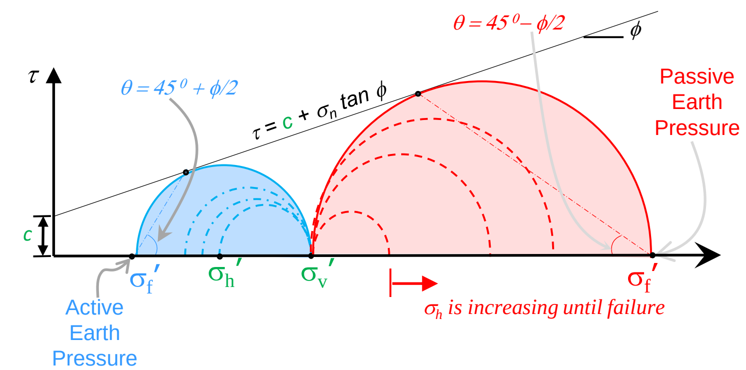
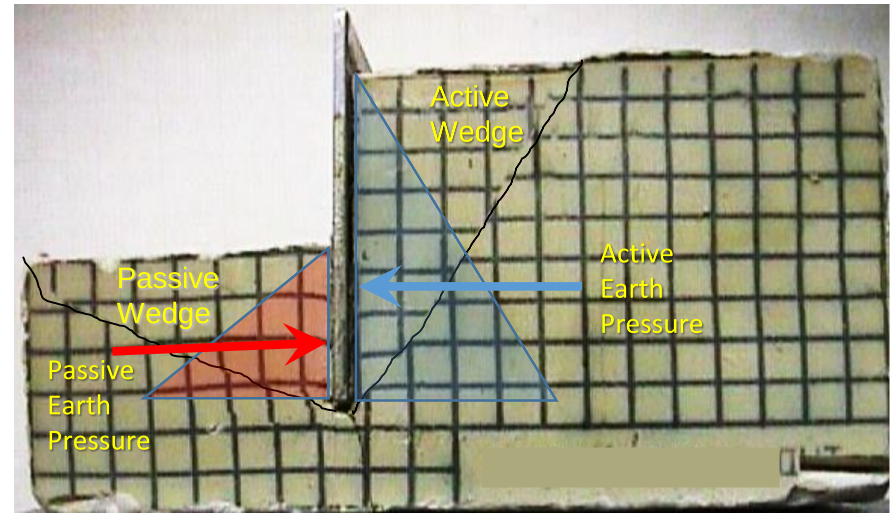
Study Guide
for
Homework # 4

Rankine's Earth Pressure Method for (c- ϕ) Soil

Rankine's Active and Passive Earth Pressure in (c- f) Soil





Rankine's Active Earth Pressure in (f) Soil with inclined backfill

$$K_a = \cos\beta \cdot \frac{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\phi)}}{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\phi)}}$$

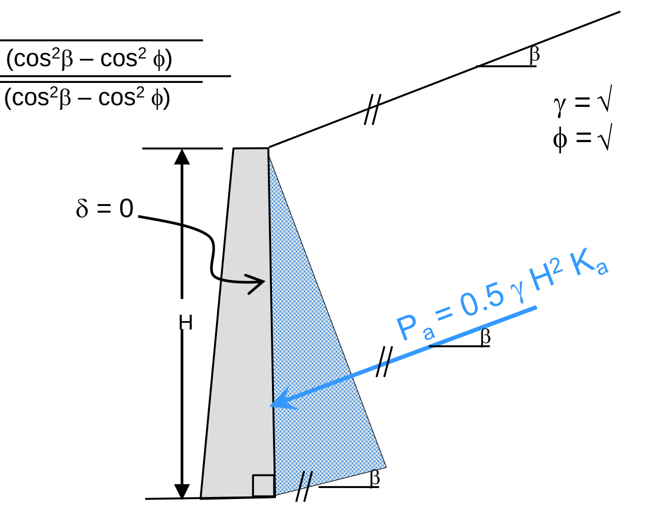


Fig.1

Active Earth Pressure

$$\sigma_f' = \sigma_V' \tan^2\left(45^o - \frac{\emptyset}{2}\right) + 2\,c\,\tan\left(45^o - \frac{\emptyset}{2}\right)$$

Or

$$\sigma_f' = \sigma_V' K_a - 2\,c\,\sqrt{K_a}$$

$$K_a = \tan^2\left(45^o - \frac{\emptyset}{2}\right) = \frac{1 - \sin\emptyset}{1 + \sin\emptyset}$$

Coefficient of active earth pressure

Passive Earth Pressure

$$\sigma_f' = \sigma_V' \tan^2\left(45^o + \frac{\emptyset}{2}\right) + 2\,c\,\tan\left(45^o + \frac{\emptyset}{2}\right)$$

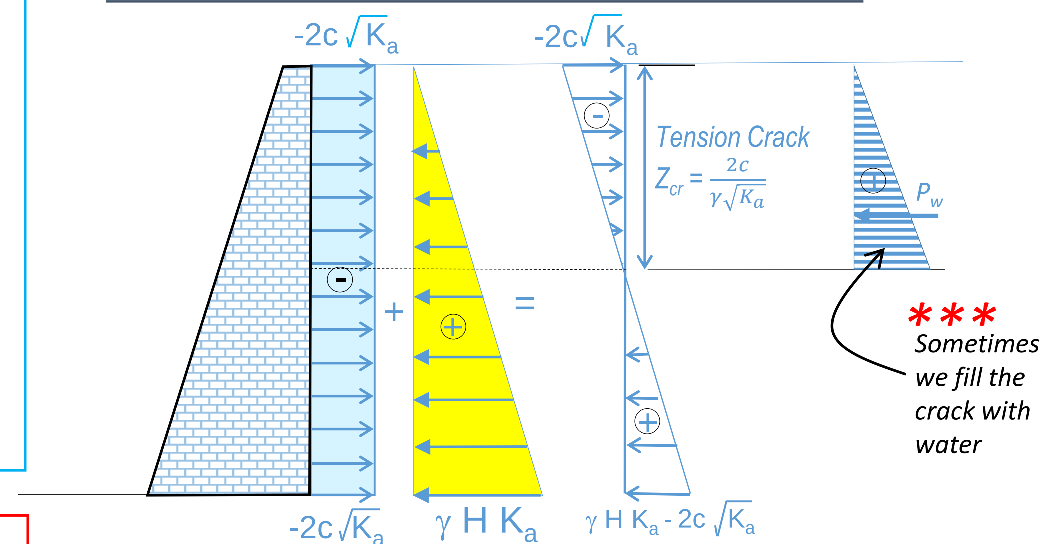
Or

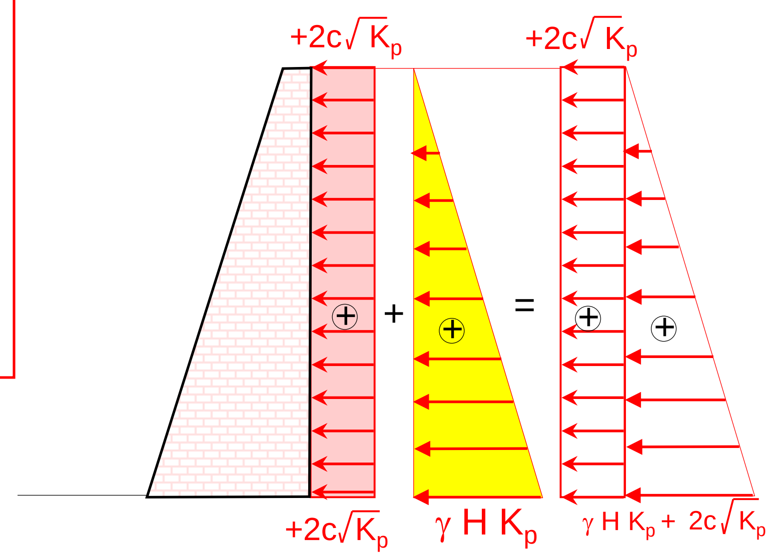
$$\sigma_f' = \sigma_V' K_p + 2\,c\,\sqrt{K_p}$$

$$K_p = \tan^2\left(45^o + \frac{\emptyset}{2}\right) = \frac{1 + \sin\emptyset}{1 - \sin\emptyset}$$

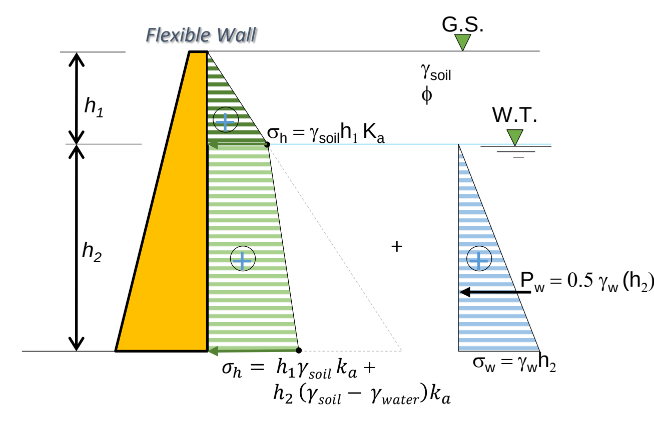
Coefficient of passive earth pressure

Effect of Cohesion of the Rankine's Active and Passive Earth Pressure

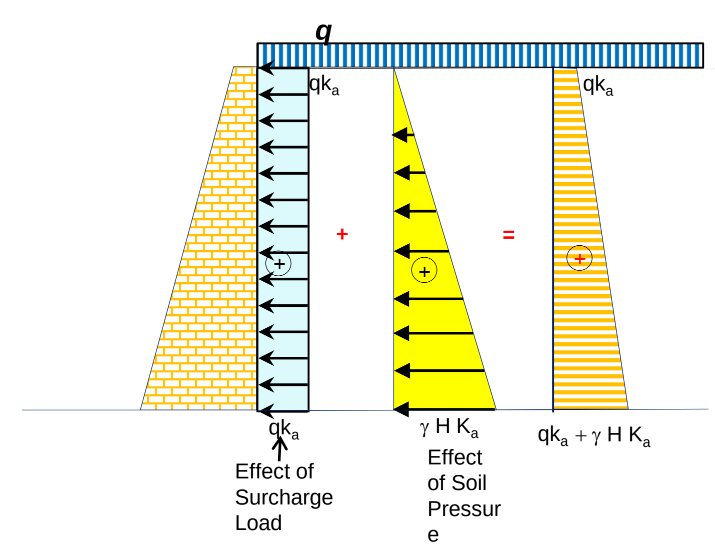




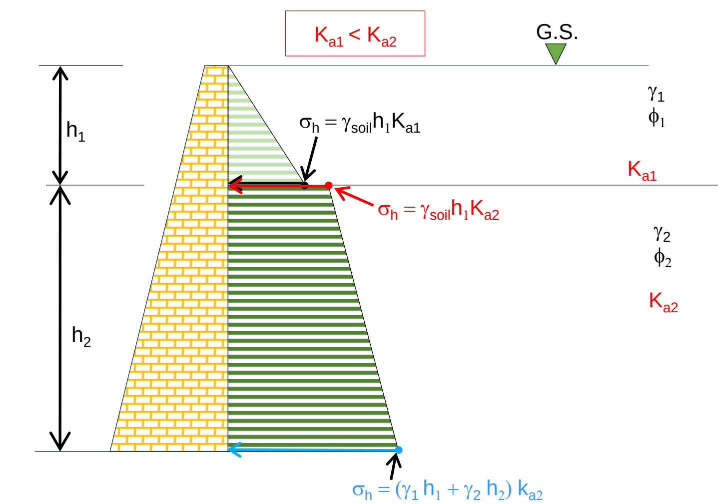
Rankine's Active Earth Pressure in f - Soil & Water Table

