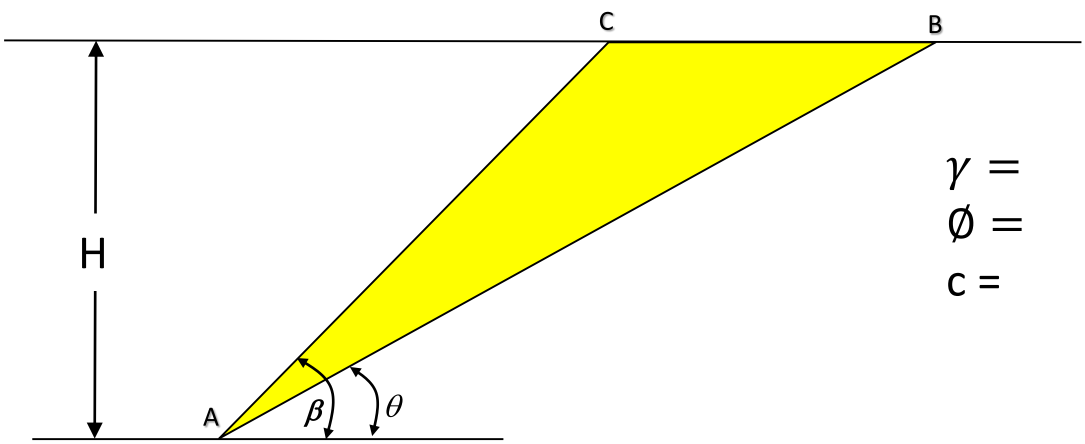
Slope Stability Analysis
Homework #5
Fall 2022

Problem 1

The following figure shows a 15-ft cut through two soil strata. The lower is a highly impermeable cohesive soil. Shearing strength data between the two strata are as follows:

Cohesion=400 psf
Angle of internal friction= 25^o
Unit weight of the upper layer= 105 pcf
Height of the slope, H= 10 ft
$\beta = 45^o$
$\theta = 30^o$

Find if the slope is safe or not



1) $F.S._c = F.S._\phi = ???$

$$C_{dev} = \frac{c}{F.S._c}$$

$$\tan\phi_{dev} = \frac{\tan\phi}{F.S._\phi}$$

$$\phi_{dev} = \tan^{-1}\left(\frac{\tan\phi}{F.S._\phi}\right)$$

① assume $F.S._\phi = 1$

$$\phi_{dev} = \tan^{-1}\left(\frac{\tan 25°}{1}\right)$$

$$= 25°$$

find $F.S._c \rightarrow C_{dev} = \frac{C}{F.S._c}$

$$H_{cr} = \frac{2c}{\gamma}\left[\frac{Sin\beta}{sin(\beta-\theta)(sin\theta - cos\theta tan\emptyset)}\right]$$

$$H_{des} = \frac{2c_d}{\gamma}\left[\frac{Sin\beta}{sin(\beta-\theta)(sin\theta - cos\theta tan\emptyset_d)}\right]$$

$$c_d = \frac{\gamma H}{2}\left[\frac{sin(\beta-\theta)(sin\theta - cos\theta tan\emptyset_d)}{Sin\beta}\right]$$

$$C_{dev} = \frac{(105)(10)}{2}\left[\frac{\sin(45°-30°)(\sin 30° - \cos 30° \tan 25°)}{\sin 45°}\right]$$

$$= (525)\left[0.0352\right]$$

$$C_{dev} = 18.4795 \text{ psf}$$

$$F.S._c = \frac{C}{C_{dev}} \rightarrow FS_c = \frac{400 \text{ psf}}{18.4795 \text{ psf}}$$

$F.S._c = 21.646$ since $F.S_c \neq F.S._\phi$, assume another value for $F.S._\phi$

