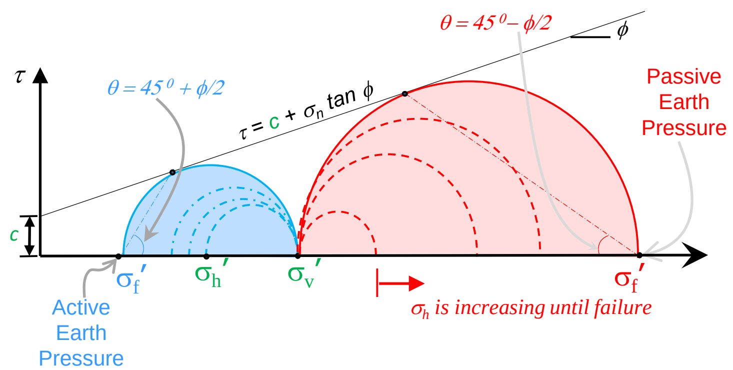
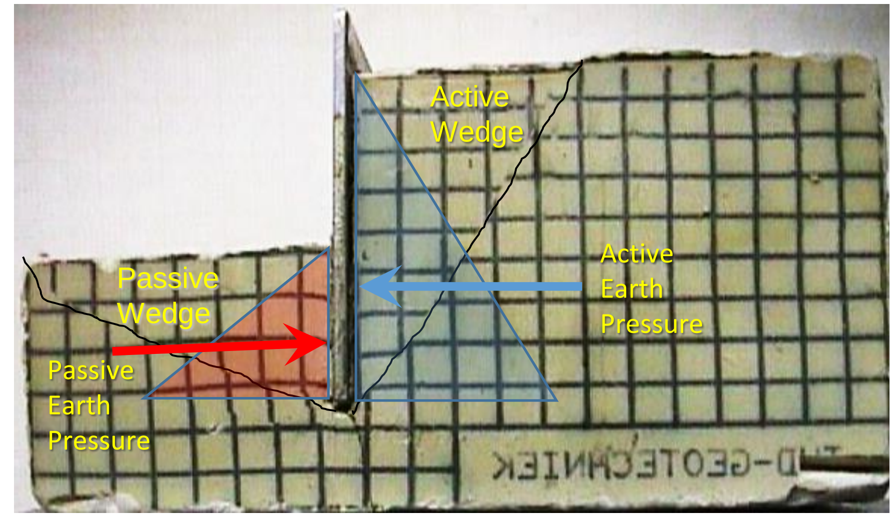
Rankine's Earth Pressure Method for (c- ϕ) Soil

Rankine's Active and Passive Earth Pressure in (c- f) Soil





Rankine's Active Earth Pressure in (f) Soil with inclined backfill

$$K_a = \cos\beta \cdot \frac{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\phi)}}{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\phi)}}$$

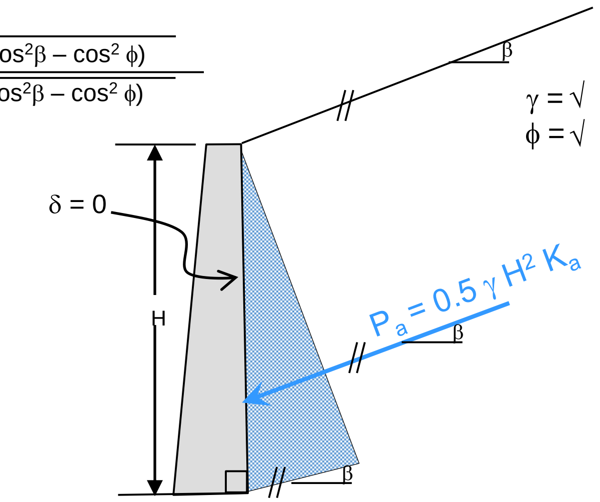


Fig.1

Active Earth Pressure

$$\sigma'_f = \sigma'_V \tan^2\left(45^o - \frac{\emptyset}{2}\right) + 2\,c\,\tan\left(45^o - \frac{\emptyset}{2}\right)$$

Or

$$\sigma'_f = \sigma'_V K_a - 2\,c\sqrt{K_a}$$

$$K_a = \tan^2\left(45^o - \frac{\emptyset}{2}\right) = \frac{1 - \sin\emptyset}{1 + \sin\emptyset}$$

Coefficient of active earth pressure

Passive Earth Pressure

$$\sigma'_f = \sigma'_V \tan^2\left(45^o + \frac{\emptyset}{2}\right) + 2\,c\,\tan\left(45^o + \frac{\emptyset}{2}\right)$$

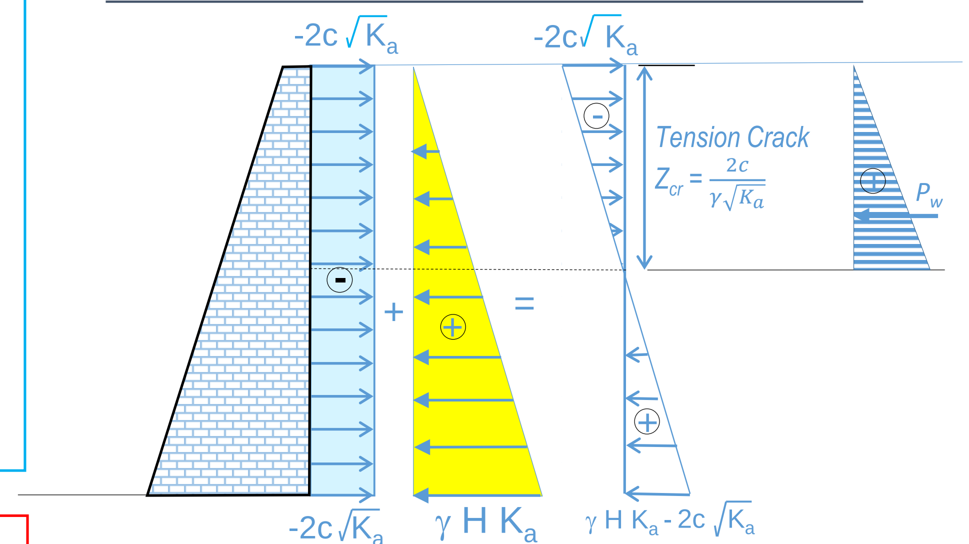
Or

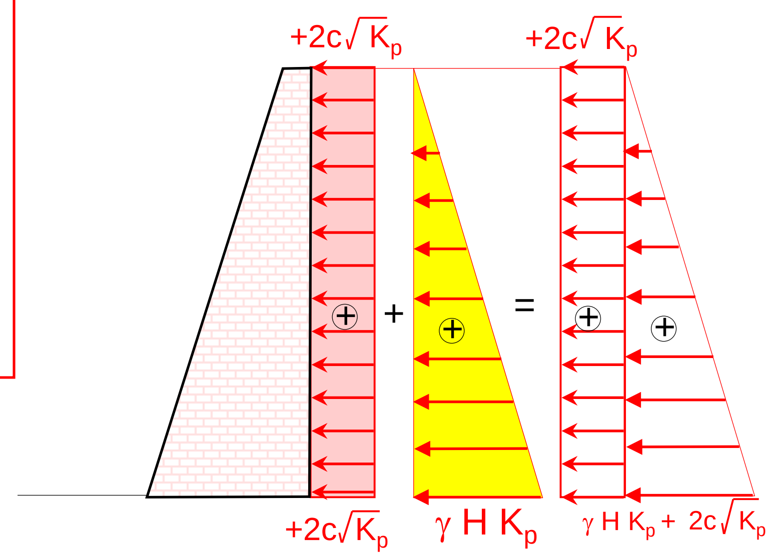
$$\sigma'_f = \sigma'_V K_p + 2\,c\sqrt{K_p}$$

$$K_p = \tan^2\left(45^o + \frac{\emptyset}{2}\right) = \frac{1 + \sin\emptyset}{1 - \sin\emptyset}$$

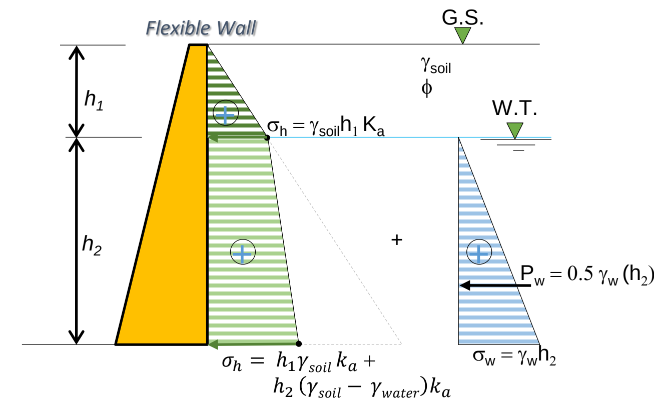
Coefficient of passive earth pressure

Effect of Cohesion of the Rankine's Active and Passive Earth Pressure

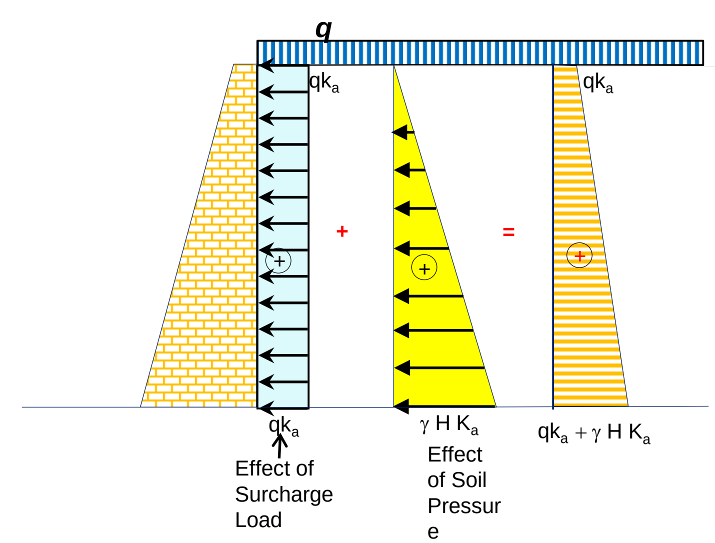




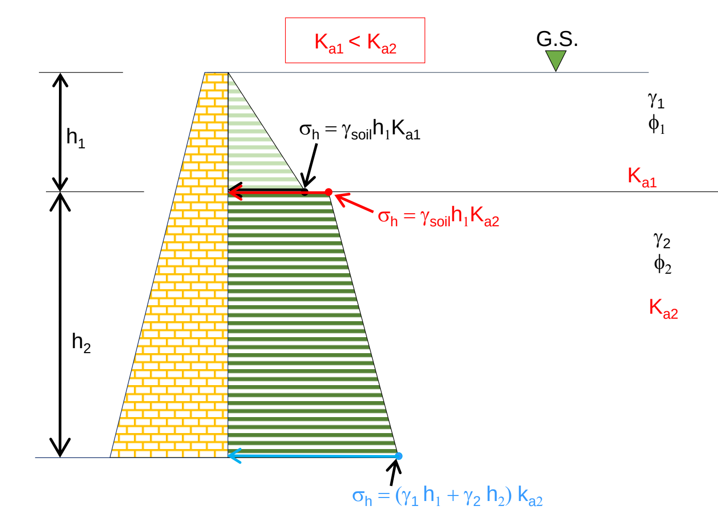
Rankine's Active Earth Pressure in f - Soil & Water Table