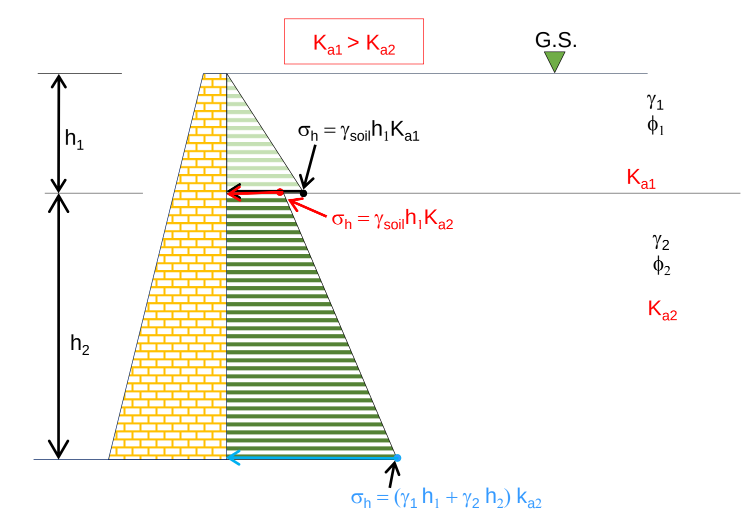


Effect of Surcharge (q) Load on Active Earth Pressure



Effect of Two Soil Layers on Active Earth Pressure





Kamal Tawfiq

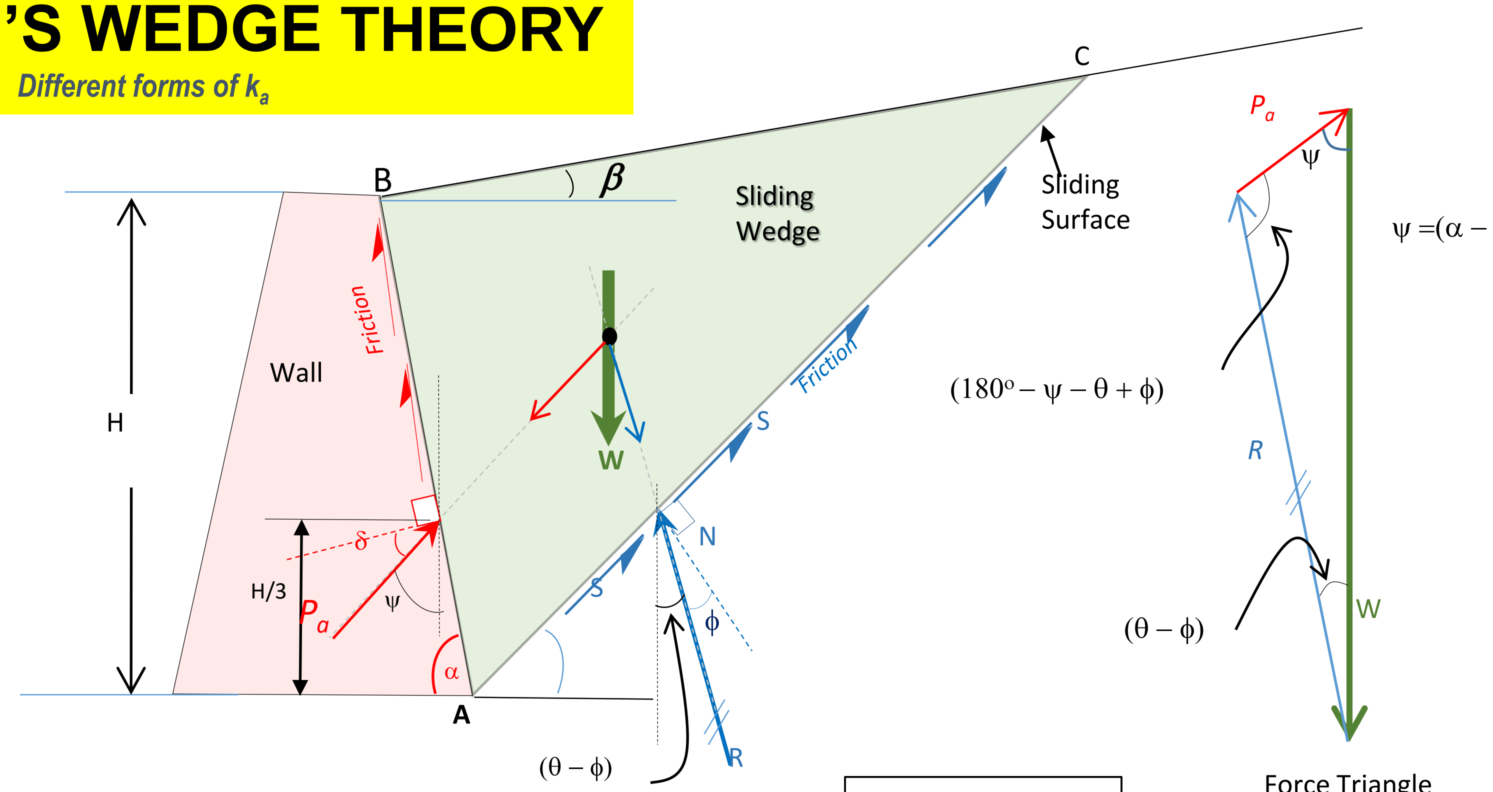
COULOMB'S WEDGE THEORY

Different forms of k_a

$$K_a = \frac{\sin^2(\alpha+\phi)}{\sin^2\alpha\ \sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$$

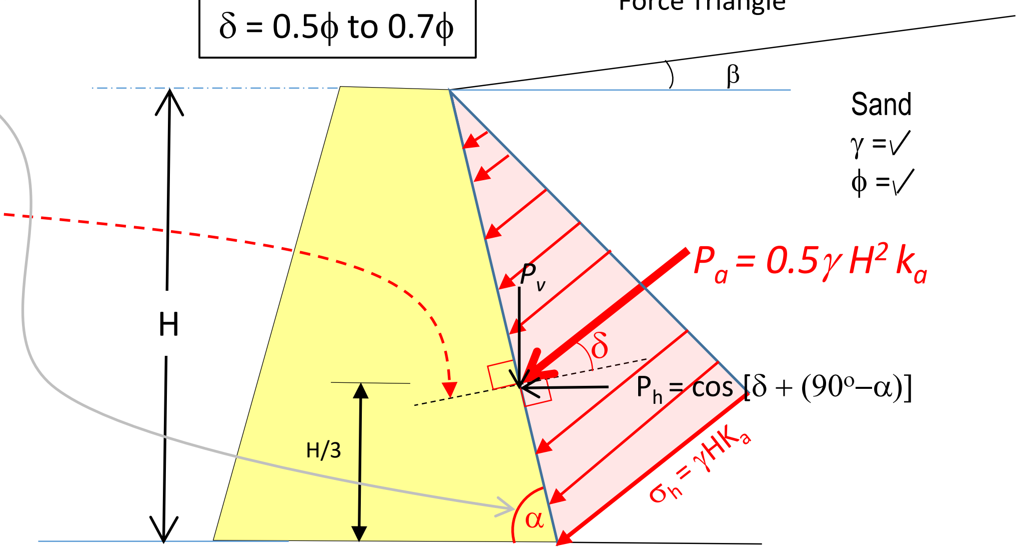
OR

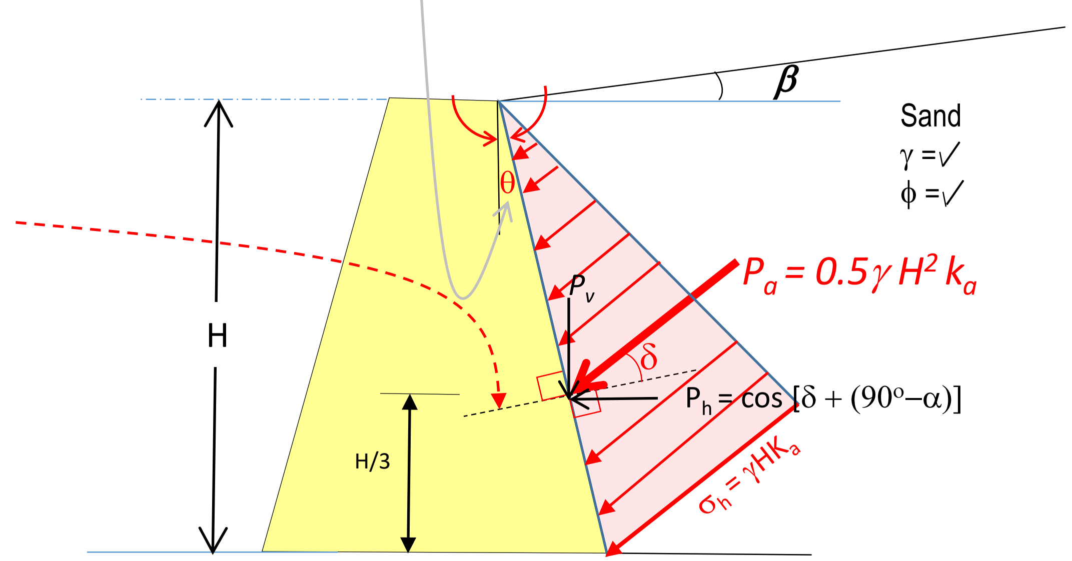
$$K_a = \frac{\cos^2(\emptyset-\theta)}{\cos^2\theta\ \cos(\delta+\theta)\left[1+\sqrt{\dfrac{\sin(\delta+\emptyset)\sin(\emptyset-\beta)}{\cos(\delta+\theta)\cos(\beta-\theta)}}\right]^2}$$