② assume $F.S._\phi = 3$

$$\phi_{dev} = \tan^{-1}\left(\frac{\tan 25°}{3}\right) = 8.835°$$

$$C_{dev} = \frac{(105)(10)}{2}\left[\frac{\sin(45°-30°)(\sin 30° - \cos 30° \tan(8.835°))}{\sin 45°}\right]$$

$$= (525)(0.1337)$$

$$= 70.214$$

$$FS_c = \frac{400 \text{ psf}}{70.214 \text{ psf}} \rightarrow F.S._c = 5.697$$

③ assume $F.S._\phi = 6$

$$\phi_{dev} = \tan^{-1}\left(\frac{\tan 25°}{6}\right) = 4.444°$$

$$C_{dev} = \frac{(105)(10)}{2}\left[\frac{\sin(45°-30°)(\sin 30° - \cos 30° \tan(4.44°))}{\sin 45°}\right]$$

$$= 83.148$$

$$F.S._c = \frac{400}{83.148} \rightarrow F.S._c = 4.8107$$

④ assume $F.S._{\phi} = 4$

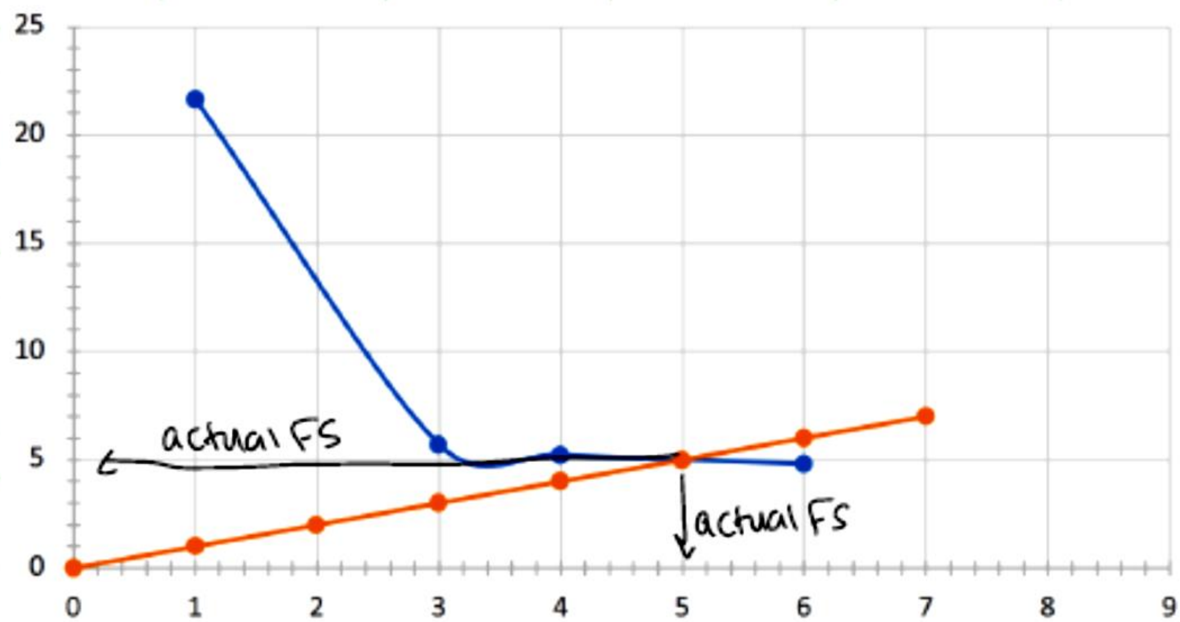
$$\phi_{dev} = \tan^{-1}\left(\frac{\tan 25^\circ}{4}\right) = 6.649$$

$$C_{dev} = \frac{(105)(10)}{2}\left[\frac{\sin(45^\circ - 30^\circ)(\sin 30^\circ - \cos 30^\circ \tan(6.649))}{\sin 45^\circ}\right]$$

$$= 76.681$$

$$FS_c = \frac{400}{76.681} \rightarrow FS_c = 5.2164$$

x Assumed FS_ϕ	y Calculated FS_c
1	21.646
3	5.697
4	5.2164
6	4.8107



$\boxed{\text{F.S.} = 5 \text{ which is } > 1.5\text{, so the slope is safe}}$

Problem 2

A 45° slope is excavated to a depth of 8 m in a deep layer of saturated clay of unit weight 19 kN/m^3: the relevant shear strength parameters are c_u = 65 kN/m^2 and ϕ_u = 0. Determine the factor of safety for the trial failure surface specified in Fig. 9.3.

