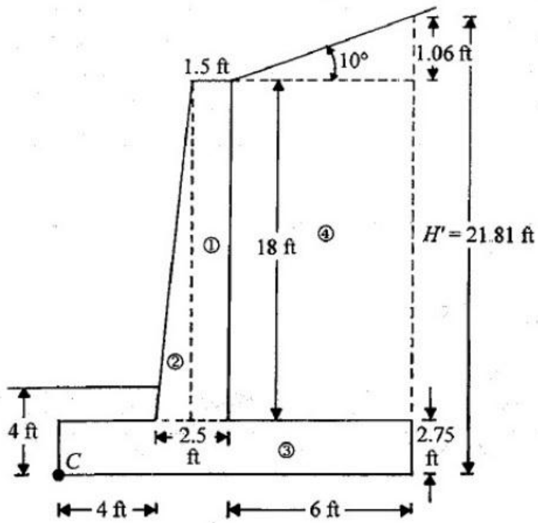
1) For the cantilever retaining wall shown in the following figure, let the following data be given:

Wall dimensions: $H = 18\ ft$ , $X_1 = 18\ in, X_2 = 30\ in, X_3 = 4\ ft, X_4 = 6\ ft, X_5 = 2.75\ ft,$ $\alpha = 10°, D = 4\ ft$

Soil properties: $\gamma_1 = 117\ Ib/ft^3, \varphi'_1 = 34°, \gamma_2 = 110\ Ib/ft^3, , \varphi'_2 = 18°, , c'_2 = 800\ Ib/ft^2$

Calculate the factor of safety with respect to overturning.

Refer to the diagram.




Using the equations, we have

$$\phi'_1 = 34°;\ \alpha = 10°;\ K_a = 0.294$$

$$P_a = \frac{1}{2}(H')^2\gamma_1 K_a = \frac{1}{2}\left(\frac{(21.81)^2(117)(0.294)}{1000}\right) = 8.18\ \text{kip/ft}$$

$$P_v = P_a \sin 10° = 1.42\ \text{kip/ft}$$

$$P_h = P_a \cos 10° = 8.06\ \text{kip/ft}$$

| Section | Weight (kip / ft) | Moment arm from $C$ (ft) | Moment about $C$ (kip-ft / ft) |
|---|---|---|---|
| 1 | $(1.5)(18)(\gamma_c) = 4.05$ | 5.75 | 23.29 |
| 2 | $½(1.0)(18)(\gamma_c) = 1.35$ | $4 + ⅔(1) = 4.67$ | 6.3 |
| 3 | $(12.5)(2.75)(\gamma_c) = 5.156$ | 6.25 | 32.23 |
| 4 | $\frac{(18+19.06)}{2}(6)(0.117) = 13.01$ | $4 + 2.5 + \frac{6}{2} = 9.5$ | 123.6 |
| | $P_v = 1.42$ | 12.5 | 17.75 |
| | $\sum 24.986$ | | $\sum 203.17$ |

$$M_o = P_h\frac{H'}{3} = (8.06)\left(\frac{21.81}{3}\right) = 58.6\ \text{kip-ft/ft}$$

$$\text{FS}_{\text{overturning}} = \frac{203.17}{58.6} = \mathbf{3.47}$$

2) For the gravity wall shown in the following figure, Calculate the factor of safety with respect to overturning, given the following data:

Wall dimensions: $H = 6m$ , $X_1 = 0.6\ m$, $X_2 = 2\ m$, $X_3 = 2\ m$, $X_4 = 0.5\ m$, $X_5 = 0.75\ m$, $X_6 = 0.8\ m$, $D = 1.5\ m$

Soil properties: $\gamma_1 = 16.5\ kN/m^3, \varphi'_1 = 32°, \gamma_2 = 18\ kN/m^3,, \varphi'_2 = 22°,, c'_2 = 40\ kN/m^2$

Use the Rankine active earth pressure in your calculation.

Refer to the figure.

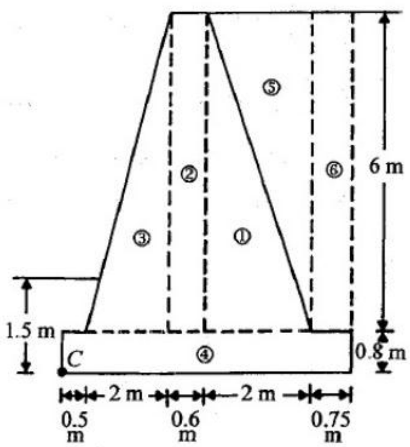
$\phi'_1 = 32°; H' = 6.8$ m

$$K_a = \tan^2\left(45 - \frac{32}{3}\right) = 0.307$$

$$P_a = P_h = \frac{1}{2}\gamma(H')^2 K_a$$

$$= \frac{1}{2}(16.5)(6.8)^2(0.307)$$

$$= 117.1 \text{ kN/m}$$




Refer to the table.

