Torque-Speed Characteristic');
grid on;
```

The resulting plot is shown below:



7-19. A 208-V four-pole 10-hp 60-Hz Y-connected three-phase induction motor develops its full-load induced torque at 3.8 percent slip when operating at 60 Hz and 208 V. The per-phase circuit model impedances of the motor are

$R_1 = 0.33\ \Omega \qquad X_M = 16\ \Omega$

$X_1 = 0.42\ \Omega \qquad X_2 = 0.42\ \Omega$

Mechanical, core, and stray losses may be neglected in this problem.

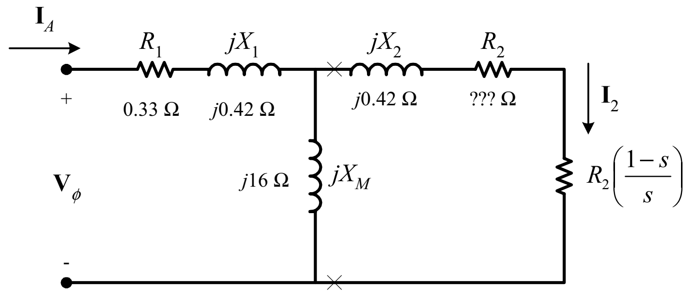
(a) Find the value of the rotor resistance R_2.

(b) Find $\tau_{\max}$, $s_{\max}$, and the rotor speed at maximum torque for this motor.

(c) Find the starting torque of this motor.

(d) What code letter factor should be assigned to this motor?

SOLUTION The equivalent circuit for this motor is



The Thevenin equivalent of the input circuit is:

$$Z_{\mathrm{TH}} = \frac{jX_M\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_M\right)} = \frac{(j16\ \Omega)(0.33\ \Omega\ +\ j0.42\ \Omega)}{0.33\ \Omega\ +\ j\left(0.42\ \Omega + 16\ \Omega\right)} = 0.313 + j0.416\ \Omega = 0.520\angle 53°\ \Omega$$

$$\mathbf{V}_{\mathrm{TH}} = \frac{jX_M}{R_1 + j(X_1 + X_M)}\mathbf{V}_\phi = \frac{(j16\,\Omega)}{0.33\,\Omega + j(0.42\,\Omega + 16\,\Omega)}(120\angle 0°\ \mathrm{V}) = 116.9\angle 1.2°\ \mathrm{V}$$

(a) If losses are neglected, the induced torque in a motor is equal to its load torque. At full load, the output power of this motor is 10 hp and its slip is 3.8%, so the induced torque is

$$n_m = (1 - 0.038)(1800\ \mathrm{r/min}) = 1732\ \mathrm{r/min}$$

$$\tau_{\mathrm{ind}} = \tau_{\mathrm{load}} = \frac{(10\ \mathrm{hp})(746\ \mathrm{W/hp})}{(1732\ \mathrm{r/min})\left(\dfrac{2\pi\ \mathrm{rad}}{1\ \mathrm{r}}\right)\left(\dfrac{60\ \mathrm{s}}{1\ \mathrm{min}}\right)} = 41.1\ \mathrm{N\cdot m}$$

The induced torque is given by the equation

$$\tau_{\mathrm{ind}} = \frac{3V_{\mathrm{TH}}^2 R_2 / s}{\omega_{\mathrm{sync}}\left[\left(R_{\mathrm{TH}} + R_2 / s\right)^2 + \left(X_{\mathrm{TH}} + X_2\right)^2\right]}$$

Substituting known values and solving for R_2 / s yields

$$41.1\ \mathrm{N\cdot m} = \frac{3(116.9\ \mathrm{V})^2 R_2 / s}{(188.5\ \mathrm{rad/s})\left[(0.313 + R_2 / s)^2 + (0.416 + 0.42)^2\right]}$$

$$7{,}747 = \frac{40{,}997\ \ R_2 / s}{\left[(0.313 + R_2 / s)^2 + 0.699\right]}$$

$$\left[(0.313 + R_2 / s)^2 + 0.699\right] = 5.292\ \ R_2 / s$$

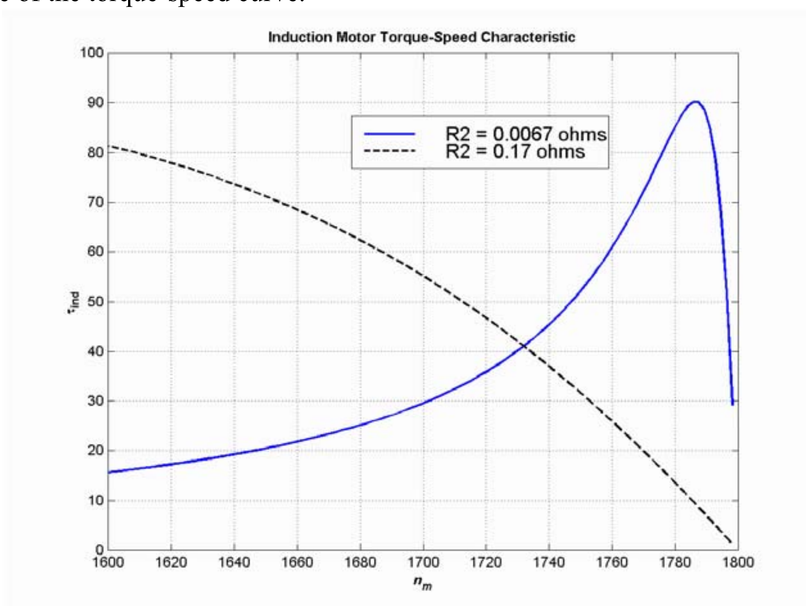
$$\left[0.098+0.626R_2/s+\left(R_2/s\right)^2+0.699\right]=5.292 \;\; R_2/s$$

$$\left(\frac{R_2}{s}\right)^2-4.666\left(\frac{R_2}{s}\right)+0.797=0$$

$$\left(\frac{R_2}{s}\right)=0.178, \;\; 4.488$$

$$R_2=0.0067\,\Omega, \;\; 0.17\,\Omega$$

These two solutions represent two situations in which the torque-speed curve would go through this specific torque-speed point. The two curves are plotted below. As you can see, only the 0.17 Ω solution is realistic, since the 0.0067 Ω solution passes through this torque-speed point at an unstable location on the back side of the torque-speed curve.