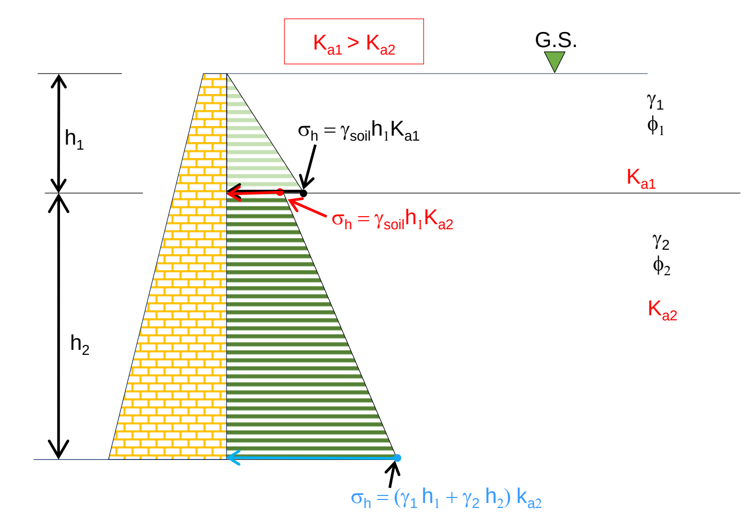


Effect of Surcharge (q) Load on Active Earth Pressure



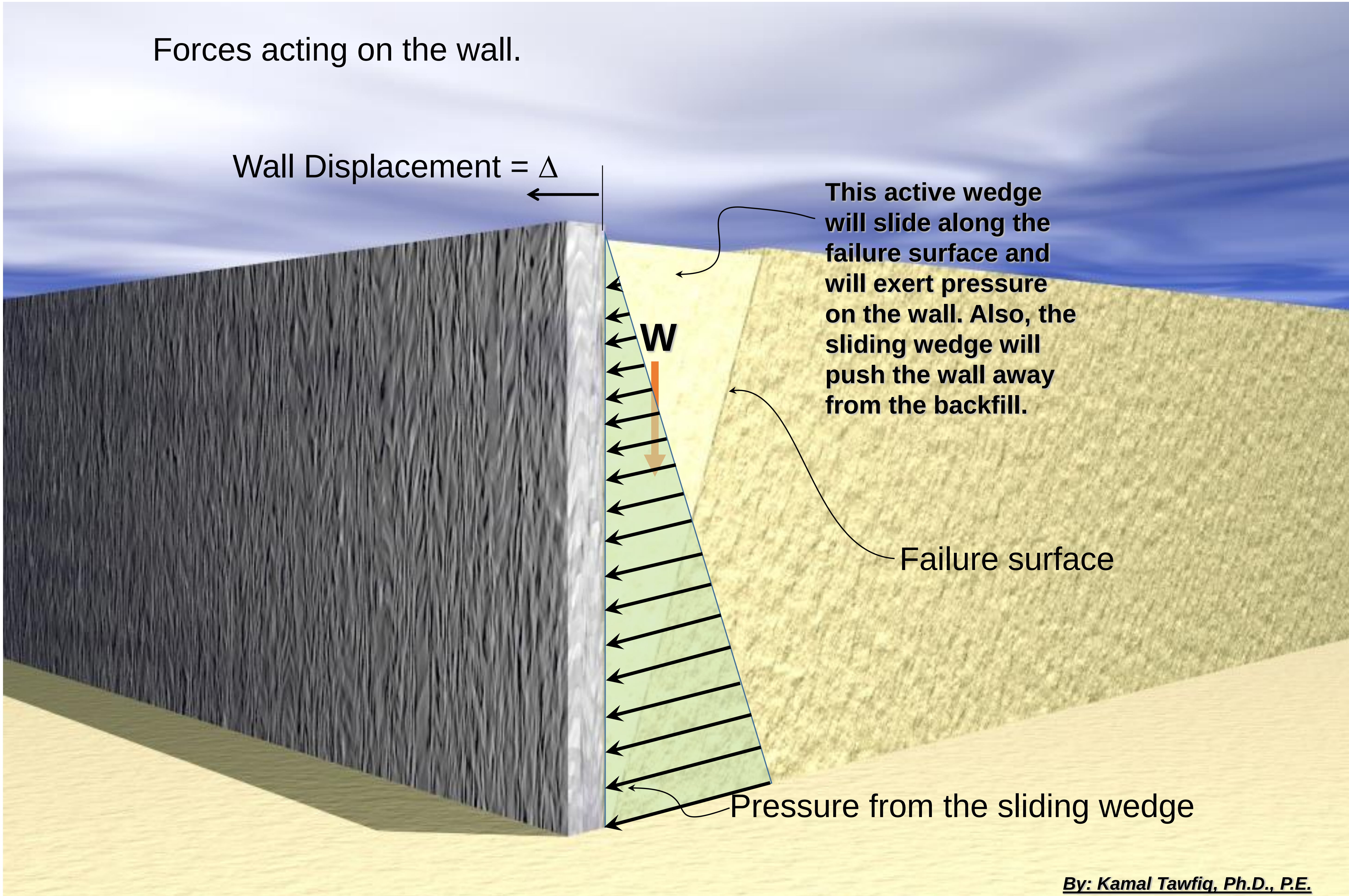
Effect of Two Soil Layers on Active Earth Pressure





Kamal Tawfiq

Coulomb Earth Pressure Method



Active Earth Pressure in ϕ – Soil (Using Rankine's Method)

Rankine's Method always assumes $\delta = 0$

Example - 1

Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30°
- Assume wall to be smooth (δ=0)

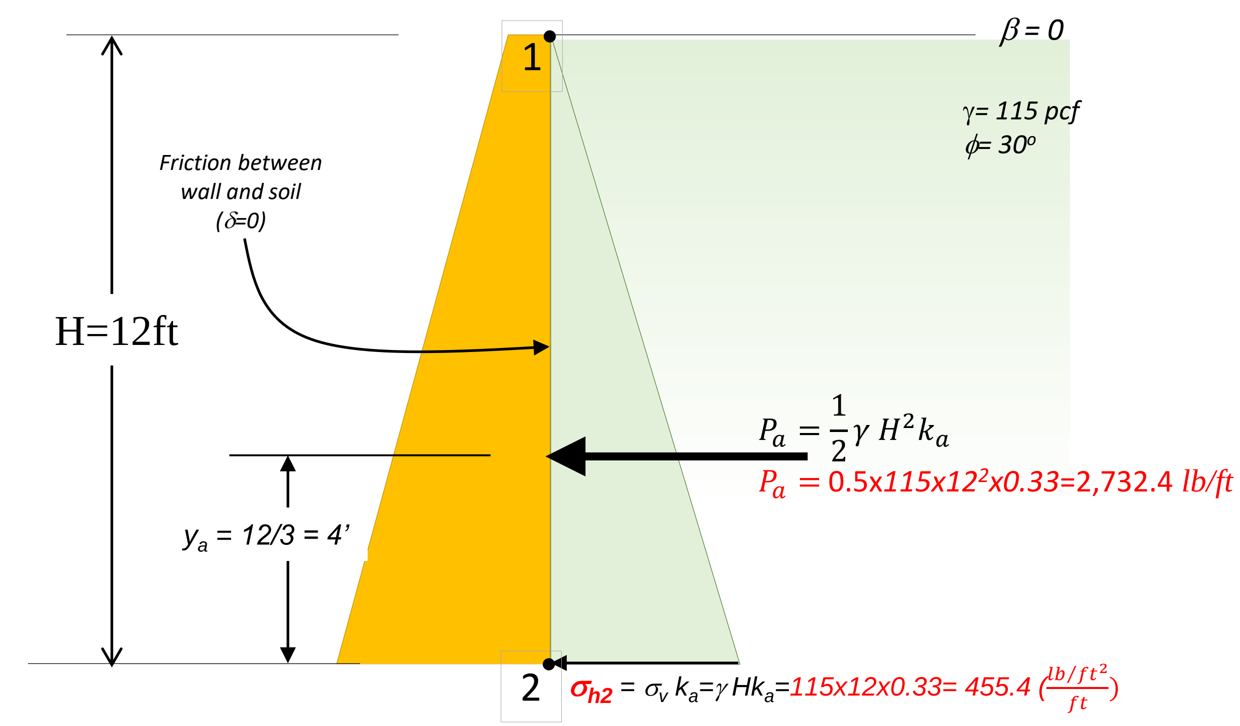
$$k_a = \frac{1 - sin\emptyset}{1 + sin\emptyset}$$

Find:
- Lateral force P_a acting on the wall

Solution:

$$k_a = \frac{1-sin\emptyset}{1+sin\emptyset} = \frac{1-sin30}{1+sin30} = 0.33$$

Point	Vertical Stress σ_v γH ($\frac{lb/ft^2}{ft}$)	Horizontal Stress σ_h $\gamma H k_a$ ($\frac{lb/ft^2}{ft}$)	$P_a = ½ \gamma H^2 k_a$ (lb/ft)	y_a(ft)
1	0	0	0.5x455.4x12=2,732.4	12/3=4
2	115x12=1,380	115x12x0.33= 455.4		