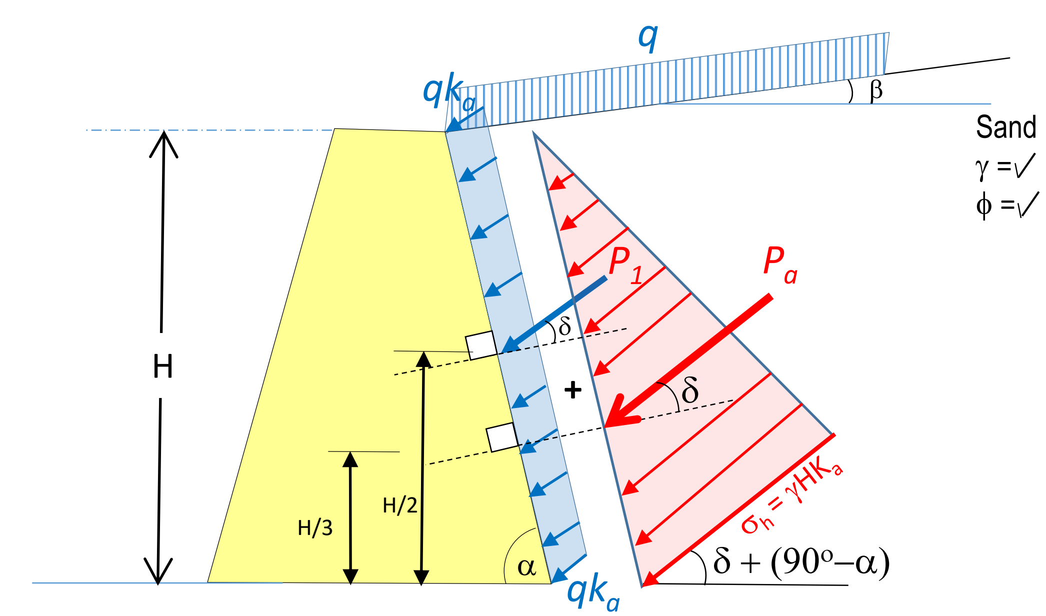


$\delta = 0.5\phi$ to 0.7ϕ

- Draw this perpendicular line first
- Then draw P_a with an angle = α

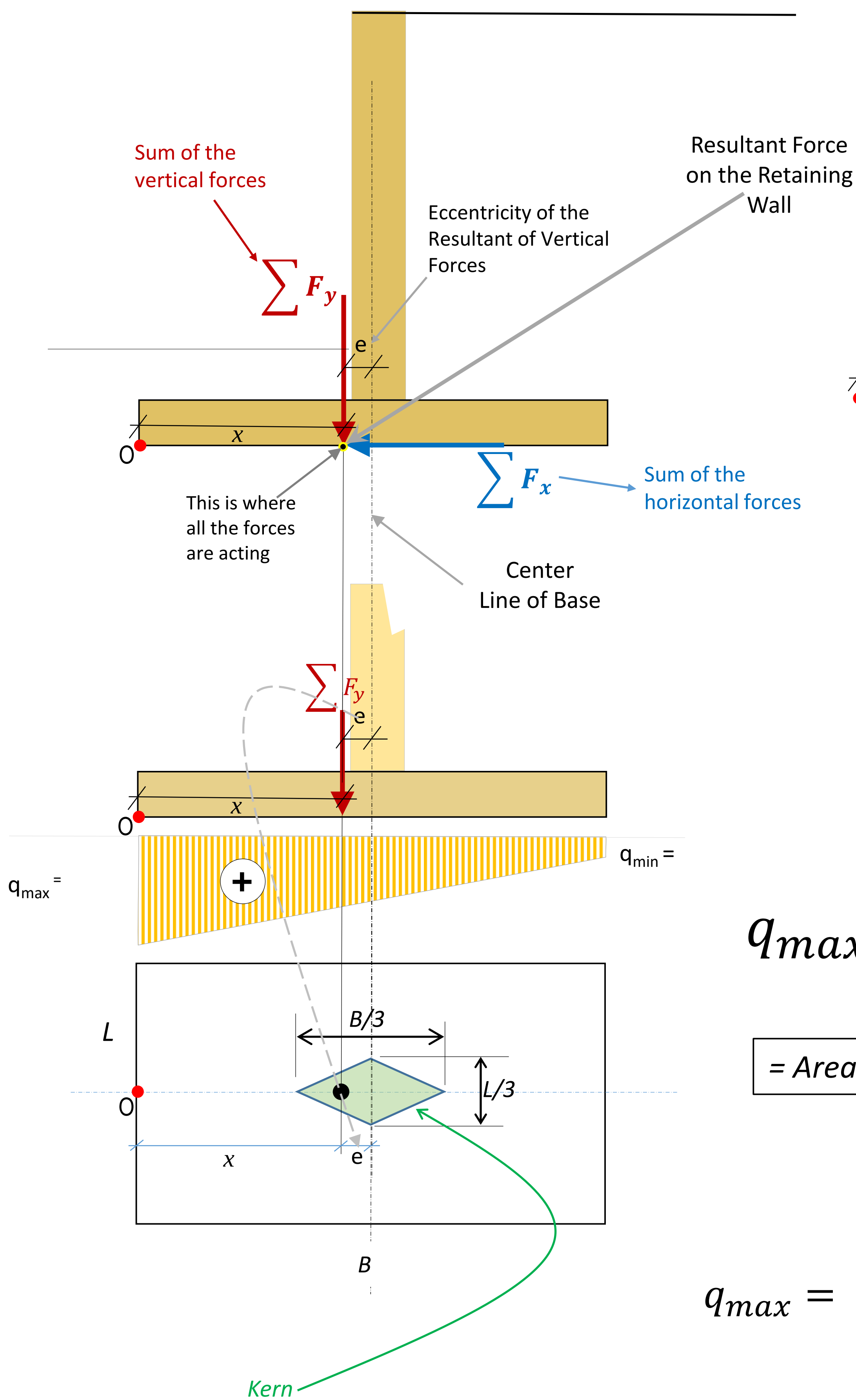




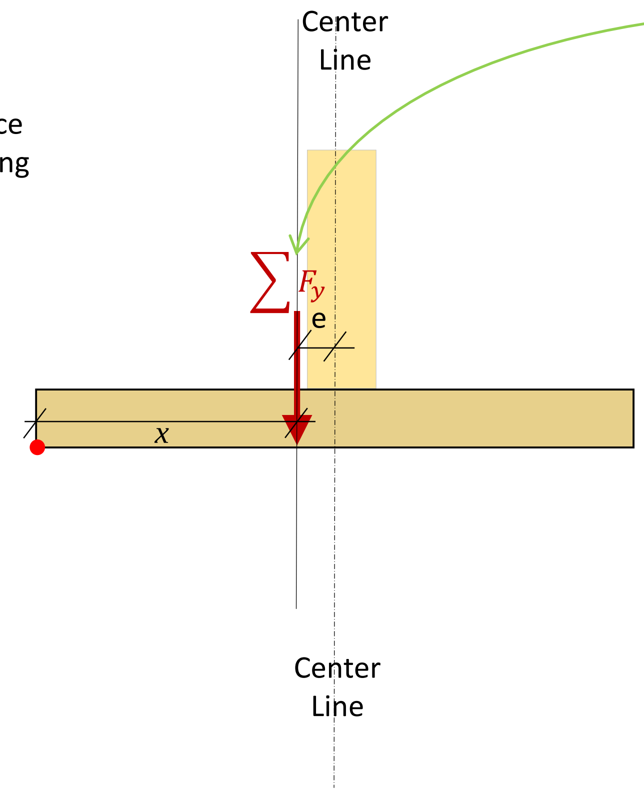


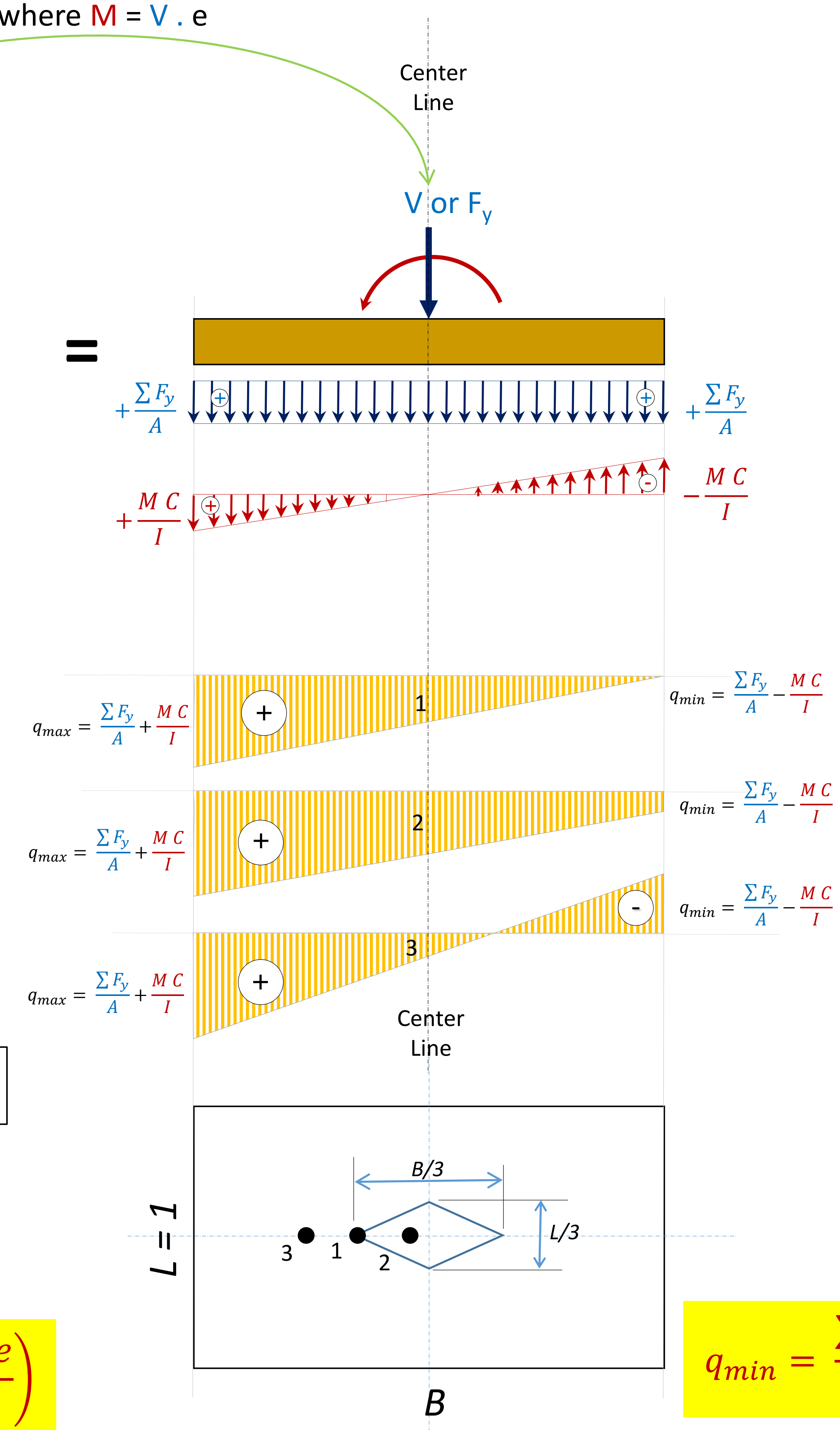
Kamal Tawfiq

Effect of Eccentricity on the Foundation



When you move the vertical force a distance =e
The you will end with a system that has V &M,
where M = V . e





$$q_{max} = \frac{\sum F_y}{A} + \frac{\overbrace{M}^{=\sum F_y . e}\ \overbrace{C}^{=B/2}}{I}$$

$= Area = B \times L$, $=1$

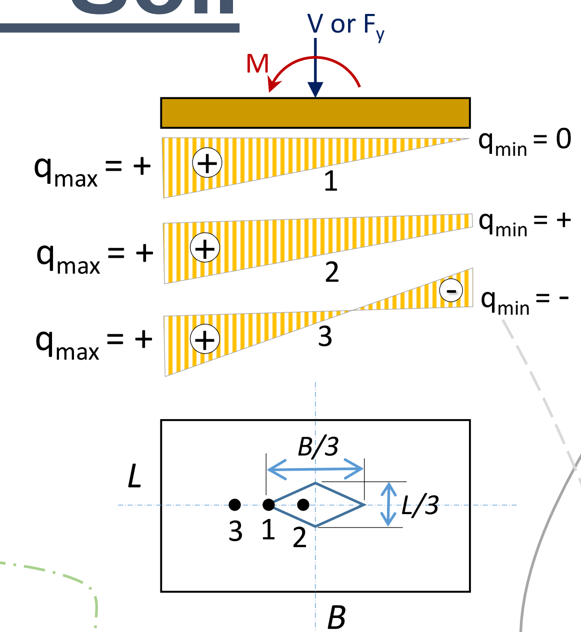
$Moment\ of\ Inertia = \frac{1\ x\ B^3}{12}$

$$q_{max} = \frac{\sum F_y}{B} + \frac{\sum F_y\,.e\,.\,0.5B\,.\cancel{12}}{1\ x\ \cancel{B^3}}$$

$$q_{max} = \frac{\sum F_y}{B}\left(1 + \frac{6\,e}{B}\right)$$

$$q_{min} = \frac{\sum F_y}{B}\left(1 - \frac{6\,e}{B}\right)$$

Active Earth Pressure in ϕ – Soil



Example -1

Given:
- Vertical retaining wall (Rigid)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30°