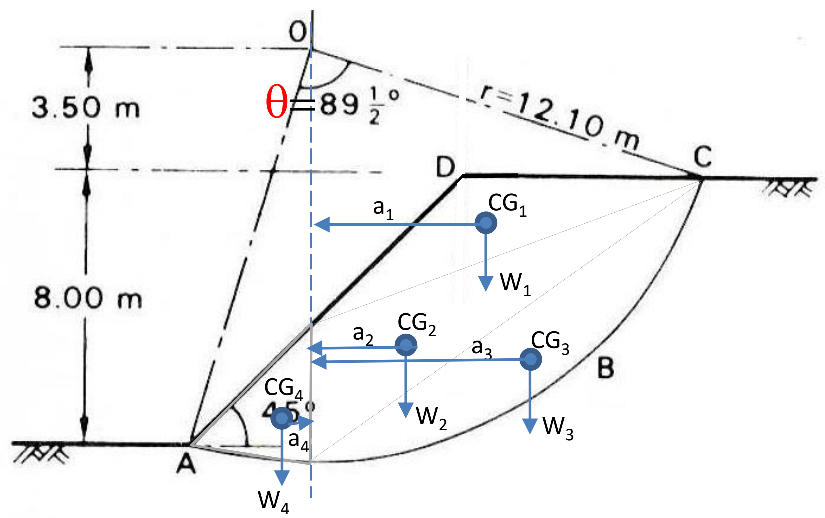
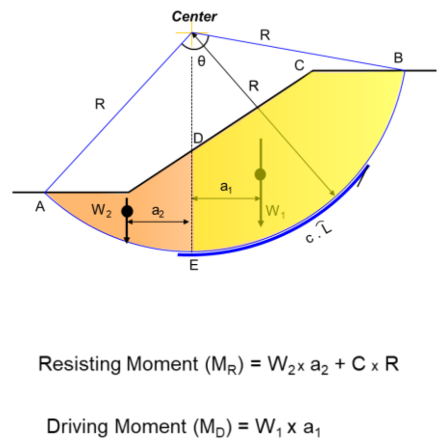