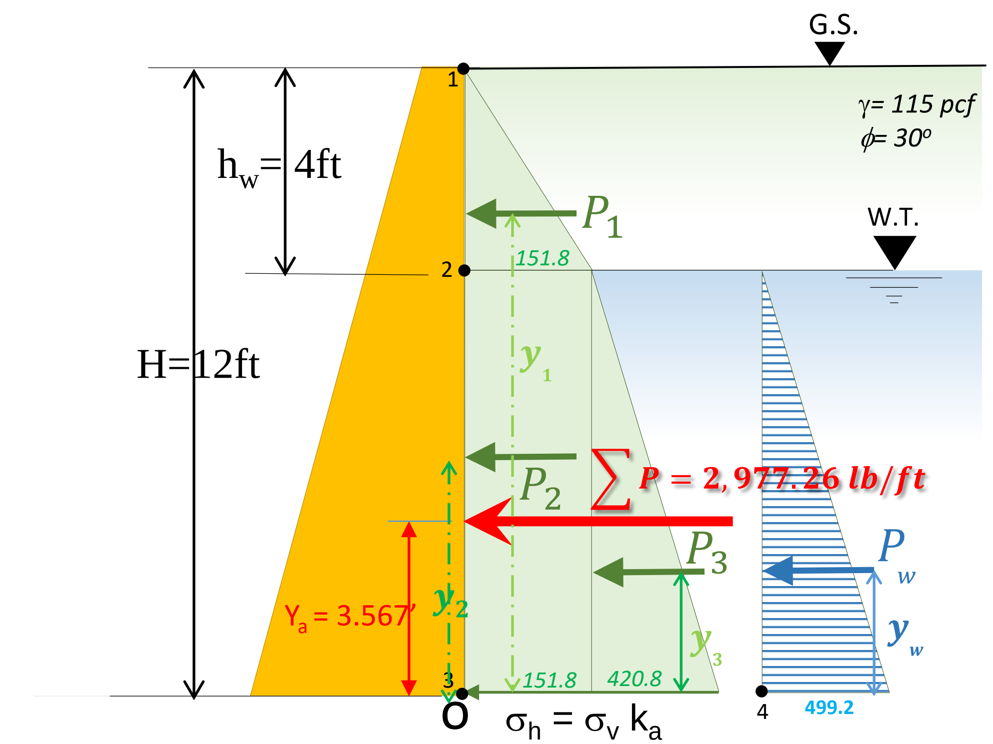
Active Earth Pressure in ϕ – Soil (Using Rankine's Method)

Rankine's Method always assumes $\delta = 0$

Example -2

$$k_a = \frac{1 - sin\emptyset}{1 + sin\emptyset}$$

Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30^o
- Assume wall to be smooth (δ=0)



Find:
- Lateral force P_a acting on the wall

Solution:
Because of the water table, the earth stress will be divided into soil and water pressure

$$k_a = \frac{1 - sin\emptyset}{1 + sin\emptyset} = \frac{1 - sin30}{1 + sin30} = 0.33$$

Point	Vertical Stress σ_v $\gamma H \left(\frac{lb/ft^2}{ft}\right)$	Horizontal Stress σ_h $\gamma H k_a \left(\frac{lb/ft^2}{ft}\right)$
1	0	0
2	115x4=460	460x0.33= 151.8
3	(115)(4)+(115-62.4)(8)= 460 + 420.8 = 880.8	[(115)(4)+(115-62.40)(8)]x0.33= 290.66
4	62.4 x 8 = 499.2	62.4 x 8 = 499.2 *(Because k_a for water =1)*

To determine the resultant Force and its point of action

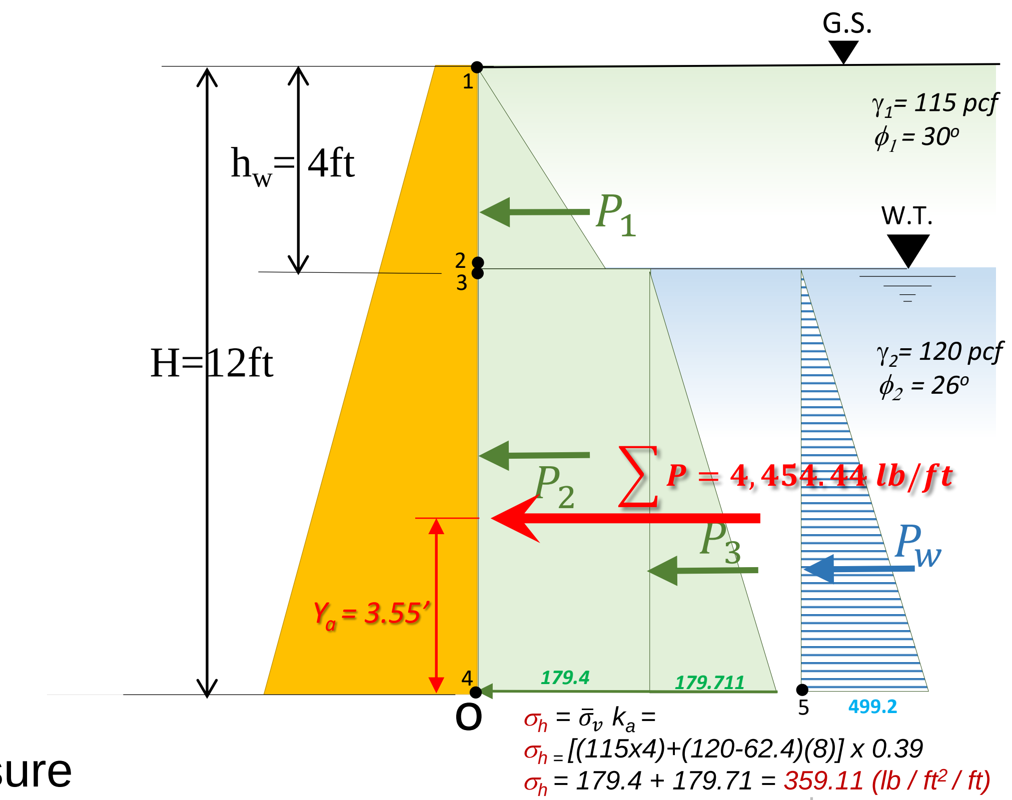
Lateral Force (lb/ft)	y_n (ft)
P_1=0.5x 151.8 x 4 = 303.6	y_1 = (4/3)+8=9.33 *(distance from P_1 to point O)*
P_2 = 151.8 x 8 = 1214.4	y_2 = 8/2=4 *(distance from P_2 to point O)*
P_3= 0.5x138.86x8 = 555.46	y_3 = 8/3=2.67 *(distance from P_3 to point O)*
P_w= 0.5x62.4x8^2 = 1996.8	y_W = 8/3=2.67 *(distance from P_w to point O)*
$\sum P = 4070.26$	Take Moment about Point O =$\sum M_o$ = 303.6x9.33 + 1214.4x4 + 555.46x2.67 + 1996.8X2.67 = 14,504.72 $y_a = \frac{\sum M_o}{\sum P} = \frac{14,504.72}{4070.26} = 3.56 ft$

Active Earth Pressure in ϕ – Soil (Using Rankine's Method)

Rankine's Method always assumes $\delta = 0$

Example -3

Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30°
- Assume wall to be smooth (δ=0)



Find:
- Lateral force P_a acting on the wall

Solution:
Because of the water table, the earth stress will be divided into soil and water pressure

$$k_{a1} = \frac{1 - sin\emptyset}{1 + sin\emptyset} = \frac{1 - sin30}{1 + sin30} = 0.33 \qquad k_{a2} = \frac{1 - sin\emptyset}{1 + sin\emptyset} = \frac{1 - sin26}{1 + sin26} = 0.39$$

Point	Vertical Stress σ_v $\gamma H\ (\frac{lb/ft^2}{ft})$	Horizontal Stress σ_h $\gamma H k_a\ (\frac{lb/ft^2}{ft})$
1	0	0
2	115x4=460	460 x 0.33 = 151.8
3	115x4 = 460	460 x 0.39 = 179.4
4	(115)(4)+(115-62.4)(8)=	γ_{sub} [(115x4)+(120-62.4)(8)] x 0.39 =359.11
5	62.4x8 =499.2	62.4 x 8 = 499.2 (Because k_a for water =1)

Lateral Force (lb/ft)	Y_n (ft)
P_1=0.5x151.8x4=303.6	Y1 = (4/3)+8 = 9.33
P_2=179.4x8=1435.2	8/2 = 4
P_3=0.5x179.71x8=718.84	8/3 = 2.67
P_w=0.5x499.2x8=1996.8	8/3 = 2.67
$\sum P$ = 4,454.44	Take Moment about Point O =$\sum M_o$ = 303.6x9.33 + 1435.2x4 + 718.84x2.67 + 1996.8X2.67 = 15,824.15 $y_a = \frac{\sum M_o}{\sum P} = \frac{15,824.15}{4,454.44} = 3.55\ ft$

Active Earth Pressure in ϕ – Soil (Using Rankine's Method)