- Assume wall to be smooth
- Angle of friction between the base and the soil δ = 20°
- Bearing Capacity of the Foundation , q_{all} =3000 psf

Determine:
The stability of the wall

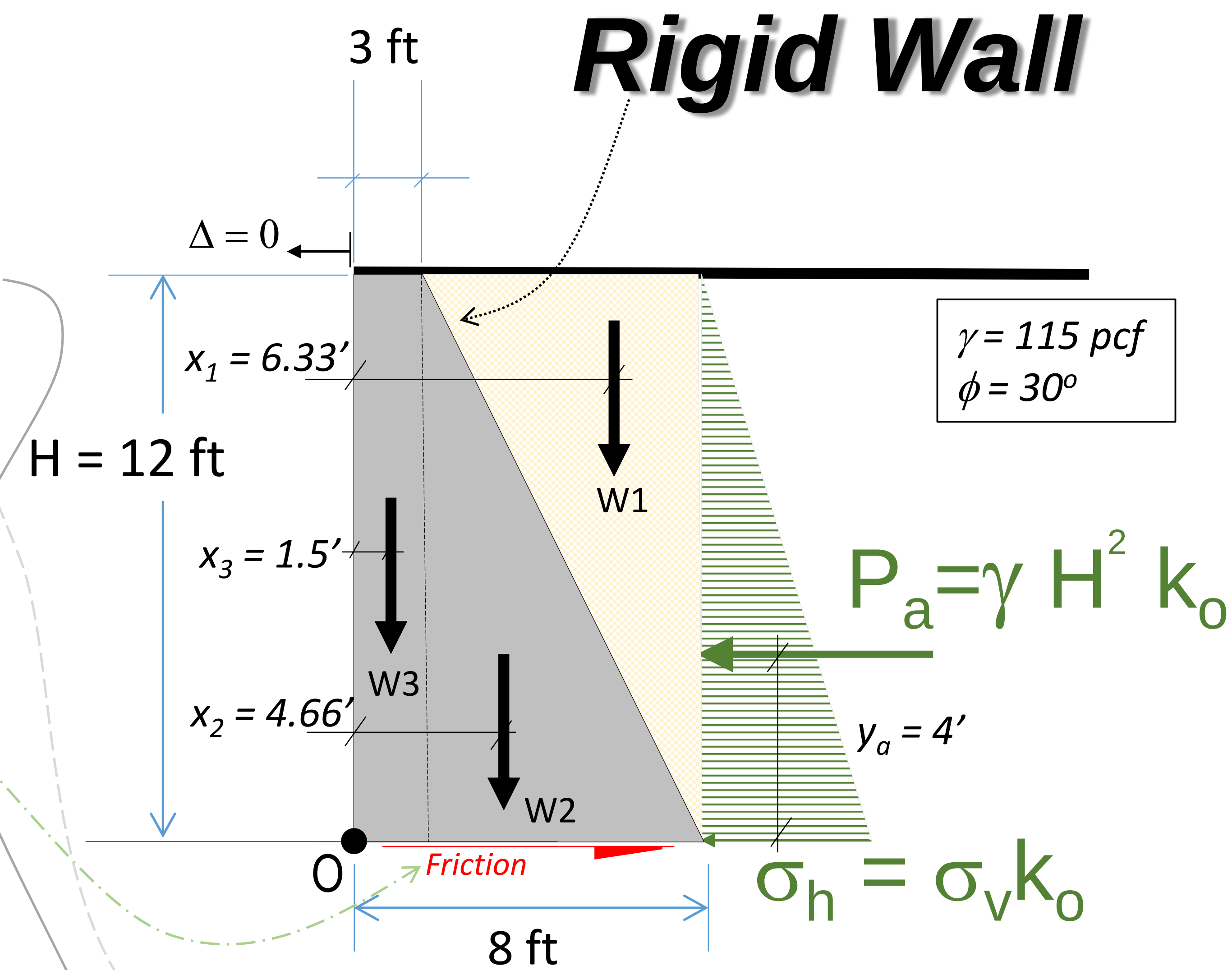
Solution:

$\sigma_h = \sigma_v\, k_o$ For Rigid Wall use k_o

$P_o = 0.5\,\gamma\ H^2\, k_o$

$K_o\ =\ 1 - \sin\phi$

$P_o = 0.5x115x12^2x0.5 = 4{,}140$ lb/ft



1- Factor of Safety Against Sliding

$$FS_{(sliding)} = \frac{\sum W \tan(20) + Pp}{P_a} \qquad P_p = 0$$

$$= \frac{[(13{,}350)\tan 20^o]}{4140} = 1.2 > 1.5\ \ Not\ OK$$

2- Factor of Safety Against Overturning

$$FS_{(overturning)} = \frac{\sum M_R}{M_D} = \frac{50908.5}{16560} = 3.1 > 1.5\ \ Ok$$

3- Factor of Safety Bearing Capacity Failure = $FS_{(BC)}$

$M_{net} = \sum M_R - \sum M_D$ = **50,908.5** -**16,560** = 34,348.5 ft.lb/ft

M_{net} = 34348.5 = $\sum F_y(X)$ = 13350 (X)

$X = (M_{net} / \sum F_y)$ = 2.57 ft

e = (8/2) – 2.57 = 1.43 ft < B/6 or 8/6 = 1.33 (Partial contact Case 3)