| Section | Weight (kN / m) | Moment arm from *C* (m) | Moment about C (kN-m / m) |
|---|---|---|---|
| 1 | $\frac{1}{2}(2)(6)(\gamma_c) = 141.48$ | 3.77 | 533.38 |
| 2 | $(0.6)(6)(\gamma_c) = 84.89$ | 2.8 | 237.69 |
| 3 | 141.48 | 1.83 | 258.9 |
| 4 | $(5.85)(0.8)(\gamma_c) = 110.35$ | 2.925 | 322.8 |
| 5 | $\frac{1}{2}(2)(6)(16.5) = 99$ | 4.43 | 438.57 |
| 6 | $(0.75)(6)(16.5) = 74.25$ | 5.475 | 405.52 |
| | $\sum 651.45$ | | $\sum 2196.86$ |

$$M_O = \frac{H'P_a}{3} = \left(\frac{6.8}{3}\right)(117.1) = 265.4 \text{ kN-m/m}$$

$$FS_{(overturning)} = \frac{\sum M_R}{\sum M_O} = \frac{2196.86}{265.4} = \mathbf{8.28}$$

3) Find the total active earth pressure per foot of the wall for the following situation:

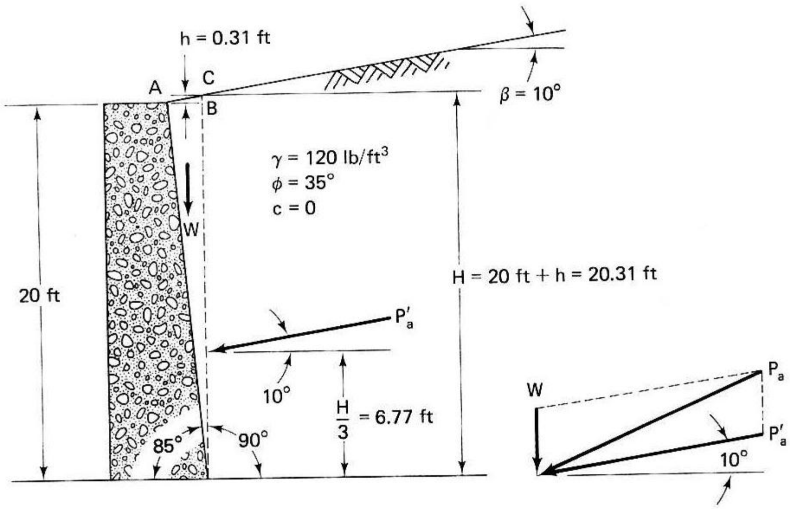



**FIGURE 10-13**

*Required*

Total active earth pressure per foot of wall.

***Solution***

As shown on Fig. 10-13,

$$\tan 5^\circ = \frac{AB}{20 \text{ ft}}$$

$$AB = (20 \text{ ft})(\tan 5^\circ) = 1.75 \text{ ft}$$

Also,

$$\tan 10^\circ = \frac{BC}{AB} = \frac{h}{1.75 \text{ ft}}$$

$$h = (1.75 \text{ ft})(\tan 10^\circ) = 0.31 \text{ ft}$$

From Eqs. (10-10) and (10-11),

$$P'_a = \tfrac{1}{2}\gamma H^2 K_a \qquad (10\text{-}10)$$

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \qquad (10\text{-}11)$$

$\gamma = 120 \text{ lb/ft}^3$

$H = 20.31 \text{ ft}$

$\beta = 10^\circ$

$\phi = 35^\circ$

$$K_a = (\cos 10^\circ)\frac{\cos 10^\circ - \sqrt{\cos^2 10^\circ - \cos^2 35^\circ}}{\cos 10^\circ + \sqrt{\cos^2 10^\circ - \cos^2 35^\circ}} = 0.282$$

$$P'_a = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(20.31 \text{ ft})^2(0.282) = 6979 \text{ lb/ft}$$

$$W = (\tfrac{1}{2})(\gamma)(AB)(H)$$

$$W = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(1.75 \text{ ft})(20.31 \text{ ft}) = 2133 \text{ lb/ft}$$

$$P_h = P'_a \cos\beta = (6979 \text{ lb/ft}) \cos 10^\circ = 6873 \text{ lb/ft}$$

$$P_v = P'_a \sin\beta = (6979 \text{ lb/ft}) \sin 10^\circ = 1212 \text{ lb/ft}$$

$$\sum V = W + P_v = 2133 \text{ lb/ft} + 1212 \text{ lb/ft} = 3345 \text{ lb/ft}$$