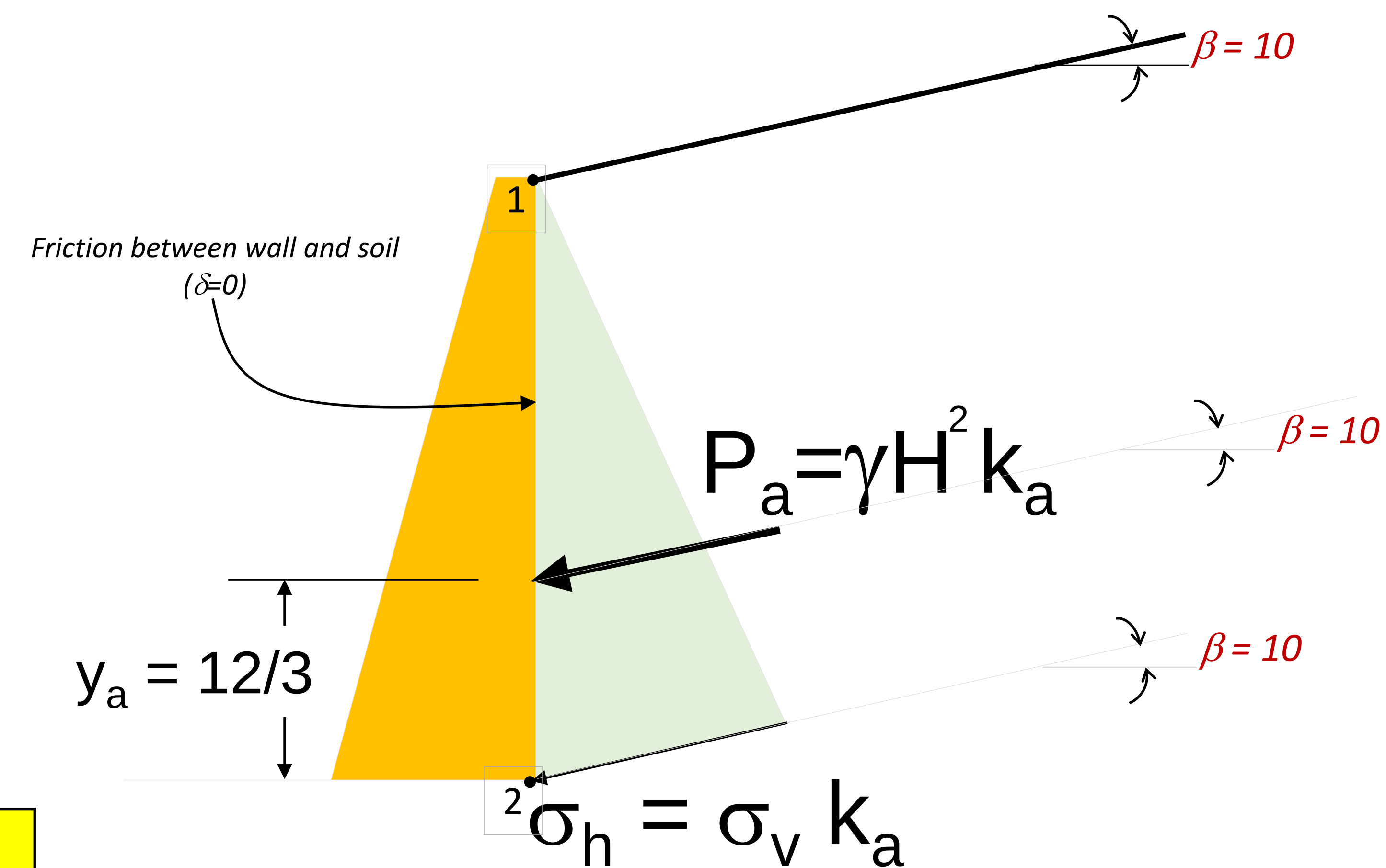
Rankine's Method always assumes $\delta = 0$

Example - 4

Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30^o
- Ground surface slope α = 10^o
- Assume wall to be smooth (δ=0)

$$k_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

Find:
- Lateral force P_a acting on the wall



Solution:

$$k_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} = \cos 10 \frac{\cos 10 - \sqrt{\cos^2 10 - \cos^2 30}}{\cos 10 + \sqrt{\cos^2 10 - \cos^2 30}} = \mathbf{0.35}$$

Point	Vertical Stress σ_v γH ($\frac{lb/ft^2}{ft}$)	Horizontal Stress σ_h $\gamma H k_a$ ($\frac{lb/ft^2}{ft}$)	$P_a = ½ \gamma H^2 k_a$ (lb/ft)	y_a(ft)
1	0	0	0.5x483x12 = 2,898	12/3=4
2	115x12=1,380	115x12x0.35= 483		

Active Earth Pressure in ϕ – Soil (Using Rankine's Method)

Rankine's Method always assumes δ = 0

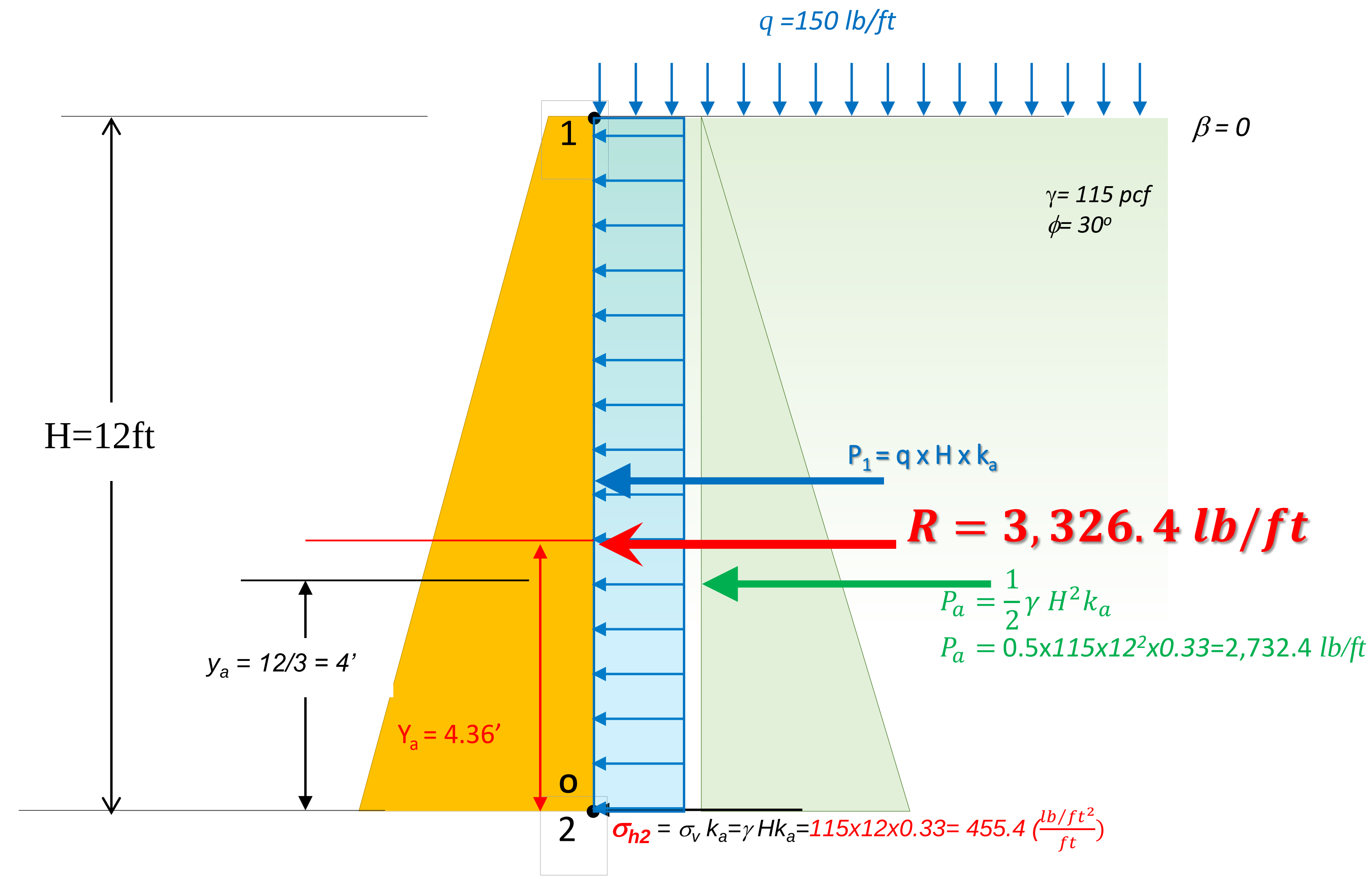
Example - 5

Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30°
- Assume wall to be smooth (δ=0)

Find:
- Lateral force P_a acting on the wall

Solution:

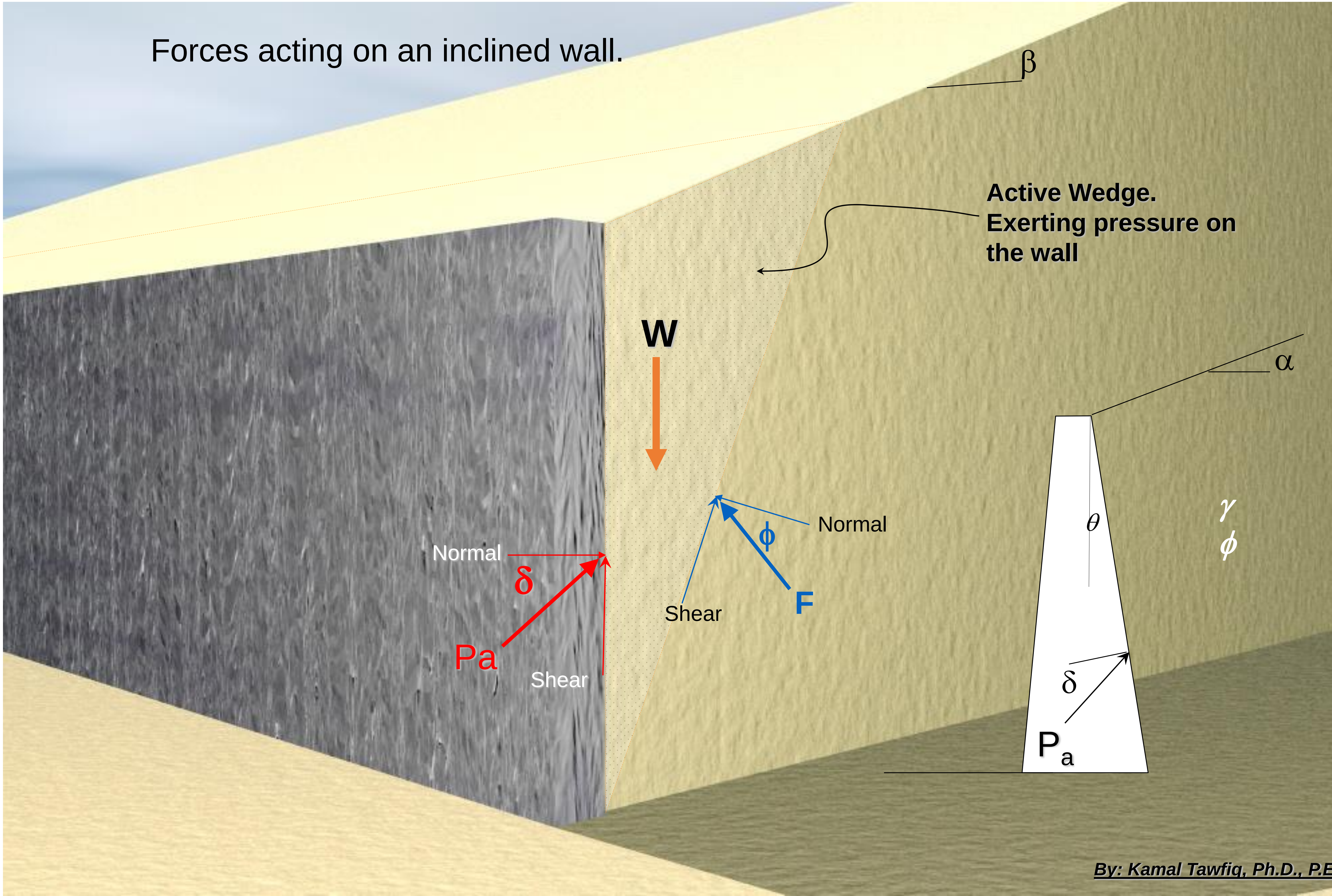
$$k_a = \frac{1-\sin\emptyset}{1+\sin\emptyset} = \frac{1-\sin 30}{1+\sin 30} = 0.33$$