(b) The slip at pullout torque can be found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model. The Thevenin equivalent of the input circuit was calculate in part *(a)*. The slip at pullout torque is

$$s_{\max}=\frac{R_2}{\sqrt{R_{\mathrm{TH}}{}^2+\left(X_{\mathrm{TH}}+X_2\right)^2}}$$

$$s_{\max}=\frac{0.17\ \Omega}{\sqrt{\left(0.313\ \Omega\right)^2+\left(0.416\ \Omega\ +0.420\ \Omega\right)^2}}=0.190$$

The rotor speed a maximum torque is

$$n_{\text{pullout}}=(1-s)\;n_{\text{sync}}=\left(1-0.190\right)\left(1800\text{ r/min}\right)=1457\text{ r/min}$$

and the pullout torque of the motor is

$$\tau_{\max}=\frac{3V_{\mathrm{TH}}^2}{2\omega_{\text{sync}}\left[R_{\mathrm{TH}}+\sqrt{R_{\mathrm{TH}}^2+\left(X_{\mathrm{TH}}+X_2\right)^2}\right]}$$

$$\tau_{\max} = \frac{3(116.9\text{ V})^2}{2(188.5\text{ rad/s})\left[0.313\,\Omega + \sqrt{(0.313\,\Omega)^2 + (0.416\,\Omega + 0.420\,\Omega)^2}\right]}$$

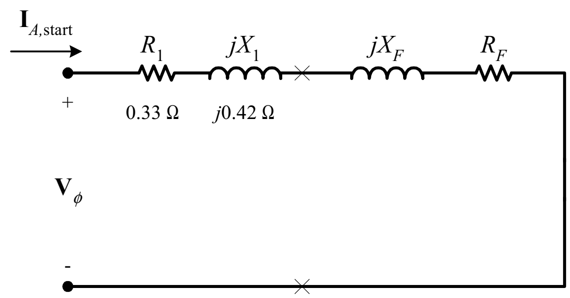
$$\tau_{\max} = 90.2\text{ N}\cdot\text{m}$$

(c) The starting torque of this motor is the torque at slip $s = 1$. It is

$$\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2/s}{\omega_{\text{sync}}\left[(R_{\text{TH}} + R_2/s)^2 + (X_{\text{TH}} + X_2)^2\right]}$$

$$\tau_{\text{ind}} = \frac{3(116.9\text{ V})^2(0.17\ \Omega)}{(188.5\text{ rad/s})\left[(0.313 + 0.17\ \Omega)^2 + (0.416 + 0.420)^2\right]} = 38.3\text{ N}\cdot\text{m}$$

(d) To determine the starting code letter, we must find the locked-rotor kVA per horsepower, which is equivalent to finding the starting kVA per horsepower. The easiest way to find the line current (or armature current) at starting is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M at starting conditions, and then calculate the starting current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M at starting conditions ($s = 1.0$) is:

$$Z_{F,\text{start}} = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j16\,\Omega} + \frac{1}{0.17 + j0.42}} = 0.161 + j0.411 = 0.442\angle 68.6^\circ\ \Omega$$

The phase voltage is $208/\sqrt{3}$ = 120 V, so line current $\mathbf{I}_{L,\text{start}}$ is

$$\mathbf{I}_{L,\text{start}} = \mathbf{I}_A = \frac{\mathbf{V}_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0^\circ\text{ V}}{0.33\,\Omega + j0.42\,\Omega + 0.161\,\Omega + j0.411\,\Omega}$$

$$\mathbf{I}_{L,\text{start}} = \mathbf{I}_A = 124\angle -59.4^\circ\text{ A}$$

Therefore, the locked-rotor kVA of this motor is

$$S = \sqrt{3}\,V_T\,I_{L,\text{rated}} = \sqrt{3}(208\text{ V})(124\text{ A}) = 44.7\text{ kVA}$$

and the kVA per horsepower is

$$\text{kVA/hp} = \frac{44.7\text{ kVA}}{10\text{ hp}} = 4.47\text{ kVA/hp}$$

This motor would have **starting code letter D**, since letter D covers the range 4.00-4.50.