(c) The voltage regulation of the transmission line is

$$\text{VR} = \frac{V_S - V_R}{V_R} \times 100\% = \frac{10000 - 8000}{8000} \times 100\% = 25\%$$

Problems 9-8 through 9-10 refer to a single phase, 8 kV, 50-Hz, 50 km-long underground cable consisting of two aluminum conductors with a 3 cm diameter separated by a spacing of 15 cm.

9-8. The single-phase transmission line referred to in Problems 9-3 through 9-7 is to be replaced by an underground cable. The cable consists of two aluminum conductors with a 3 cm diameter, separated by a center-to-center spacing of 15 cm. As before, assume that the 50 Hz ac resistance of the line is 5% greater than its dc resistance, and calculate the series impedance and shunt admittance of the line in ohms per km and siemens per km. Also, calculate the total impedance and admittance for the entire line.

SOLUTION The series inductance per meter of this transmission line is given by Equation (9-22).

$$l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right) \quad \text{H/m} \tag{9-22}$$

where $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m .

$$l = \frac{\mu_0}{\pi}\left(\frac{1}{4} + \ln\frac{0.15 \text{ m}}{0.015 \text{ m}}\right) = \frac{4\pi \times 10^{-7} \text{ H/m}}{\pi}\left(\frac{1}{4} + \ln\frac{0.15 \text{ m}}{0.015 \text{ m}}\right) = 1.021 \times 10^{-6} \text{ H/m}$$

Therefore the inductance of this transmission line will be

$$L = \left(1.021 \times 10^{-6} \text{ H/m}\right)\left(50{,}000 \text{ m}\right) = 0.0511 \text{ H}$$

The inductive reactance of this transmission line is

$$X = j\omega L = j2\pi f L = j2\pi\left(50 \text{ Hz}\right)\left(0.0511 \text{ H}\right) = j16.05\ \Omega$$

The resistance of this transmission line is the same as for the overhead transmission line calculated previously: $R_{\text{AC}} = 2.1\ \Omega$. The total series impedance of this entire line would be $Z = 2.1\ + j16.05\ \Omega$, so the impedance per kilometer would be

$$Z = \left(2.1\ + j16.05\ \Omega\right)/\left(50 \text{ km}\right) = 0.042 + j0.321\ \Omega/\text{km}$$

The shunt capacitance per meter of this transmission line is given by Equation (9-41).

$$c = \frac{\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \tag{9-41}$$

$$c = \frac{\pi\left(8.854 \times 10^{-12} \text{ F/m}\right)}{\ln\left(\frac{0.15}{0.015}\right)} = 1.21 \times 10^{-11} \text{ F/m}$$

Therefore the capacitance per kilometer will be

$$c = 1.21 \times 10^{-8} \text{ F/km}$$

The shunt admittance of this transmission line per kilometer will be

$$y_{sh} = j2\pi f c = j2\pi\left(50 \text{ Hz}\right)\left(1.21 \times 10^{-8} \text{ F/km}\right) = j3.80 \times 10^{-6} \text{ S/km}$$

Therefore the total shunt admittance will be

$$Y_{sh} = \left(j3.80 \times 10^{-6} \text{ S/km}\right)\left(50 \text{ km}\right) = j1.90 \times 10^{-4} \text{ S}$$

9-9. The underground cable is operating with the receiving side of the line open-circuited. The sending end voltage is 8 kV at 50 Hz. How much charging current is flowing in the line? How does this charging current in the cable compare to the charging current of the overhead transmission line?

9-16. A 50 Hz three phase transmission line is 300 km long. It has a total series impedance of $23 + j75\ \Omega$ and a shunt admittance of $j500\ \mu\text{S}$. It delivers 150 MW at 220 kV, with a power factor of 0.88 lagging. Find the voltage at the sending end using *(a)* the short line approximation. *(b)* The medium-length line approximation. *(c)* The long line equation. How accurate are the short and medium-length line approximations for this case?