Point	Vertical Stress σ_v γH ($\frac{lb/ft^2}{ft}$)	Horizontal Stress σ_h $\gamma H k_a$ ($\frac{lb/ft^2}{ft}$)
1	150	150 x 0.33 = 49.5
2	150	150 x 0.33 = 49.5
3	**115x12**	**115x12x0.33 = 455.4**

Lateral Force P (lb/ft)	Distance to O y_a(ft)
P_1 = 150 x 0.33 x 12 = 594	**12/2= 6**
P_a = 0.5x115x12²x0.33 = 2,732.4	**12/3 = 4**
R = $\sum P$ = 3,326.4	Take Moment about Point O =$\sum M_o$ = 594x6 +2732.4 x (12/3) = 14,493.6 $y_a = \frac{\sum M_o}{\sum P} = \frac{14,493.6}{3,326.4} = \mathbf{4.36} ft$

Coulomb Earth Pressure Method



COULOMB'S WEDGE THEORY

W = weight of the soil wedge
R = resultant of the shear and normal forces on the failure surface BC
P_a = the active force per unit length of the wall. The direction of Pa is inclined at an angle δ to the normal drawn and the face of the wall that supports the soil
δ = the angle of friction between the soil and the wall

W = g (area of wedge *ABC*)

From the triangles of forces,

$$\frac{P_a}{\sin(\theta - \emptyset)} = \frac{W}{\sin(180^o - \psi - \theta + \phi)}$$

$$P_a = \frac{W \sin(\theta - \emptyset)}{\sin(180^o - \psi - \theta + \phi)}$$

Substituting for *W*,

$$P_a = \frac{1}{2} \cdot \frac{\gamma H^2}{\sin^2 \alpha} \cdot \frac{\sin(\theta - \emptyset)}{\sin(180^o - \psi - \theta + \phi)} \cdot \frac{\sin(\theta + \alpha) . \sin(\alpha + \beta)}{\sin(\theta - \beta)}$$

The maximum value of *Pa* is obtained by equating the first derivative of *Pa* with respect to θ to zero; or

$(\partial P_a)/\partial\theta = 0$, and substituting the corresponding value of θ.

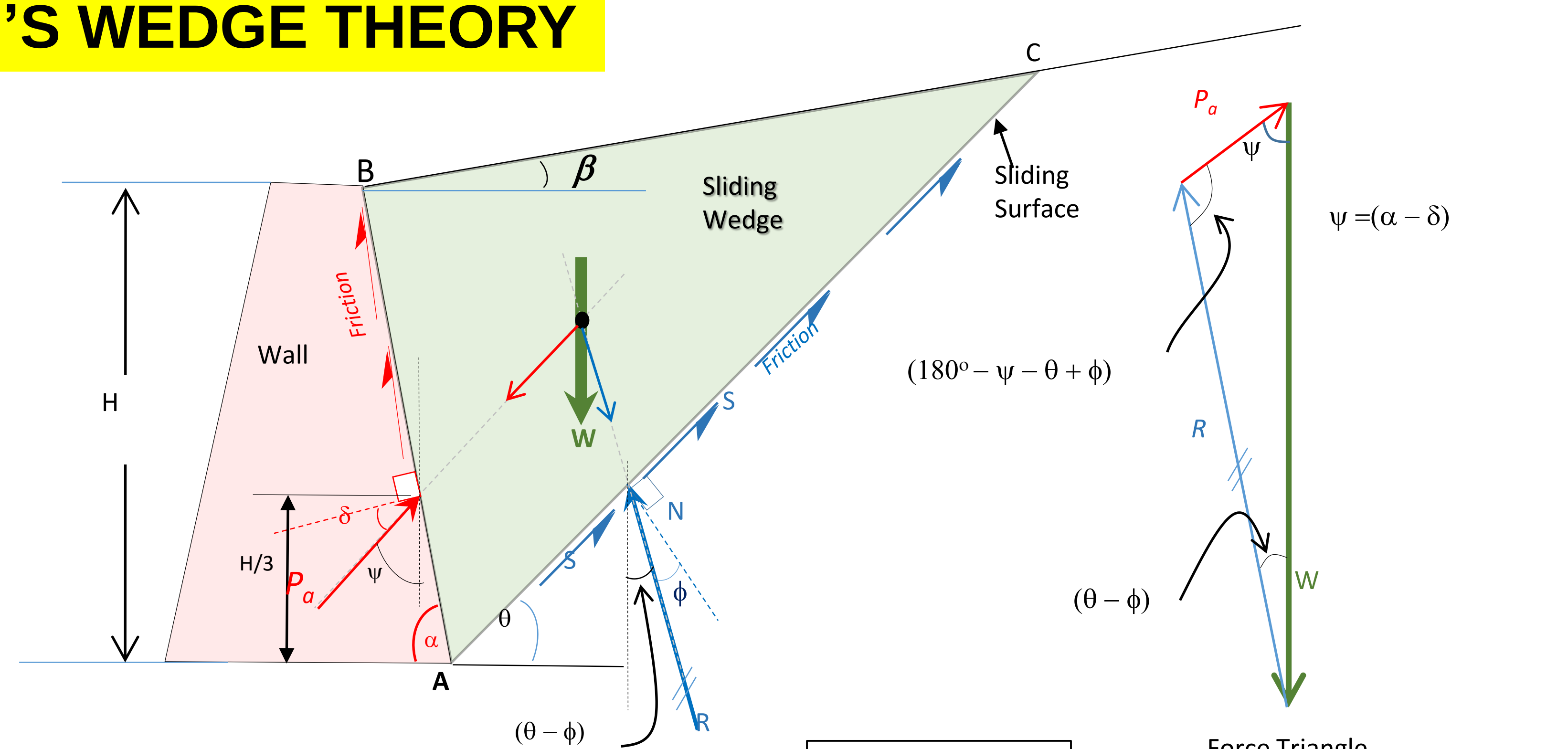
The value of P_a so obtained is written as

$$P_a = \frac{1}{2} . \gamma H^2 . \frac{sin^2(\alpha + \phi)}{sin^2\alpha \ sin(\alpha - \delta) \left[1 + \sqrt{\frac{\sin(\emptyset + \delta)\sin(\emptyset - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2}$$

This is usually written as

$$P_a = \frac{1}{2} . \gamma H^2 . \ K_a$$

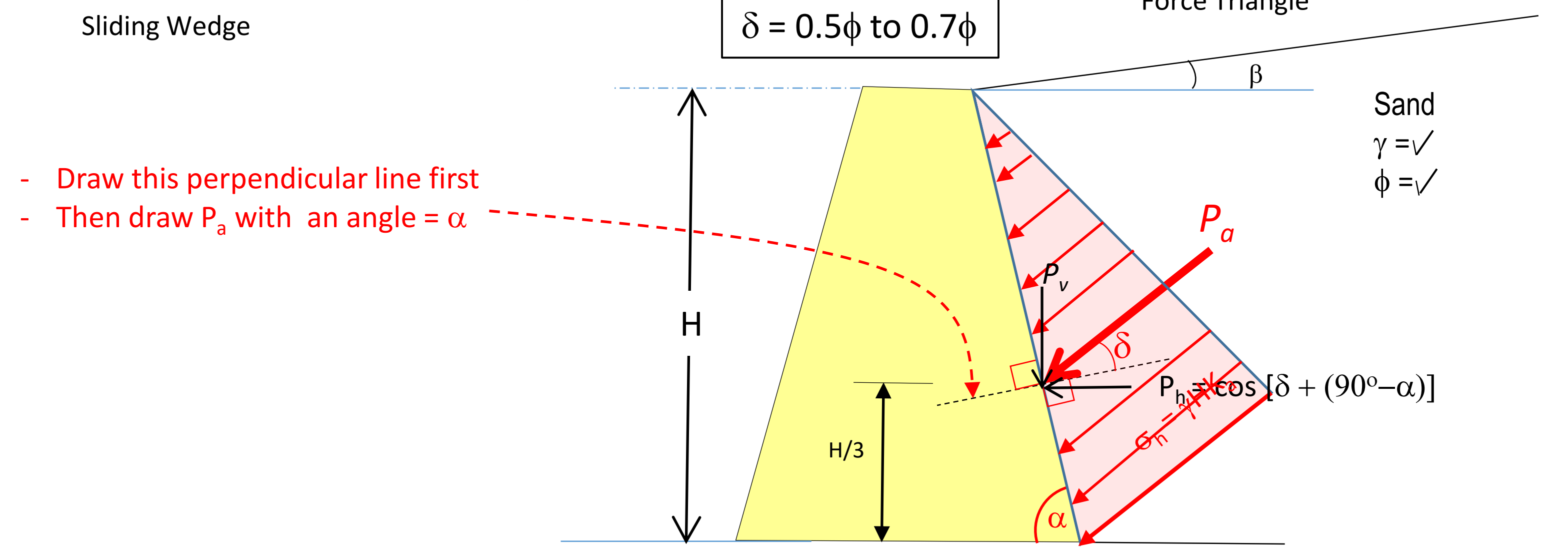
Where K_a being the coefficient of active earth pressur $= \dfrac{sin^2(\alpha + \phi)}{sin^2\alpha \ sin(\alpha - \delta) \left[1 + \sqrt{\frac{\sin(\emptyset + \delta)\sin(\emptyset - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2}$