Case 3 : is not acceptable

No need to determine q_{max} and q_{min}

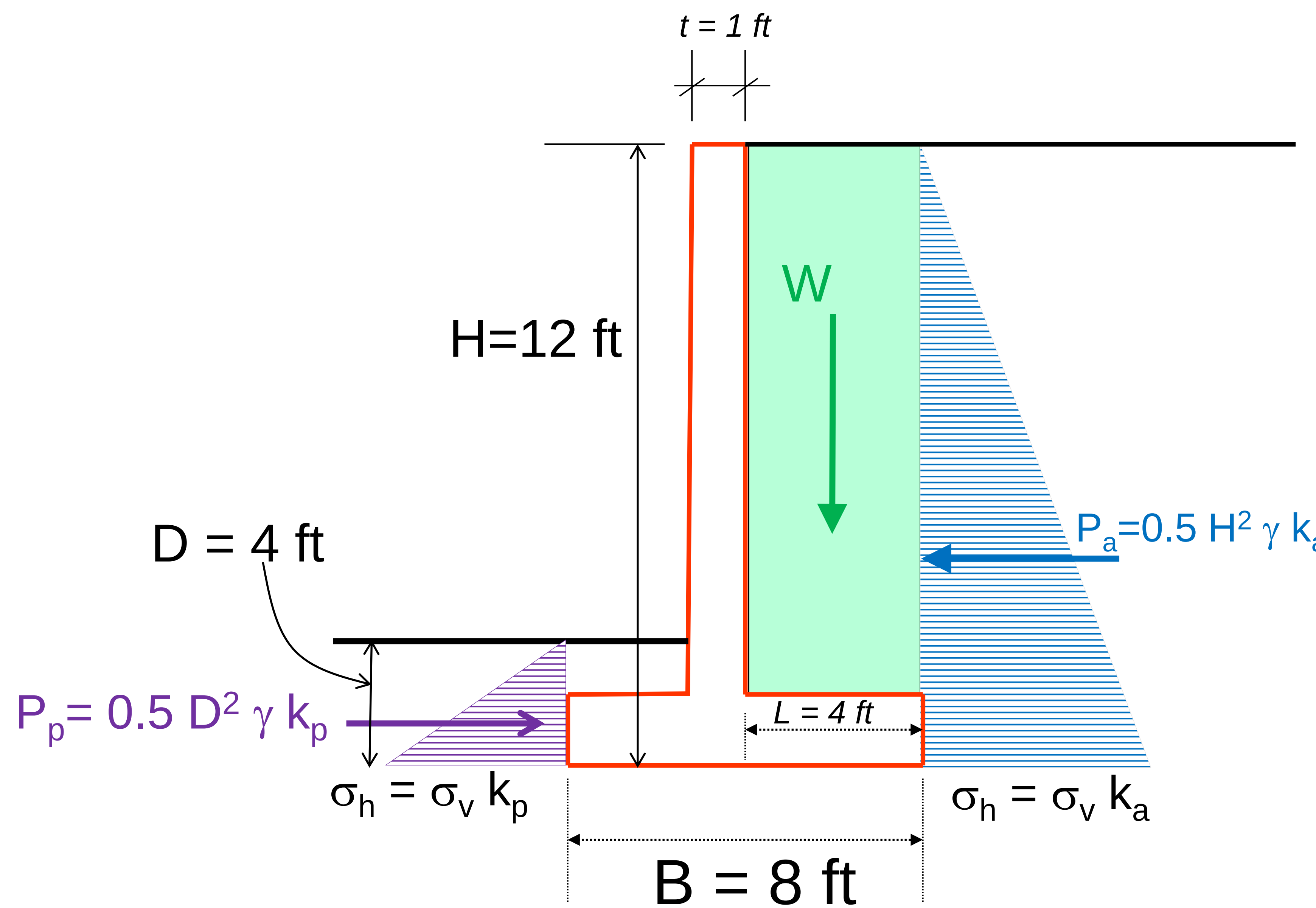
X - Force (lb)/ft)	Vertical Distance (ft)	$F_y = \sum W$ (lb)/ft	Moment Arm X (ft)	Driving Moment (ft.lb)/ft	Resisting Moment (ft.lb)/ft
P_a= 4,140	4			4,140 x 4 = 16,560	
		W_1= 0.5x5x12x115 = 3,450	6.33		3,450x6.33=21,838.5
		W_2 =0.5x5x12x150 = 4,500	4.66		4,500x4.66 = 20,970
		W_3 = 3x12x150 = 5400	1.5		5,400 x 1.5 = 8,100
P_p = 0	0				o
		13,350		**16,560**	**50,908.5**

Active & Passive Earth Pressure in ϕ – Soil

Example -2

Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30°
- Assume wall to be smooth
- $\gamma_{concrete}$ = 150 lb/ft^3
- D = 4 ft

Find:
- Resultant Force of the Wall



Solution:

$\sigma_h = \sigma_v k_a$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \qquad K_p = \frac{1 + \sin\phi}{1 - \sin\phi}$$

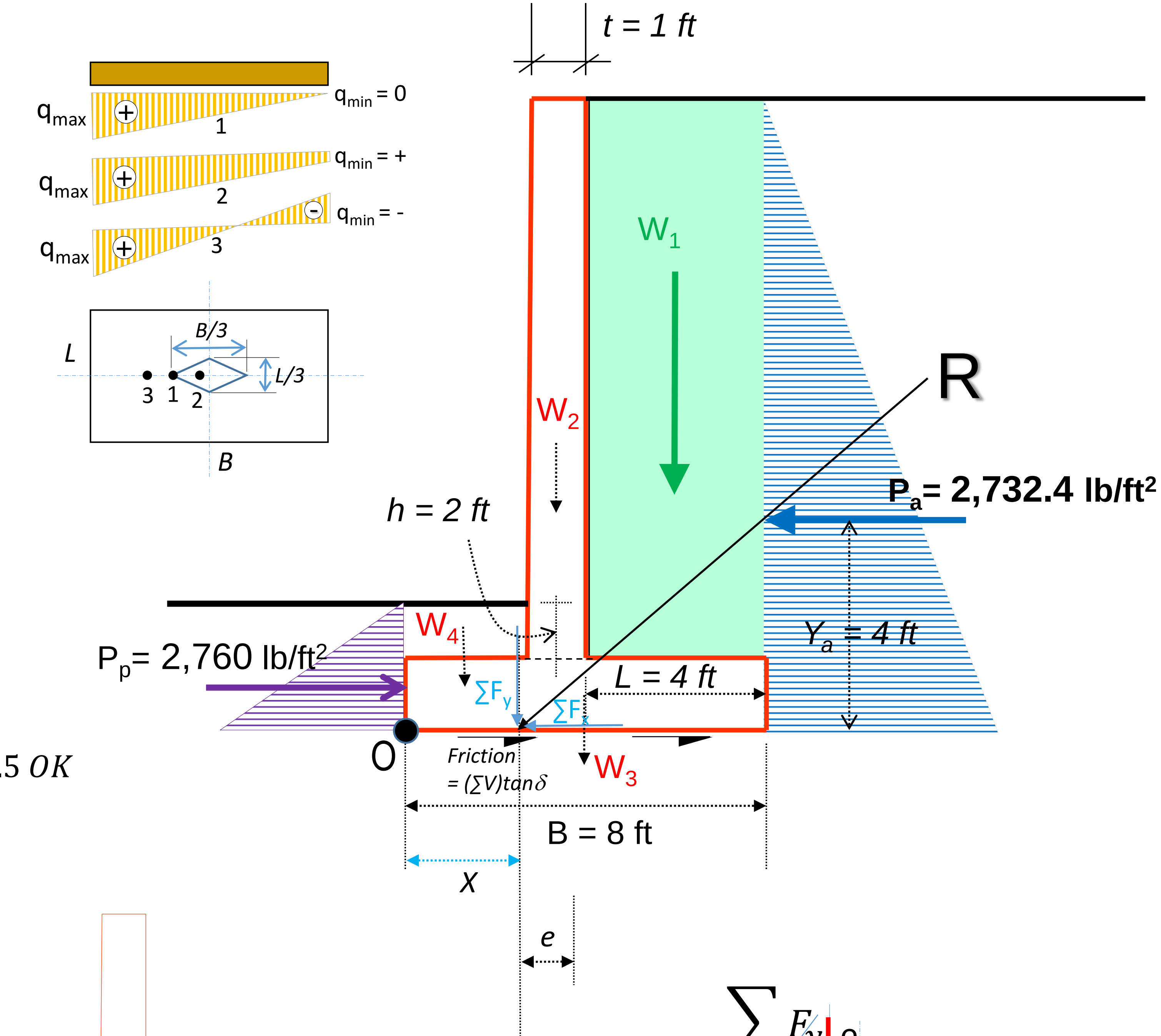
$P_a = 0.5\,\gamma\; H^2 k_a$

$P_a = 0.5 \times 12^2 \times 115 \times 0.33 = 2{,}732.4$ lb/ft^2

$P_p = 0.5\, D^2\, \gamma\, k_p$

$P_p = 0.5 \times 4^2 \times 115 \times 3 = 2{,}760$ lb/ft^2

F_x (lb)/ft	Y (ft)	F_y (lb)/ft	Moment Arm X (ft)	Driving Moment (ft.lb)/ft	Resisting M (ft.lb)/ft
P_a = 2,732.4	4			2,732.4x4 = 10,929.6	
		W_1= 4x10x115 = 4,600	6		4600x6=27,600
		W_2 =1x10x150 = 1,500	3.5		1,500x3.5=5,250
		W_3 = 8x1x150=1,200	4		1,200x4=4,800
		W_4 =2x3x115 = 690	1.5		690x1.5= 1,035
P_p= 2,760	1.33				2,760x1.33= 3,680
		7,990		10,929.6	42,365



1- Factor of Safety Against Sliding = $FS_{(sliding)}$ = $\frac{\sum F_y \tan(20)+Pp}{P_a} = \frac{[(7{,}990)\tan 20^o]+2{,}760}{2{,}732.4} = 2.1 > 1.5\ OK$

δ δ

2- Factor of Safety Against Overturning = $FS_{(overturning)}$ = $\frac{\sum M_{Resisting}}{M_{Driving}} = \frac{42{,}365}{10{,}929.6} = 3.8 > 1.5\ Ok$