SOLUTION

(a) In the short line approximation, the shunt admittance is ignored. The *ABCD* constants for this line are:

$$A = 1 \quad B = Z$$
$$C = 0 \quad D = 1 \tag{9-67}$$

$$A = 1$$

$$B = Z = 23 + j75\ \Omega = 78.4\angle 73°\ \Omega$$

$$C = 0\ \text{S}$$

$$D = 1$$

The receiving end line voltage is 220 kV, so the rated phase voltage is 220 kV / $\sqrt{3}$ = 127 kV, and the current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3}V_{LL}} = \frac{150{,}000{,}000\ \text{W}}{\sqrt{3}\left(220{,}000\ \text{V}\right)} = 394\ \text{A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 127\angle 0°\ \text{kV}$, and the phase current at the receiving end will be $\mathbf{I}_R = 394\angle -28.4°\ \text{A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = (1)(127\angle 0°\ \text{kV}) + (78.4\angle 73°\ \Omega)(394\angle -28.4°\ \text{A})$$

$$\mathbf{V}_S = 151\angle 8.2°\ \text{kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = (0\ \text{S})(133\angle 0°\ \text{kV}) + (1)(394\angle -28.4°\ \text{A})$$

$$\mathbf{I}_S = 394\angle -28.4°\ \text{A}$$

(b) In the medium length line approximation, the shunt admittance divided into two equal pieces at either end of the line. The *ABCD* constants for this line are:

$$A = \frac{ZY}{2} + 1 \qquad B = Z$$
$$C = Y\left(\frac{ZY}{4} + 1\right) \quad D = \frac{ZY}{2} + 1 \tag{9-73}$$

$$A = \frac{ZY}{2} + 1 = \frac{(23 + j75\ \Omega)(j0.0005\ \text{S})}{2} + 1 = 0.9813\angle 0.34°$$

$$B = Z = 23 + j75\ \Omega = 78.4\angle 73°\ \Omega$$

$$C = Y\left(\frac{ZY}{4} + 1\right) = (j0.0005\ \text{S})\left[\frac{(23 + j75\ \Omega)(j0.0005\ \text{S})}{4} + 1\right]$$

$$C = 4.953 \times 10^{-4} \angle 90.2° \text{ S}$$

$$D = \frac{ZY}{2} + 1 = \frac{(23 + j75\ \Omega)(j0.0005\text{ S})}{2} + 1 = 0.9813\angle 0.34°$$

The receiving end line voltage is 220 kV, so the rated phase voltage is 220 kV / $\sqrt{3}$ = 127 kV, and the current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3}V_{LL}} = \frac{150{,}000{,}000\text{ W}}{\sqrt{3}(220{,}000\text{ V})} = 394\text{ A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 127\angle 0°\text{ kV}$, and the phase current at the receiving end will be $\mathbf{I}_R = 394\angle -28.4°\text{ A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = 148.4\angle 8.7°\text{ kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = 361\angle -19.2°\text{ A}$$

(c) In the long transmission line, the *ABCD* constants are based on modified impedances and admittances:

$$Z' = Z\frac{\sinh \gamma d}{\gamma d} \qquad (9\text{-}74)$$

$$Y' = Y\frac{\tanh(\gamma d/2)}{\gamma d/2} \qquad (9\text{-}75)$$

and the corresponding *ABCD* constants are

$$A = \frac{Z'Y'}{2} + 1 \qquad B = Z'$$

$$C = Y'\left(\frac{Z'Y'}{4} + 1\right) \qquad D = \frac{Z'Y'}{2} + 1 \qquad (9\text{-}76)$$

The propagation constant of this transmission line is $\gamma = \sqrt{yz}$

$$\gamma = \sqrt{yz} = \sqrt{\left(\frac{j500 \times 10^{-6}\text{ S}}{300\text{ km}}\right)\left(\frac{23 + j75\ \Omega}{300\text{ km}}\right)} = 0.00066\angle 81.5°$$

$$\gamma d = (0.00066\angle 81.5°)(300\text{ km}) = 0.198\angle 81.5°$$

The modified parameters are

$$Z' = Z\frac{\sinh \gamma d}{\gamma d} = (23 + j75\ \Omega)\frac{\sinh(0.198\angle 81.5°)}{0.198\angle 81.5°} = 77.9\angle 73°\ \Omega$$

$$Y' = Y\frac{\tanh(\gamma d/2)}{\gamma d/2} = 5.01 \times 10^{-4}\angle 89.9°\text{ S}$$

and the *ABCD* constants are

$$A = \frac{Z'Y'}{2} + 1 = 0.983\angle 0.33°$$

$$B = Z' = 77.9\angle 73° \ \Omega$$

$$C = Y'\left(\frac{Z'Y'}{4} + 1\right) = 4.97\angle 90.1° \text{ S}$$

$$D = \frac{Z'Y'}{2} + 1 = 0.983\angle 0.33°$$

The receiving end line voltage is 220 kV, so the rated phase voltage is 220 kV / $\sqrt{3}$ = 127 kV, and the current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3}V_{LL}} = \frac{150,000,000 \text{ W}}{\sqrt{3}\left(220,000 \text{ V}\right)} = 394 \text{ A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 127\angle 0° \text{ kV}$, and the phase current at the receiving end will be $\mathbf{I}_R = 394\angle -28.4° \text{ A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = 148.2\angle 8.7° \text{ kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = 361.2\angle -19.2° \text{ A}$$

The short transmission line approximate was rather inaccurate, but the medium and long line models were both in good agreement with each other.