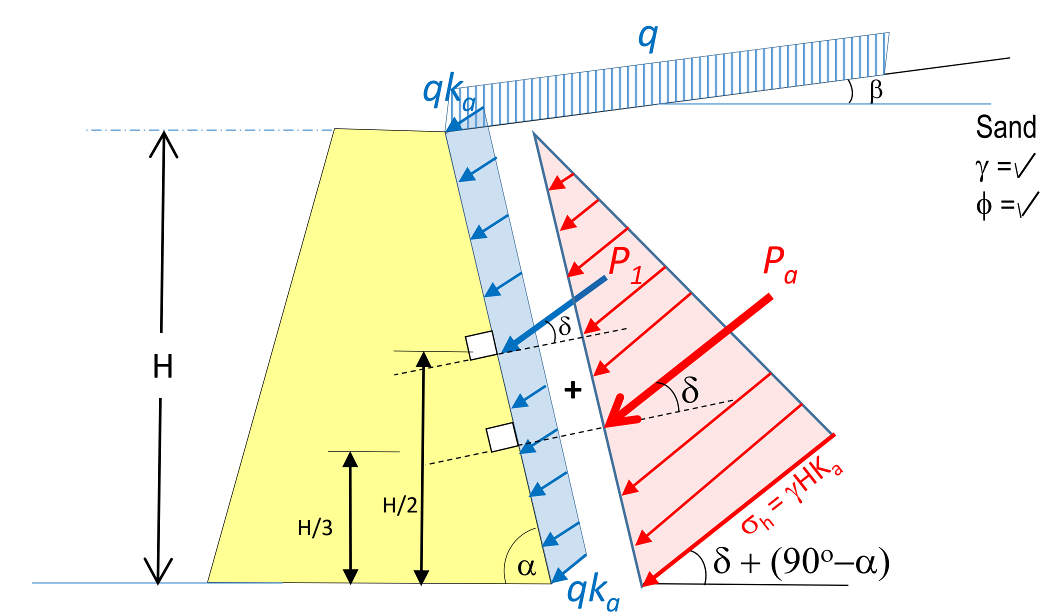


Sliding Wedge

Force Triangle

δ = 0.5ϕ to 0.7ϕ

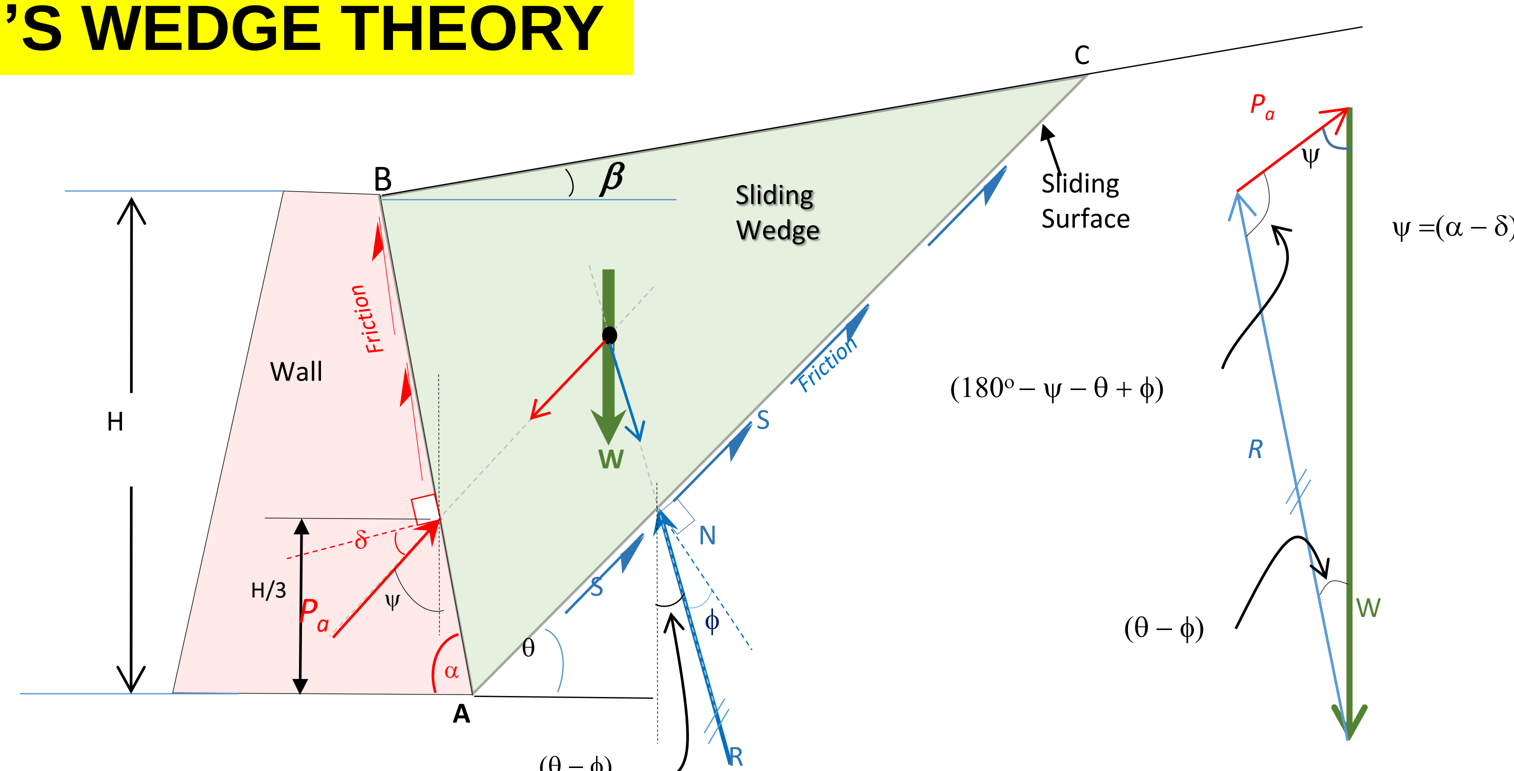




Kamal Tawfiq

COULOMB'S WEDGE THEORY

$$K_a = \frac{sin^2(\alpha + \phi)}{sin^2\alpha \; sin(\alpha - \delta)\left[1 + \sqrt{\dfrac{\sin(\emptyset + \delta)\sin(\emptyset - \beta)}{\sin(\alpha - \delta)\sin(\alpha + \beta)}}\right]^2}$$