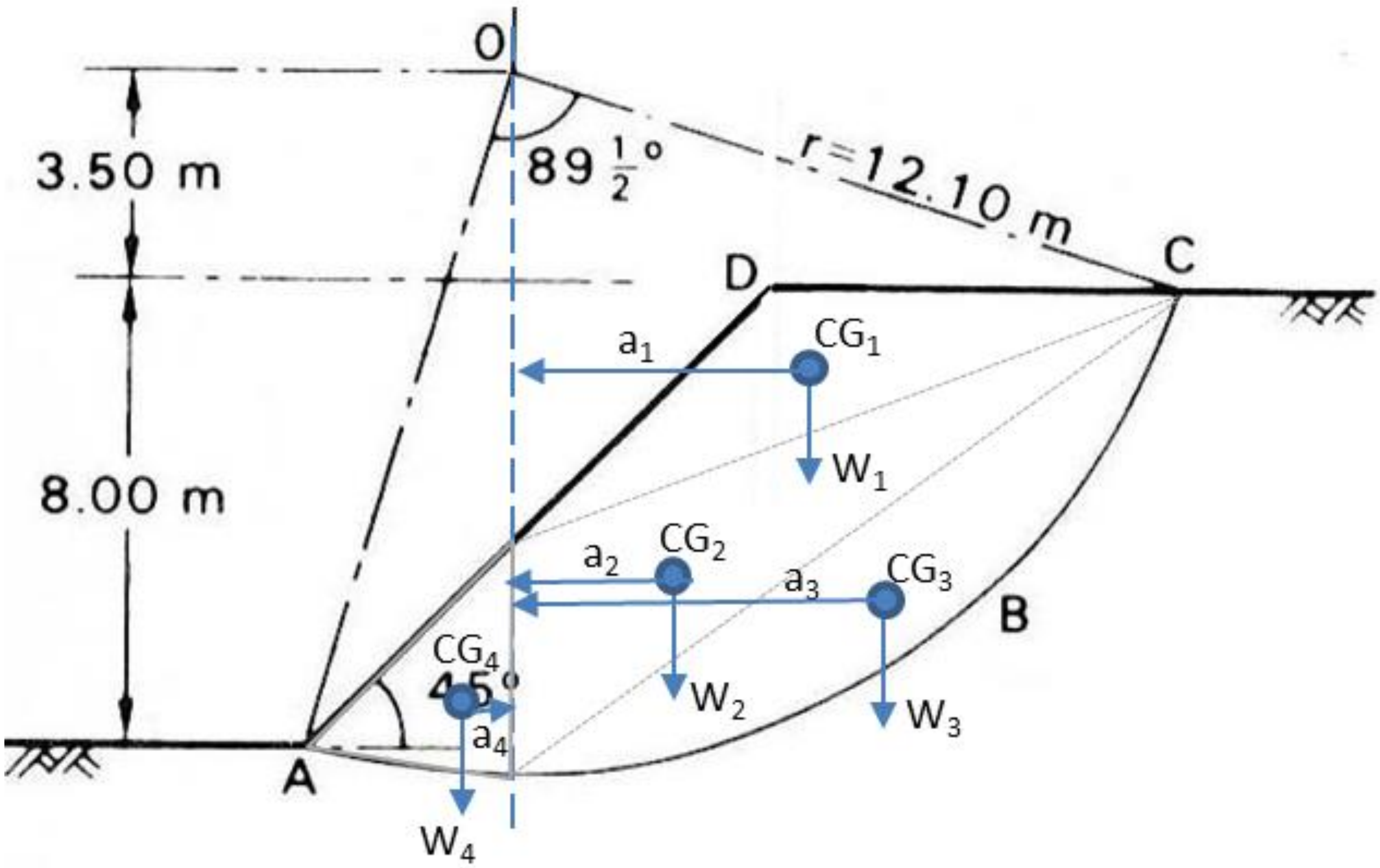