3- Factor of Safety Bearing Capacity Failure = $FS_{(BC)}$

$M_{net} = \sum M_r - \sum M_o$ = 42,365 -10,929.6 = 31,435.4 ft.lb/ft

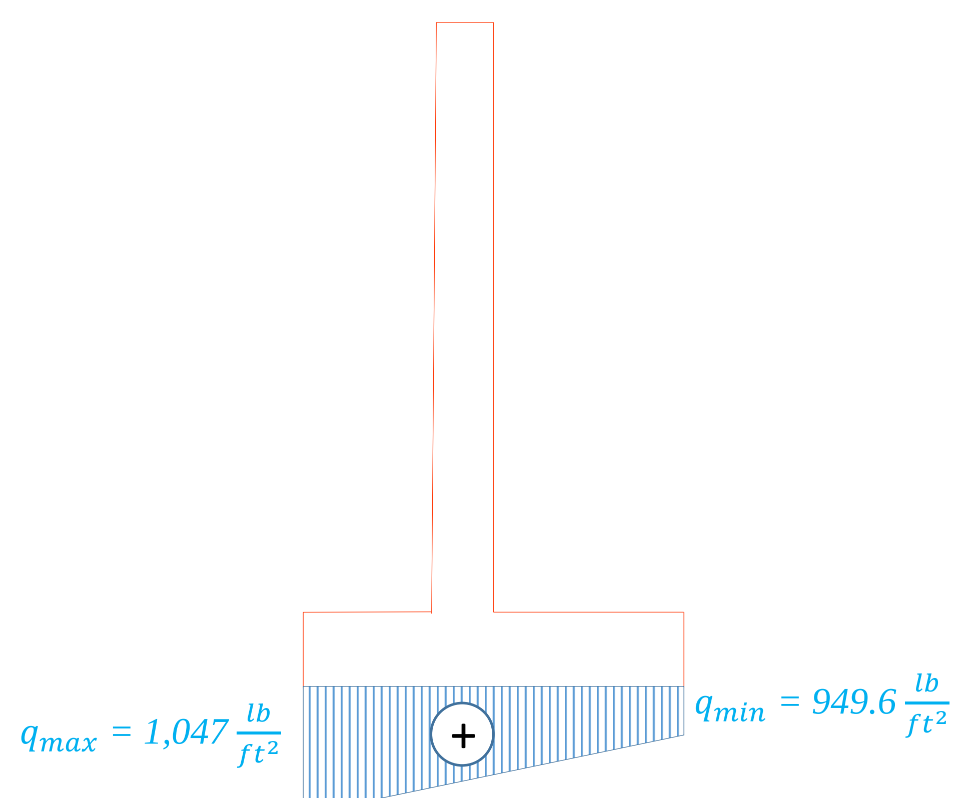
M_{net} = 31,435.4 = $\sum F_y(X)$ = 7,990.(X)

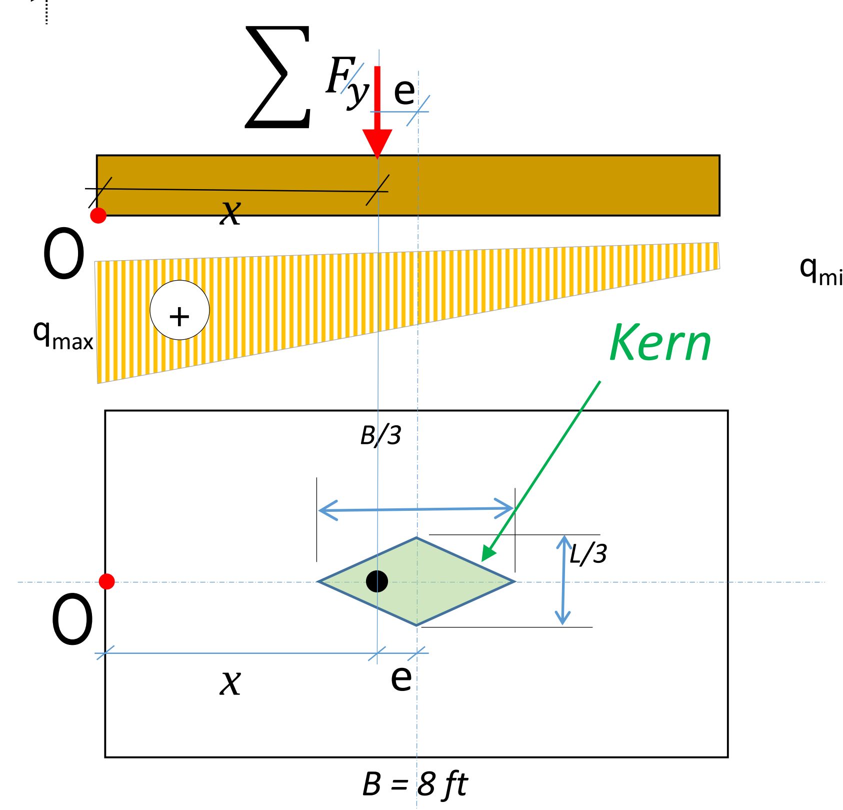
$X = (M_{net} / \sum F_y)$ = 3.93 ft

e = (8/2) – 3.93 = 0.0656 ft < B/6 or 8/6 = 1.33 (Full contact)

$$q_{max} = \frac{\sum F_y}{B}\left(1+\frac{6\,e}{B}\right) = \frac{7{,}990}{8}\left(1+\frac{(6)(0.0656)}{8}\right) = 1{,}047.8\,\frac{lb}{ft^2} < \quad q_{all} = 3{,}000\,\frac{lb}{ft^2}$$

$$q_{min} = \frac{\sum F_y}{B}\left(1-\frac{6\,e}{B}\right) = \frac{7{,}990}{8}\left(1-\frac{(6)(0.0656)}{8}\right) = 949.6\,\frac{lb}{ft^2}$$





Example 3

Given

The cross section of a cantilever retaining wall is shown in Figure 1. Calculate the factors of safety with respect to overturning, sliding, and bearing capacity.

Solution

From the figure,

$H^* = H_1 + H_2 + T_1 = 8 \tan 10° + 19.5 + 2 = 22.91$ ft

The Rankine active force per unit length of wall =

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}} \qquad K_a = \cos 10 \frac{\cos 10 - \sqrt{\cos^2 10 - \cos^2 30}}{\cos 10 + \sqrt{\cos^2 10 - \cos^2 30}} = 0.34$$

$$P_a = \text{Lateral Pressure from Surcharge} + \text{Lateral Pressure from Soil}$$

$$P_{a1} = q H^* k_a$$

$$P_{a2} = \frac{1}{2}\gamma H^{*2} k_a$$

$$P_{a1} = 120x22.9x0.34 = 934.32\, lb/ft$$

$$P_{v1} = 934.32 \sin(10^\circ) = 162.24\ lb/ft$$

$$P_{h1} = 934.32 \cos(10^\circ) = 920.13\ lb/ft$$

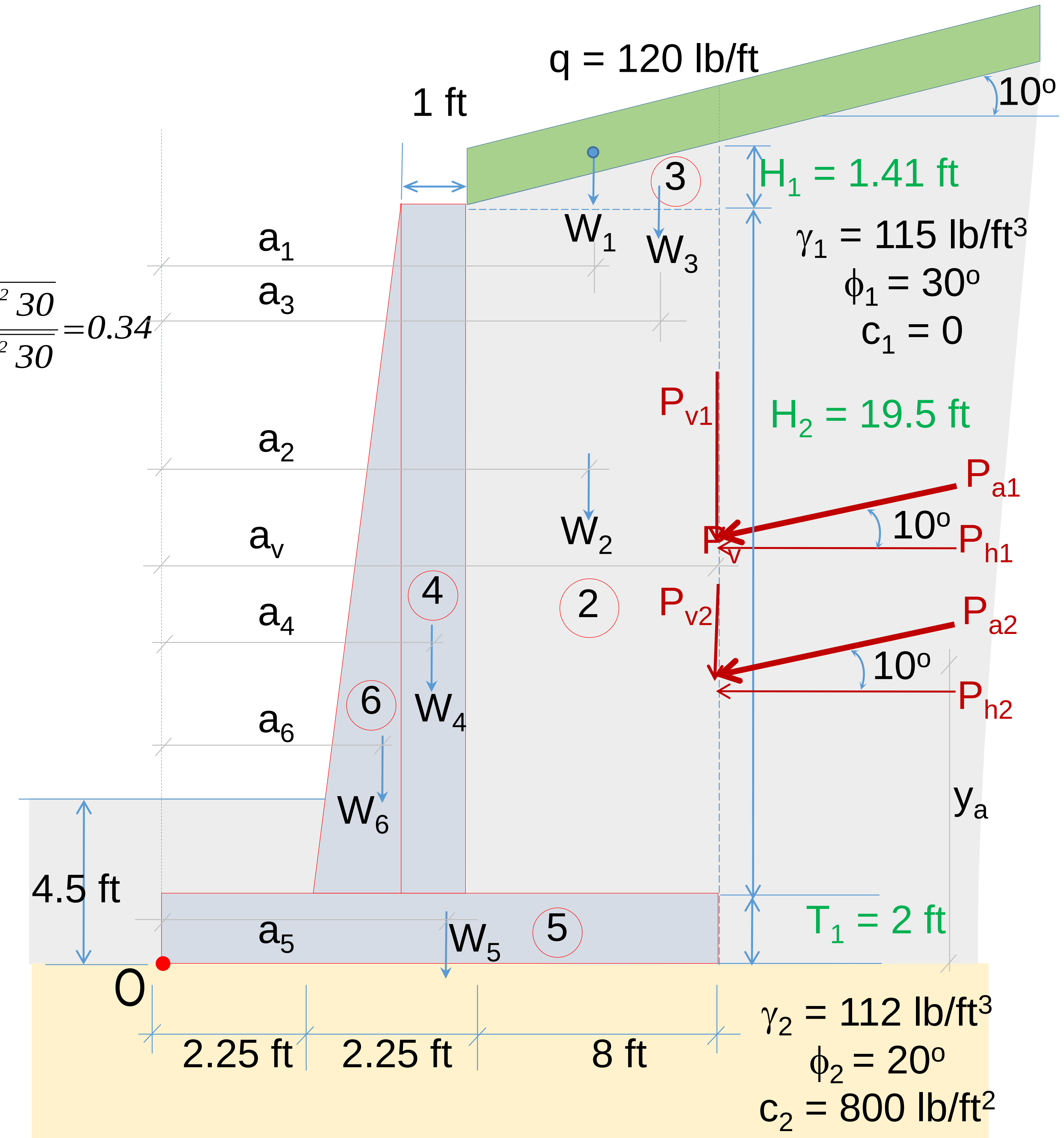
$$Y_{a1} = \frac{22.9}{2} = 11.45\ ft$$

$$P_{a2} = 0.5x115x22.9^2\,x0.34 = 10252.22 = 11186.54\, lb/ft$$

$$P_{v2} = 10{,}252.2 \sin(10^\circ) = 1780.28\ lb/ft$$

$$P_{h2} = 10{,}252.2 \cos(10^\circ) = 10096.45\ lb/ft$$

$$Y_{a1} = \frac{22.9}{3} = 7.63\ ft$$



Driving Pressure $(lb/ft^2)/ft$	Resisting Pressure $(lb/ft^2)/ft$	Weight/Unit Length (lb/ft)	Moment Arm from Point O (ft)	Driving Moment (ft.lb/ft)	Resisting Moment (ft.lb/ft)
P_{h1} = 920.13			11.45	920.13x11.45 = 10535.49	
P_{h2} = 10096.45			7.64	10096.45x7.64 = 77136.88	
		W_1 = 120 x 8 = 960	8.5		960x8.5 = 8,160
		W_2 = 8X19.2X115 = 17,664	8.5		17,664x8.5 = 150,144
		W_3 = 0.5X1.41X8X115 = 648.6	9.83		648.6x9.83 = 6,375.74
		W_4 = 1X19.5X150 = 2,925	4.0		2925x4 = 11,700
		W_5 = 12.5X2X150 = 3,750	6.25		3750x6.25 = 23,437.5
		W_6 = 0.5X1.25X19.2X150 = 1,800	3.08		1800x3.08 = 5,544
		∑Pv = 162.24+1780.28 = 1,942.52	12.5		1942.52x12.5 = 24,281.5
$\sum P_h$ = 11,016.58	0	$\sum F_y$ = 29,690.12		$\sum M_D$ = 87,672.37	$\sum M_R$ = 229,642.74