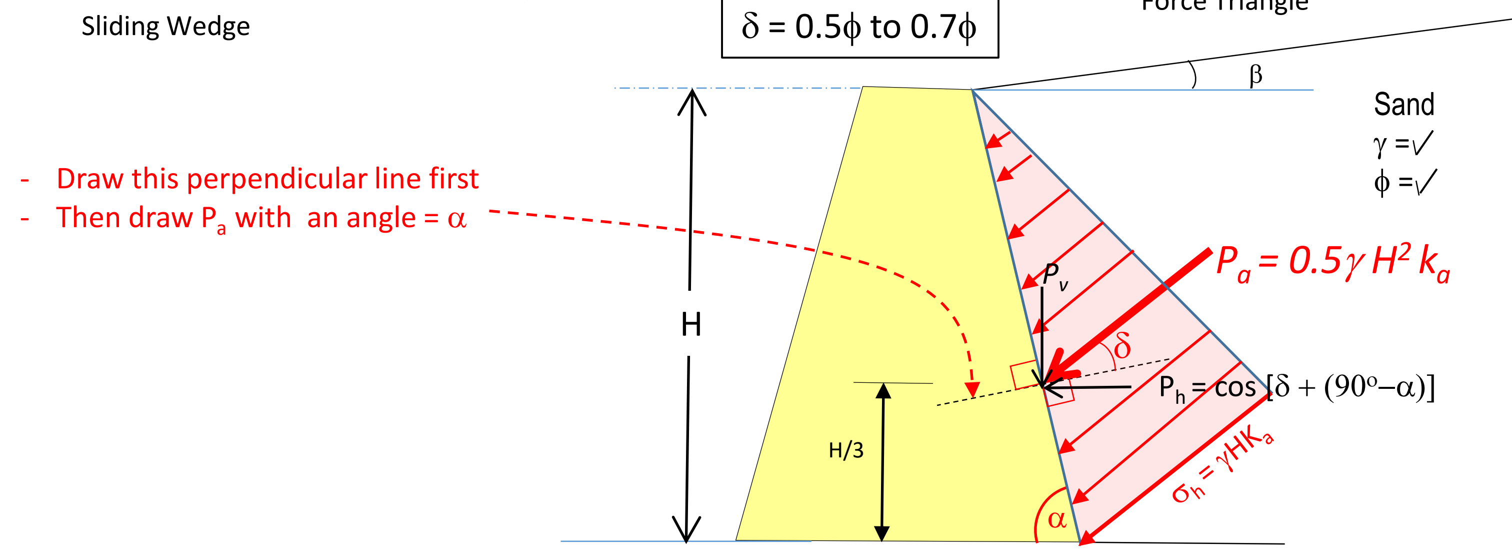
Force Triangle

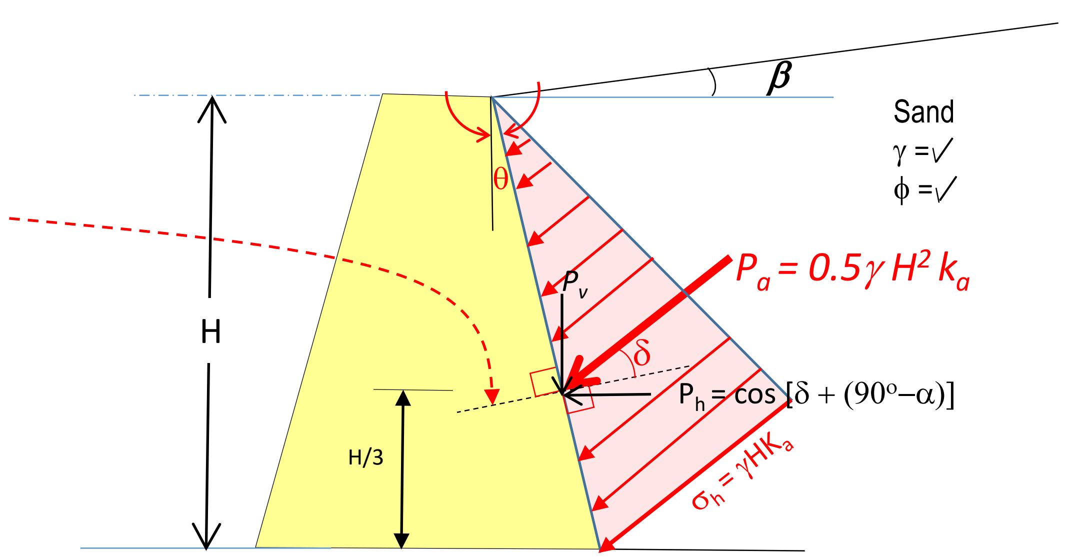
Sliding Wedge

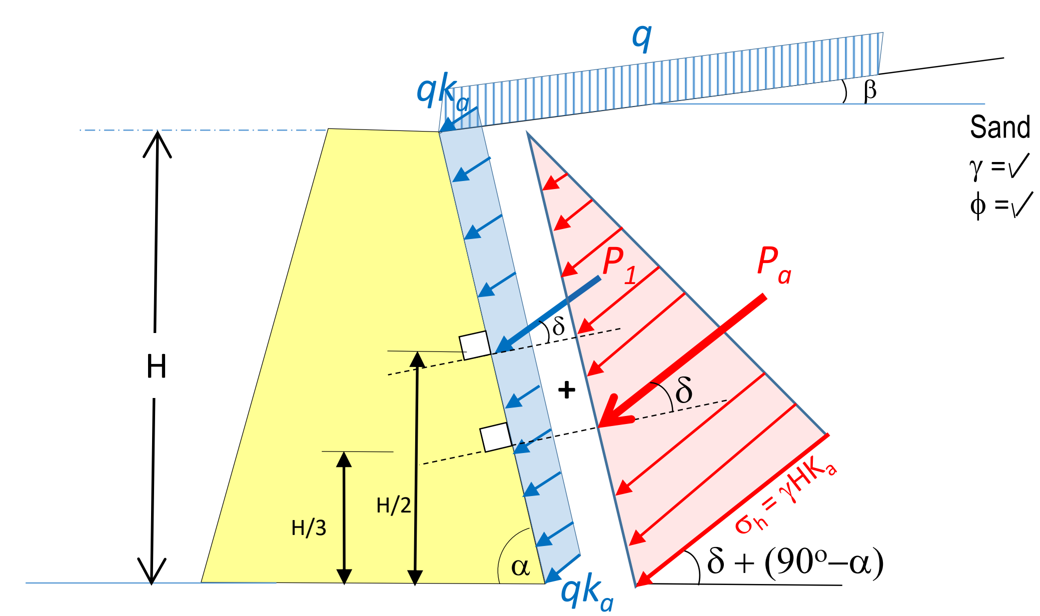
δ = 0.5φ to 0.7φ

OR

$$K_a = \frac{cos^2(\emptyset - \theta)}{cos^2\theta \; cos(\delta + \theta)\left[1 + \sqrt{\dfrac{\sin(\delta + \emptyset)\sin(\emptyset - \beta)}{\cos(\delta + \theta)\cos(\beta - \theta)}}\right]^2}$$





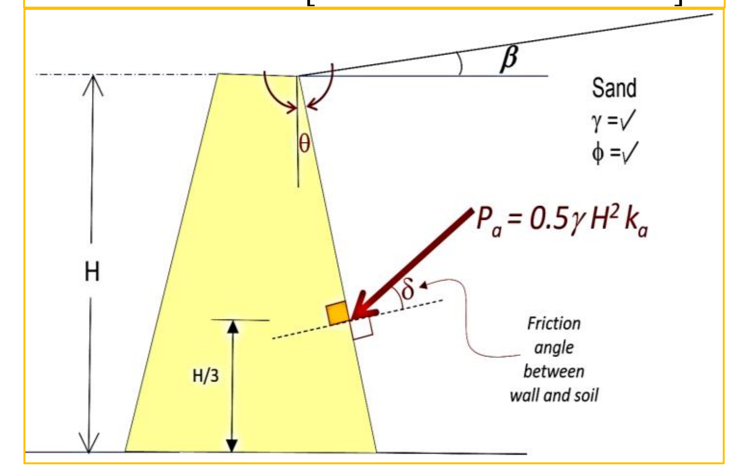


Kamal Tawfiq

Active Earth Pressure in ϕ – Soil (Using Coulomb's Method)

Example - 6

$$K_a = \frac{cos^2(\emptyset - \theta)}{cos^2\theta \;\; cos(\delta+\theta)\left[1+\sqrt{\frac{\sin(\delta+\emptyset)\sin(\emptyset-\beta)}{\cos(\delta+\theta)\cos(\beta-\theta)}}\right]^2}$$

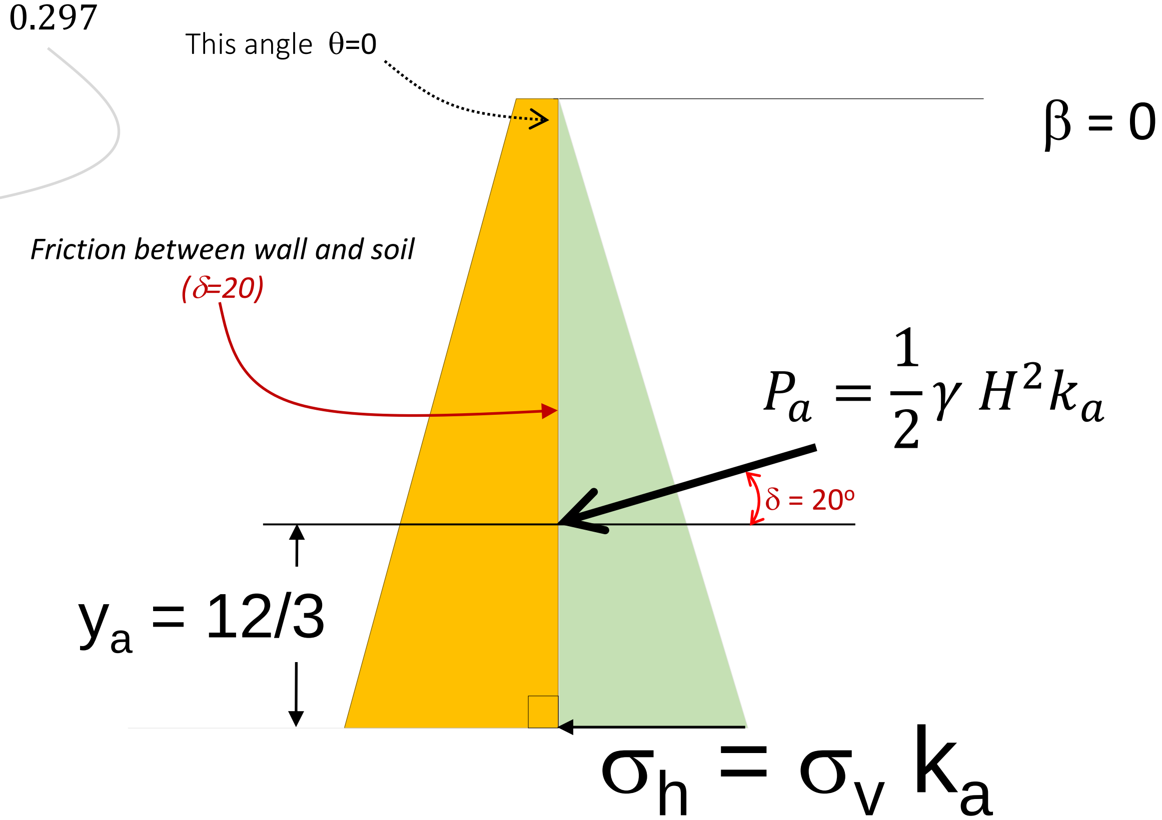


Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30º
- Assume wall to be smooth (δ=20)

Find:
- Lateral force P_a acting on the wall

Solution:

$$K_a = \frac{cos^2(\emptyset - \theta)}{cos^2\theta \;\; cos(\delta+\theta)\left[1+\sqrt{\frac{\sin(\delta+\emptyset)\sin(\emptyset-\beta)}{\cos(\delta+\theta)\cos(\beta-\theta)}}\right]^2} = \frac{cos^2(30-0)}{cos^2 0 \;\; cos(20+0)\left[1+\sqrt{\frac{\sin(20+30)\sin(30-0)}{\cos(20+0)\cos(0-0)}}\right]^2} = 0.297$$



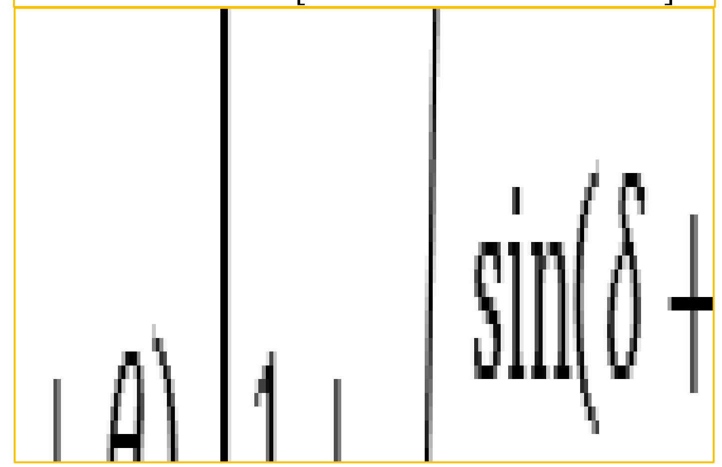
Point	Vertical Stress σ_v $\gamma H\ (\frac{lb/ft^2}{ft})$	Horizontal Stress σ_h $\gamma H k_a\ (\frac{lb/ft^2}{ft})$	$P_a = ½\ \gamma H^2 k_a$ (lb/ft)	y_a(ft)
1	0	0	0.5x409.86x12=2,459.2	12/3=4
2	115x12=1,380	115x12x0.297= 409.86		

About 10% less than the Rankine's Earth Pressure

Active Earth Pressure in ϕ – Soil (Using Coulomb's Method)

Example - 7

$$K_a = \frac{cos^2(\emptyset - \theta)}{cos^2\theta \;\; cos(\delta + \theta)\left[1 + \sqrt{\frac{\sin(\delta + \emptyset)\sin(\emptyset - \beta)}{\cos(\delta + \theta)\cos(\beta - \theta)}}\right]^2}$$