9-17. A 60 Hz, three phase, 110 kV transmission line has a length of 100 miles and a series impedance of $0.20 + j0.85$ Ω/mile and a shunt admittance of 6×10^{-6} S/mile. The transmission line is supplying 60 MW at a power factor of 0.85 lagging, and the receiving end voltage is 110 kV.

(a) What are the voltage, current, and power factor at the receiving end of this line?
(b) What are the voltage, current, and power factor at the sending end of this line?
(c) How much power is being lost in this transmission line?
(d) What is the current angle δ of this transmission line? How close is the transmission line to its steady-state stability limit?

SOLUTION This transmission line may be considered to be a medium-line line. The impedance Z and admittance Y of this line are:

$$Z = \left(0.20 + j0.85 \ \Omega/\text{mile}\right)\left(100 \text{ miles}\right) = 20 + j85 \ \Omega$$

$$Y = \left(j6 \times 10^{-6}/\text{mile}\right)\left(100 \text{ miles}\right) = j0.0006 \text{ S}$$

The *ABCD* constants for this line are:

$$A = \frac{ZY}{2} + 1 = \frac{\left(20 + j85 \ \Omega\right)\left(j0.0006 \text{ S}\right)}{2} + 1 = 0.9745\angle 0.35°$$

$$B = Z = 20 + j85 \ \Omega = 87.3\angle 76.8° \ \Omega$$

$$C = Y\left(\frac{ZY}{4} + 1\right) = \left(j0.0006 \text{ S}\right)\left[\frac{\left(20 + j85 \ \Omega\right)\left(j0.0006 \text{ S}\right)}{4} + 1\right] = 0.00059\angle 90.2° \text{ S}$$

$$D = \frac{ZY}{2} + 1 = \frac{(20 + j85\ \Omega)(j0.0006\ \text{S})}{2} + 1 = 0.9745\angle 0.35^\circ$$

(a) Assuming that the receiving end voltage is at 0°, the receiving end phase voltage and current are.

$$\mathbf{V}_R = 110\angle 0^\circ\ \text{kV}/\sqrt{3} = 63.5\angle 0^\circ\ \text{kV}$$

$$I_R = \frac{P}{\sqrt{3}V\cos\theta} = \frac{60\ \text{MW}}{\sqrt{3}(110\ \text{kV})(0.85)} = 370\ \text{A}$$

$$\mathbf{I}_R = 370\angle -31.7^\circ\ \text{A}$$

The receiving end power factor is 0.85 lagging. The receiving end line voltage and current are 110 kV and 370 A, respectively.

(b) The sending end voltage and current are given by

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R = A(63.5\angle 0^\circ\ \text{kV}) + B(370\angle -31.7^\circ\ \text{A})$$

$$\mathbf{V}_S = 87.8\angle 15.35^\circ\ \text{kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R = C(63.5\angle 0^\circ\ \text{kV}) + D(370\angle -31.7^\circ\ \text{A})$$

$$\mathbf{I}_S = 342.4\angle -26^\circ\ \text{A}$$

The sending end power factor is $\cos[15.35^\circ - (-26^\circ)] = \cos(41.4^\circ) = 0.751\ \text{lagging}$. The sending end line voltage and current are $\sqrt{3}(87.8\ \text{kV}) = 152\ \text{kV}$ and 342 A, respectively.

(c) The power at the sending end of the transmission line is

$$P_S = 3V_{\phi,S}I_{\phi,S}\cos\theta = 3(87{,}800)(342)(0.751) = 67.7\ \text{MW}$$

The power at the receiving end of the transmission line is

$$P_R = 3V_{\phi,R}I_{\phi,R}\cos\theta = 3(63{,}500)(370)(0.85) = 59.9\ \text{MW}$$

Therefore the losses in the transmission line are approximately 7.8 MW.

(d) The angle δ is 15.35°. It is about 1/4 of the way to the line's static stability limit.