1- Factor of Safety Against Sliding = $FS_{(sliding)} = \frac{\sum F_y \tan(10) + C + Pp}{P_a} = \frac{[(29{,}690.12)\tan 10^o] + [800x12.5] + 0}{11{,}016.58} = \frac{5235.17 + 10000 + 0}{11{,}016.58} = 1.38 < 1.5 \;\; Not\ OK$

Add Passive Resistance

P_p = 0.5 x 4.5^2 x 115 x 3 = 3493.13 lb/ft

2- Factor of Safety Against Sliding = $FS_{(sliding)} = \frac{\sum F_y \tan(10) + C + P_p}{P_a} = \frac{[(29{,}690.12.4)\tan 10^o] + [8x12.5] + 3493.13}{11{,}016.58} = \frac{5235.17 + 10000 + 3493.13}{11{,}016.58} = 1.7 < 1.5 \; OK$

3- Factor of Safety Against Overturning = $FS_{(overturning)} = \frac{\sum M_R}{\sum M_D} = \frac{229{,}642.74}{87{,}672.37} = 2.6 > 1.5 \qquad OK$

4- Factor of Safety Bearing Capacity Failure = $FS_{(BC)}$ =

$M_{net} = \sum M_R - \sum M_D = 225{,}559.86 - 87{,}672.37 = 137{,}887.5$ ft.lb/ft

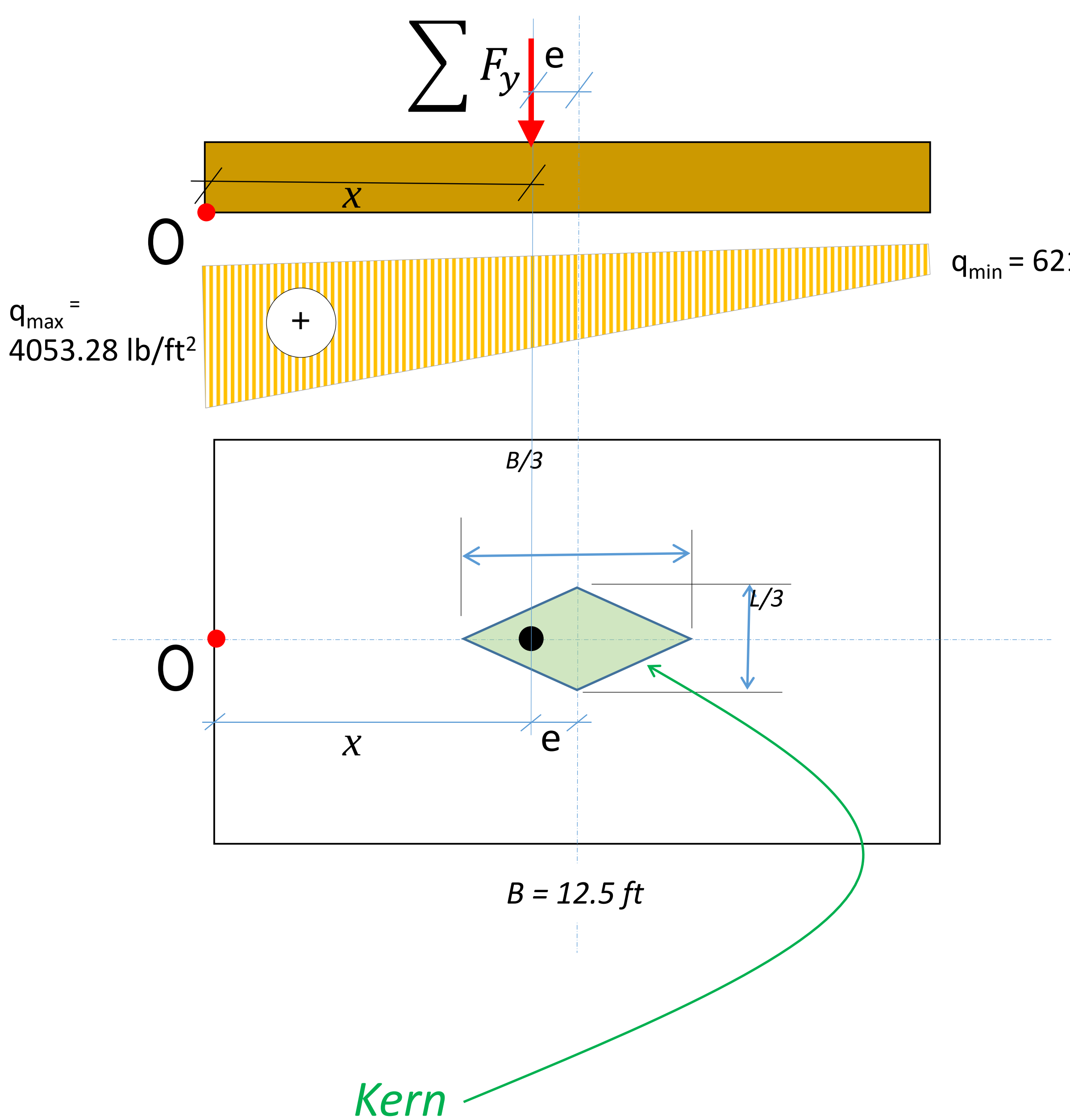
$M_{net} = 137{,}887.5 = \sum F_y(X) = 29{,}212.4 \ (X)$

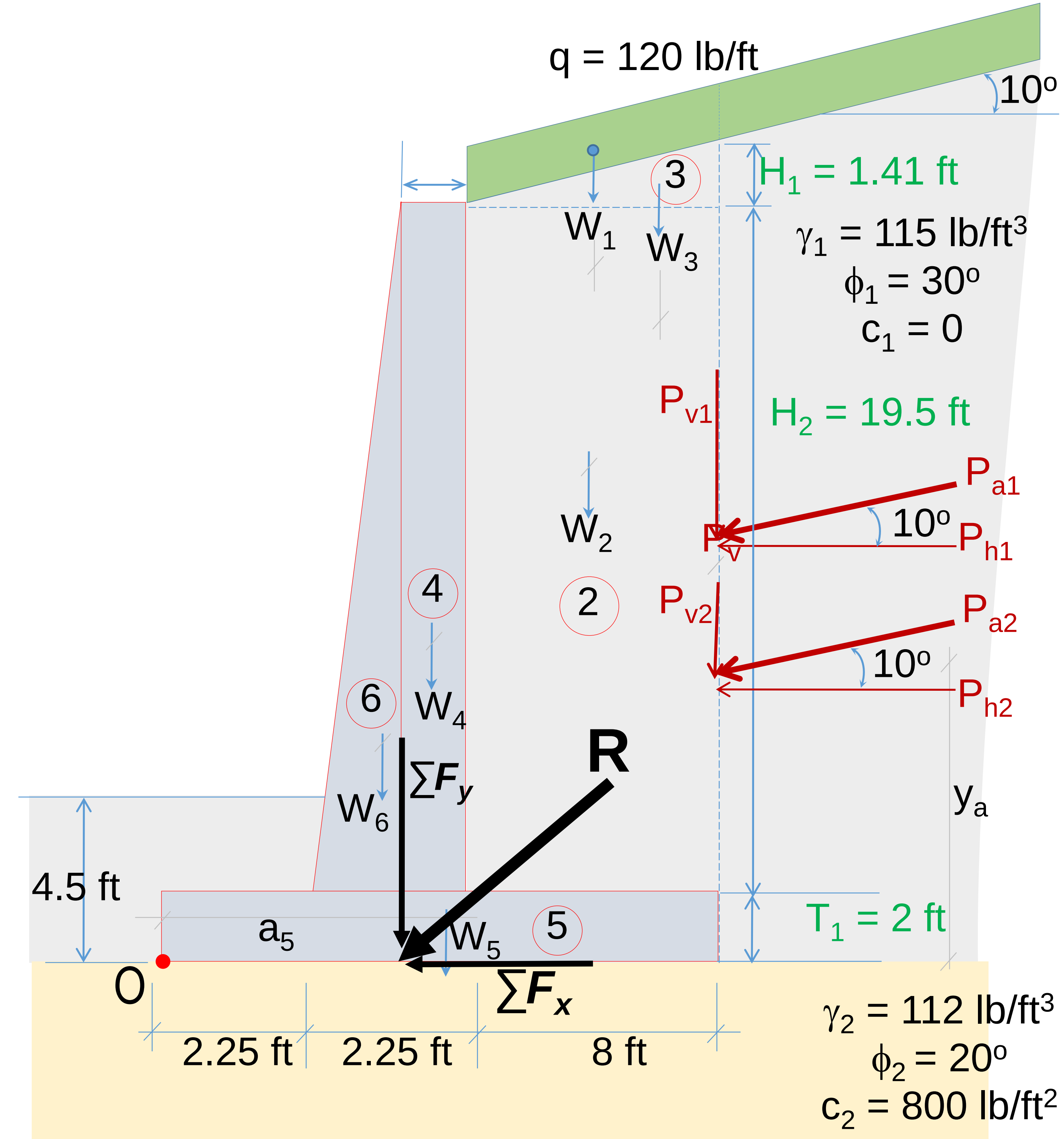
$X = (M_{net} / \sum F_y) = 4.72$ ft

e = (12.5/2) – 4.72 = 1.53 ft < B/6 or 12.5/6 = 2.083 (Full contact)

$$q_{max} = \frac{\sum F_y}{B}\left(1+\frac{6\,e}{B}\right) = \frac{29{,}212.4}{12.5}\left(1+\frac{(6)(1.53)}{12.5}\right) = 4053.28\,\frac{lb}{ft^2} > \quad q_{all} = 3{,}000\,\frac{lb}{ft^2} \qquad \text{No good}$$

$$q_{min} = \frac{\sum F_y}{B}\left(1-\frac{6\,e}{B}\right) = \frac{29{,}212.4}{12.5}\left(1-\frac{(6)(1.53)}{12.5}\right) = 621\,\frac{lb}{ft^2}$$