$$\sum H = P_h = 6873 \text{ lb/ft}$$

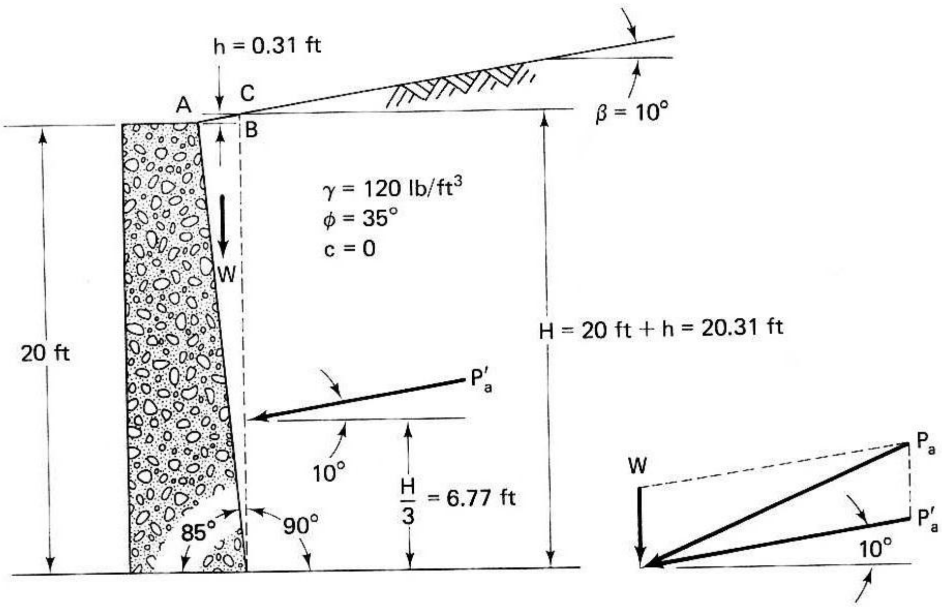
$$\text{Total active earth pressure } (P_a) = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$$

$$= \sqrt{(3345 \text{ lb/ft})^2 + (6873 \text{ lb/ft})^2}$$

$$= 7640 \text{ lb/ft}$$

Problem 1)

For the following situation, considering the angle of wall friction between backfill and wall ($\delta$) is 20°, find the active earth pressure per foot of wall by Coulomb theory.




***Solution***

From Eqs. (10-10) and (10-16),

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \qquad (10\text{-}10)$$

$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2\alpha \sin(\alpha - \delta)\left[1 + \sqrt{\dfrac{\sin(\phi + \delta)\sin(\phi - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2} \qquad (10\text{-}16)$$

$\gamma = 120$ lb/ft³

$H = 20$ ft

$\alpha = 180° - 95° = 85°$

$\phi = 35°$

$\delta = 20°$

$\beta = 10°$

$$K_a = \frac{\sin^2(85° + 35°)}{\sin^2(85°)\sin(85° - 20°)\left[1 + \sqrt{\dfrac{\sin(35° + 20°)\sin(35° - 10°)}{\sin(85° - 20°)\sin(85° + 10°)}}\right]^2}$$

$K_a = 0.318$

$P_a = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(20 \text{ ft})^2(0.318) = 7630$ lb/ft

Problem 2)

Given:

1- A smooth vertical wall is 20 $ft$ high and retains a cohesionless soil with $\gamma = 120 \frac{lb}{ft3}$ and $\varphi = 28°$.
2- The top of the soil is horizontal and level with the top of the wall.
3- The soil surface carries a uniformly distributed load of $1000 \frac{lb}{ft^2}$.

Required:

1- Total active earth pressure on the wall per linear foot of wall.
2- Point of action of the total active earth pressure by Rankine theory.

***Solution***

From Eqs. (10-10) and (10-14) (for level backfill),

$$P_a = \tfrac{1}{2}\gamma H^2 K_a \tag{10-10}$$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \tag{10-14}$$

$$K_a = \frac{1 - \sin 28°}{1 + \sin 28°} = 0.361$$

$$P_a = (\tfrac{1}{2})(120 \text{ lb/ft}^3)(20 \text{ ft})^2(0.361) = 8660 \text{ lb/ft}$$

Point of action for $P_a = H/3 = 20 \text{ ft}/3 = 6.67$ ft from the base of the wall.
From Eq. (10-18),

$$P' = qHK_a \tag{10-18}$$

$$P' = (1000 \text{ lb/ft}^2)(20 \text{ ft})(0.361) = 7220 \text{ lb/ft}$$

Point of action for $P' = H/2 = 20 \text{ ft}/2 = 10$ ft from the base of the wall.

1. Total active earth pressure $= P_a + P' = 8660 \text{ lb/ft} + 7220 \text{ lb/ft}$ $= 15{,}880$ lb/ft.

2. Let the point of application of the total active earth pressure be "$h$" ft above the base of the wall. "$h$" is obtained by taking moments of forces (i.e., $P_a$ and $P'$) at the base of the wall.

$$(15{,}880 \text{ lb/ft})(h) = (8660 \text{ lb/ft})(6.67 \text{ ft}) + (7220 \text{ lb/ft})(10 \text{ ft})$$

$$h = 8.18 \text{ ft}$$

Hence, total active earth pressure acts at 8.18 ft above the base of the wall.