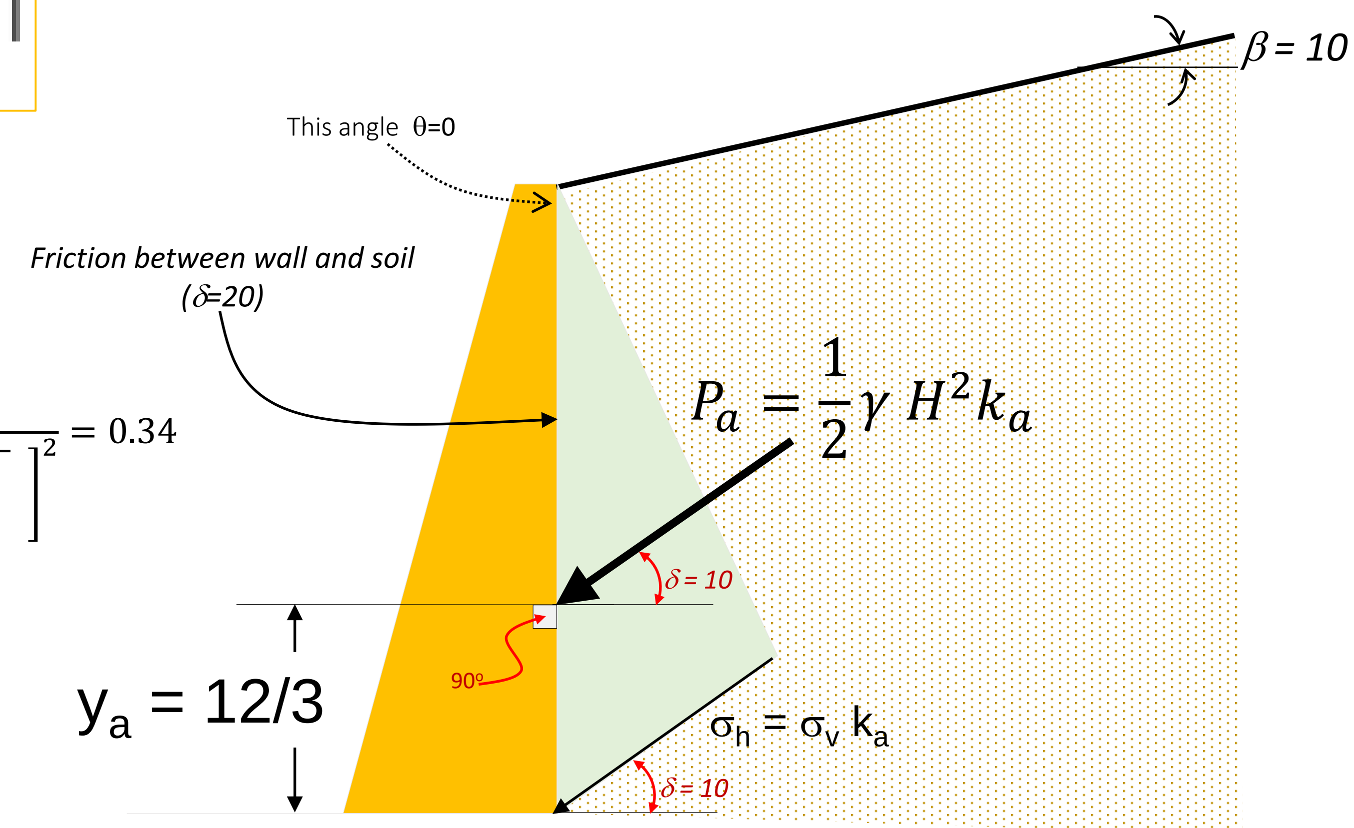


Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30º
- Ground surface slope α = 10º
- Assume wall to be smooth (δ=20)

Find:
- Lateral force P_a acting on the wall

Solution:

$$K_a = \frac{cos^2(\emptyset - \theta)}{cos^2\theta \;\; cos(\delta + \theta)\left[1 + \sqrt{\frac{\sin(\delta + \emptyset)\sin(\emptyset - \beta)}{\cos(\delta + \theta)\cos(\beta - \theta)}}\right]^2} = \frac{cos^2(30 - 0)}{cos^2 0 \;\; cos(20 + 0)\left[1 + \sqrt{\frac{\sin(20 + 30)\sin(30 - 10)}{\cos(20 + 0)\cos(10 - 0)}}\right]^2} = 0.34$$



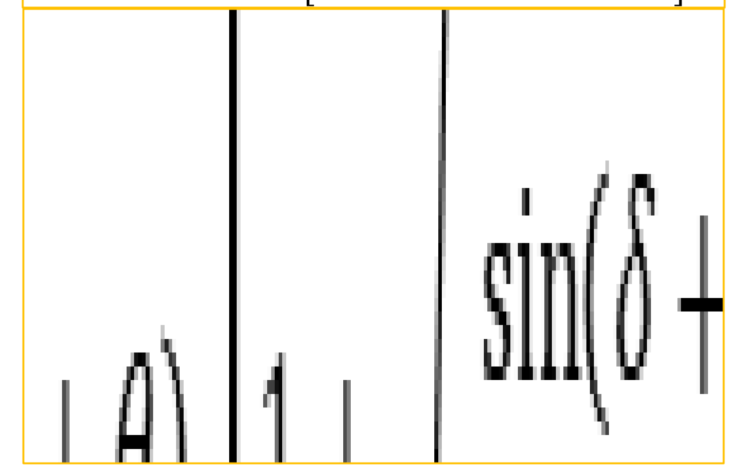
Point	Vertical Stress σ_v γH ($\frac{lb/ft^2}{ft}$)	Horizontal Stress σ_h $\gamma H k_a$ ($\frac{lb/ft^2}{ft}$)	P_a = ½ $\gamma H^2 k_a$(lb/ft)	y_a(ft)
1	0	0	0.5x469.2x12 = 2,815.2	12/3 = 4
2	115x12=1,380	115x12x0.34= 469.2		

About 3% less than the Rankine's Earth Pressure

Active Earth Pressure in ϕ – Soil (Using Coulomb's Method)

Example - 8

$$K_a = \frac{cos^2(\phi-\theta)}{cos^2\theta \;\; cos(\delta+\theta)\left[1+\sqrt{\frac{\sin(\delta+\phi)\,\sin(\phi-\beta)}{\cos(\delta+\theta)\,\cos(\beta-\theta)}}\right]^2}$$

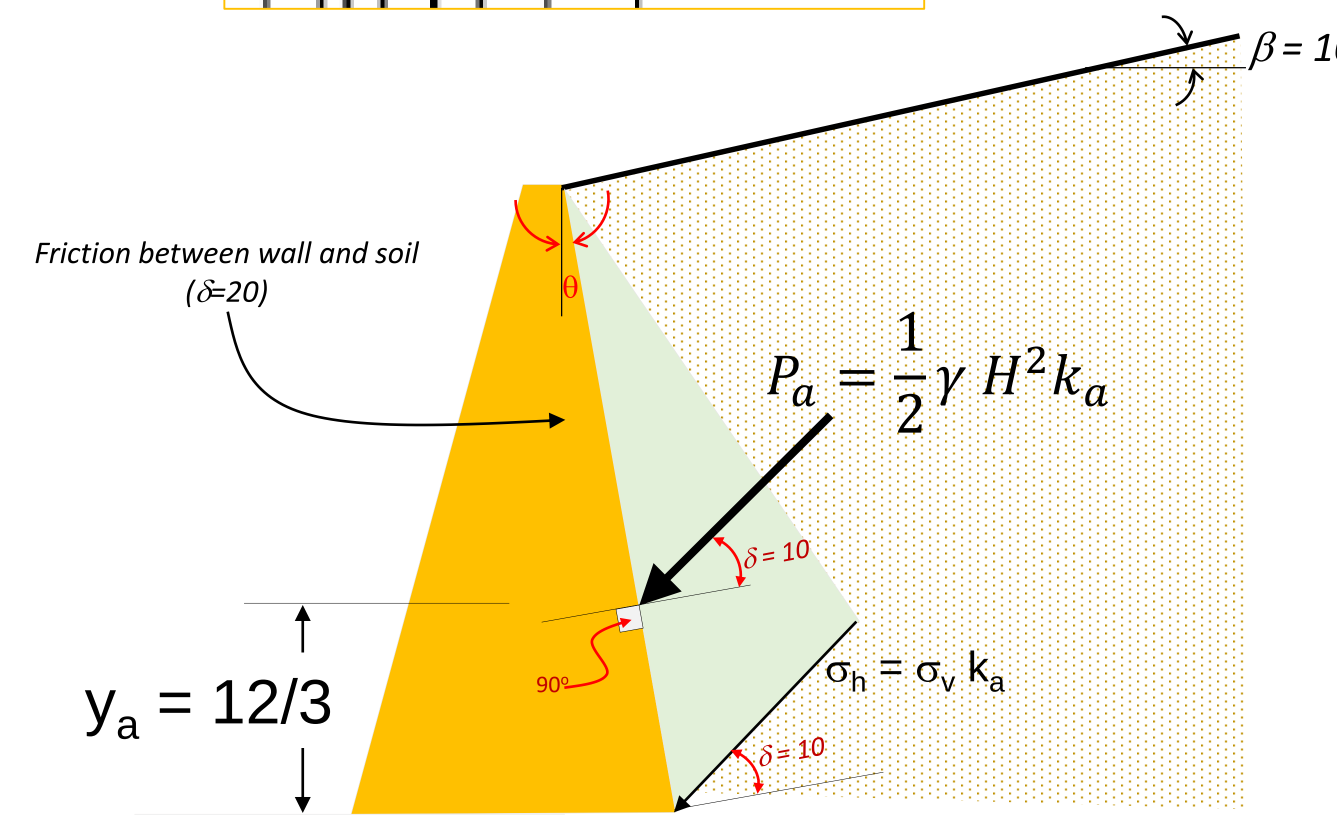


Given:
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30º
- Ground surface slope α = 10º
- Assume wall to be smooth (δ=20)
- The wall batter angle θ = 10º

Find:
- Lateral force P_a acting on the wall

Solution:

$$K_a = \frac{cos^2(\phi-\theta)}{cos^2\theta \;\; cos(\delta+\theta)\left[1+\sqrt{\frac{\sin(\delta+\phi)\,\sin(\phi-\beta)}{\cos(\delta+\theta)\,\cos(\beta-\theta)}}\right]^2} = \frac{cos^2(30-10)}{cos^2 10 \;\; cos(20+10)\left[1+\sqrt{\frac{\sin(20+30)\,\sin(30-10)}{\cos(20+10)\,\cos(10-10)}}\right]^2} = 0.475$$



Point	Vertical Stress σ_v γH ($\frac{lb/ft^2}{ft}$)	Horizontal Stress σ_h $\gamma H k_a$ ($\frac{lb/ft^2}{ft}$)	P_a = ½ $\gamma H^2 k_a$(lb/ft)	y_a(ft)
1	0	0	0.5x655.5x12 = 3,933	12/3 = 4
2	115x12=1,380	115x12x0.475= 655.5		

About 28% more than Vertical wall ($\theta = 0$)