Example 4

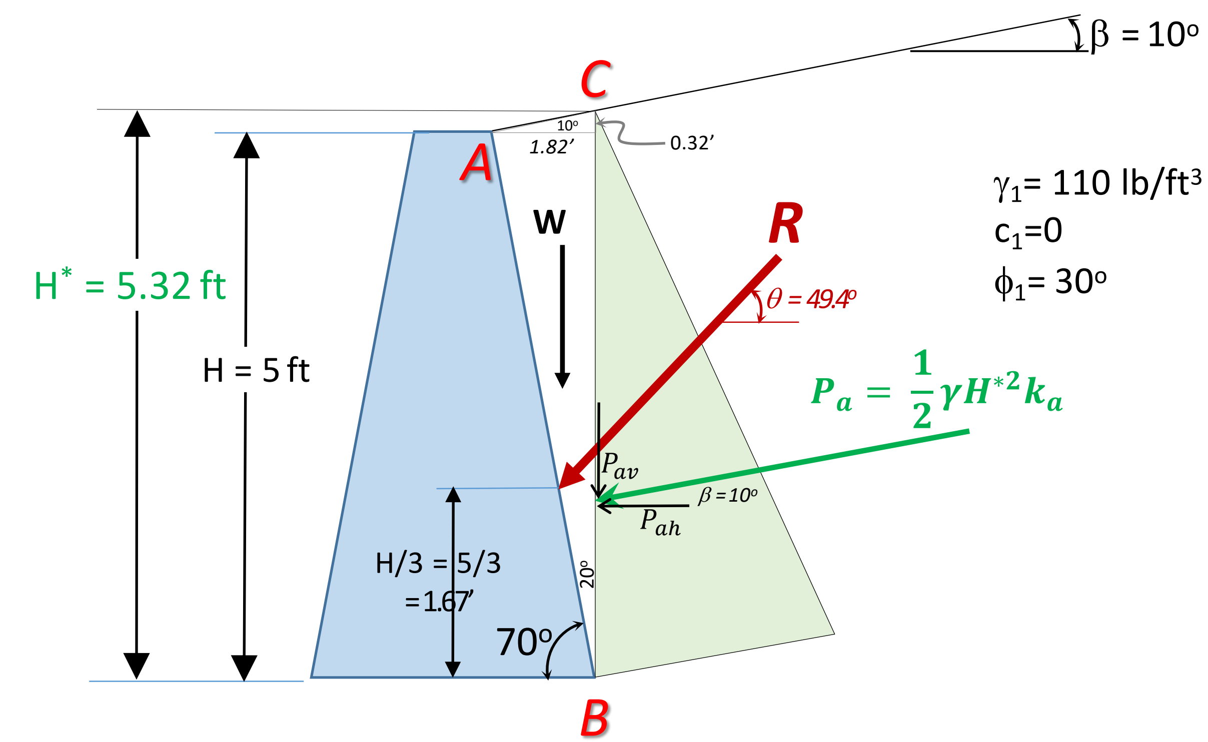
Given:
The cross section of a retaining wall as shown in Figure 4.

Find:
Calculate the earth pressure using Rankine's and Coulomb Earth Pressure Methods

Solution:

1- Using Rankine's Method

Because of the slope in the backfill



$$k_a = \cos\beta . \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} = \cos 10 . \frac{\cos 10 - \sqrt{\cos^2 10 - \cos^2 30}}{\cos 10 + \sqrt{\cos^2 10 - \cos^2 30}} = \mathbf{0.35}$$

H* = 5'+0.32' = 5.32'

$$P_a = \frac{1}{2}\gamma H^{*2} k_a = 0.5 \ x \ 110 \ x \ 5.32^2 \ x \ 0.35 = 544.82 \ \frac{lb}{ft} \begin{cases} P_{av} = 544.82 \sin 10^o = 94.61 \ lbft \\ P_{ah} = 544.82 \cos 10^o = 536.54 \text{ lb/ft} \end{cases}$$

W = Area ABC x γ_{soil} = 0.5 x 5.32 x 1.82 x 110 = 532.53 $\frac{lb}{ft}$

$$R = \sqrt{(P_{av} + W)^2 + P_{ah}^2} = \sqrt{(94.61 + 532.53)^2 + (536.54)^2} = 825.34 \frac{lb}{ft}$$

$$\theta = \tan^{-1}\left[\frac{(P_{av} + W)}{P_{ah}}\right] = \tan^{-1}\left[\frac{627.14}{536.54}\right] = 49.4^o$$

H/3 = 5/3 = 1.67'

Example: Clayey – Sand (SC)

Active Earth Pressure in c- ϕ Soil (Using Rankine's Method)

Rankine's Method always assumes δ = 0

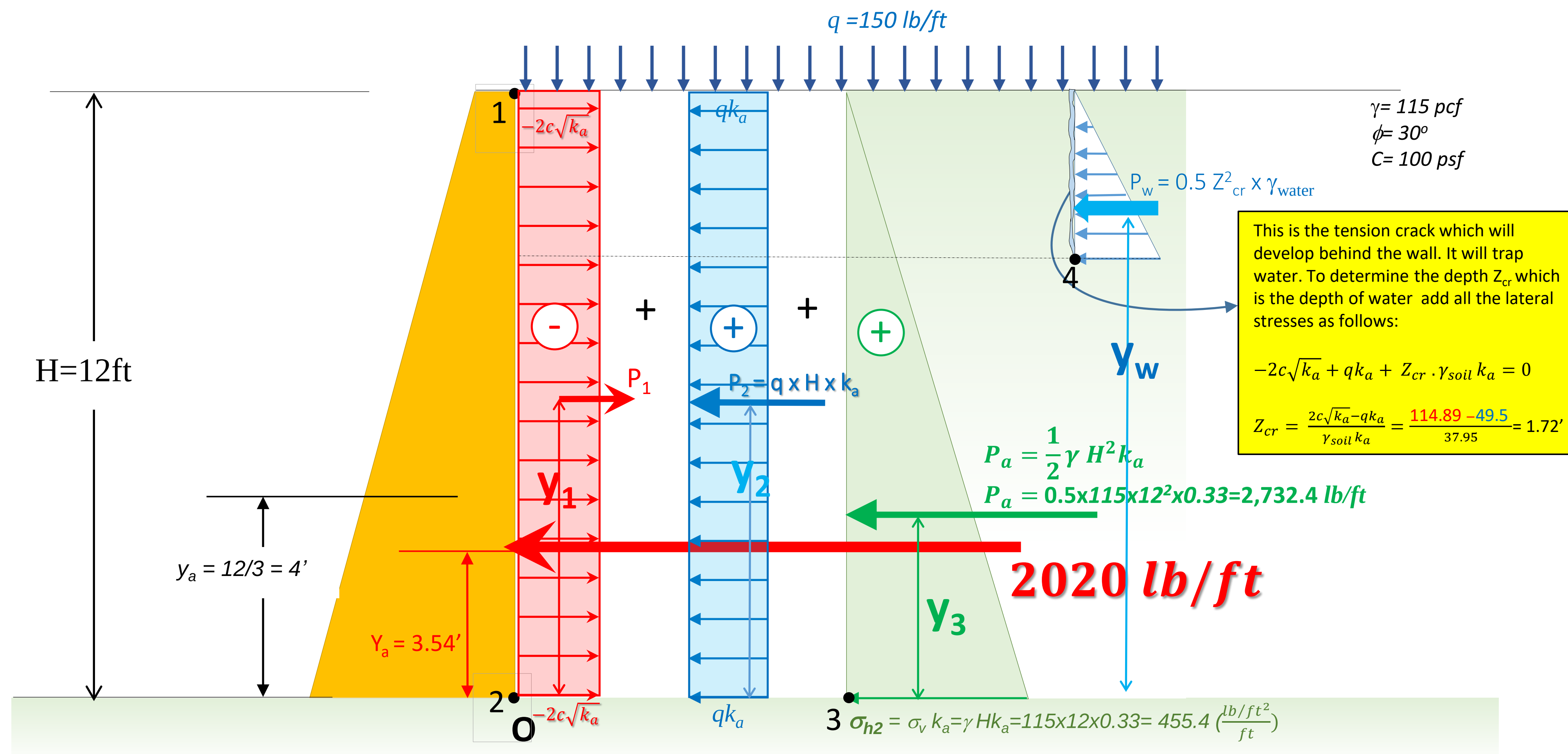
Example - 5

Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30^o
- Soil Cohesion c = 100 lb/ft^2
- Assume wall to be smooth (δ=0)

Find:
- Lateral force P_a acting on the wall

Solution:

$$k_a = \frac{1-sin\emptyset}{1+sin\emptyset} = \frac{1-sin30}{1+sin30} = 0.33$$



Point	Vertical Stress σ_v ($\frac{lb/ft^2}{ft}$)	Horizontal Stress σ_h ($\frac{lb/ft^2}{ft}$)
1	0	-2c$\sqrt{k_a} = -2(100)\sqrt{0.33}$ = -114.89
2	0	-2c$\sqrt{k_a} = -2(100)\sqrt{0.33}$ =-114.89
1	q =150	qk_a = 150 x 0.33 = 49.5
2	q = 150	qk_a= 150 x 0.33 = 49.5
3	γH= 115x12	γHk_a = 115x12x0.33 = 455.4
4	Z_{cr} x γ_{water} =1.72x62.4	P_w = 0.5 (1.72)^2 x 62.4 = 92.30

To determine the resultant Force and its point of action

Lateral Force P (lb/ft)	Distance to O (ft)
P_1 = -114.89 x 12 = - 1,378.7	y_1 = 12/2= 6
P_2 = 49.5 x 12 = 594	y_2 = 12/2= 6
P_a = 0.5 x 455.4 x 12 = 2,732.4	y_3 = 12/3 = 4
P_w= 92.30	Y_w = 10.85
R = $\sum P$ = 2040	Take Moment about Point O = $\sum M_o$ = -1,378.7x6 + 594x6 +2732.4 x (12/3)+ 92.3x10.85= 7222.88 ft-lb $y_a = \frac{\sum M_o}{\sum P} = \frac{7222.88}{2040} = 3.54ft$