Figure 9.3 Example 9.1.



1. Draw the problem to scale
2. Divide the shapes to smaller areas (1, 2, 3, and 4)
3. Find the weight of each area (For example $A_1 x \gamma_{soil} = W_1$)
4. Find where is the center of gravity for each area (CG_1, CG_2, …)
5. Using the scale, measure the arms (a1, a2, a3, ..)
6. For A_4 you can approximate the area to triangle.
7. Now determine the driving moments ($W_1 x a_1 + W_2 x a_2 + W_3 x a_3$)
8. Determine the resisting moments ($W_4 x a_4$ + c x L_{curve})

9. $L_{curve} = \theta$ x r … θ in radians



O
3.50 m
89 1/2°
r=12.10 m
C
D
8.00 m
a1
CG1
W1
CG2
a2
a3
CG3
B
CG4
45°
a4
W2
W3
A
W4

Problem 2

A 45° slope is excavated to a depth of 8 m in a deep layer of saturated clay of unit weight 19 kN/m^3: the relevant shear strength parameters are c_u = 65 kN/m^2 and ϕ_u = 0. Determine the factor of safety for the trial failure surface specified in Fig. 9.3.

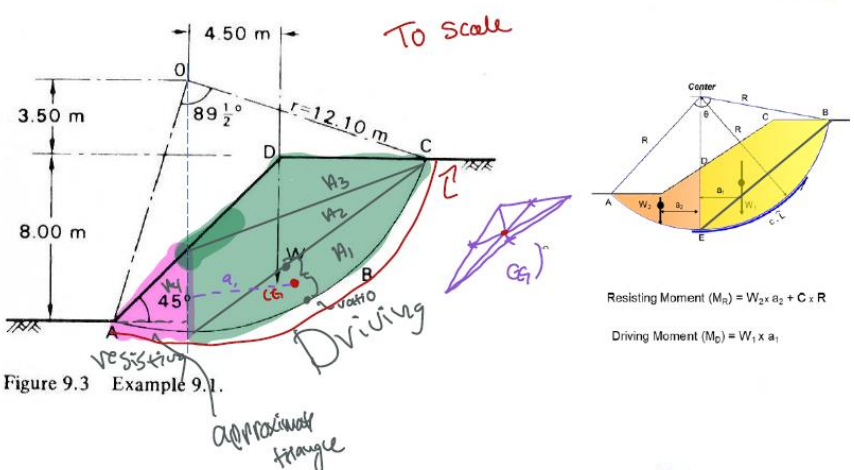


Figure 9.3 Example 9.1.

$$\frac{A_4 \gamma a_4 + C \times l \times r}{A_1 \gamma a_2 + A_2 \gamma a_2 + \ldots}$$

give CG and A

Find a

2.

A 45° slope is excavated to a depth of 8 m in a deep layer of saturated clay of unit weight 19 kN/m^3: the relevant shear strength parameters are c_u = 65 kN/m^2 and ϕ_u = 0. Determine the factor of safety for the trial failure surface specified in Fig. 9.3.

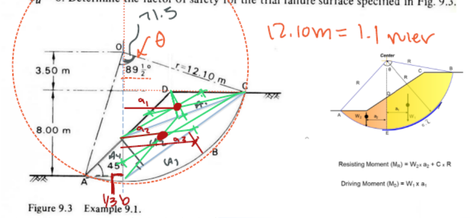
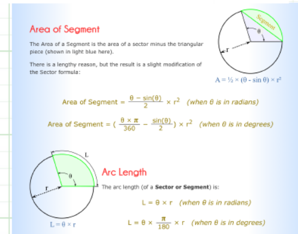


Figure 9.3 Example 9.1.

$\gamma = 19\ kN/m^3$

$C_u = 65\ kN/m^2$

$\phi_u = 0$



arc length = $\theta \times r$ ← radians

$\left(12.10 \times 89.5 \frac{\pi}{180}\right) = \underline{18.9m}$

Area 1

$\frac{12.10}{x} = \frac{1.1}{.2}$

$h = 2.2$

$b_1 = 5.5$

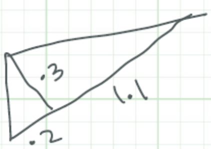
$b_2 = 6.60$

$= .5(5.5)(2.2) + .5(6.6)(2.2)$

$\underline{Area\ 1 = 13.31\ m^2}$

$a_1 = .55 \rightarrow 6.05$

Area 2



$\frac{1}{2}(2.2)(3.3) + \frac{1}{2}(3.3)(12.1)$

$\underline{A_2 = 23.6\ m^2}$

$a_2 = .4 \rightarrow 4.4$

Area 3 $= 21.93$

$$\left(\frac{71.3 \times \pi}{360} - \frac{\sin(71.5)}{2}\right) \times 12.1^2$$

centroid $= \frac{4b}{3\pi}$ ← height circle $= .25 \rightarrow 2.75$

$= 1.167m$

$= .106$ on ruler

$a_3 = .6 \rightarrow 6.6m$

Area 4

$\frac{1}{2}(3.3)(3.85)$

$= 6.35$

$a_4 = 1/3 b =$

$\frac{1}{3}(3.3) = 1.1$

$$\frac{A_4 \gamma a_4 + C \times l \times r}{A_1 \gamma a_2 + A_2 \gamma a_2 + \ldots}$$

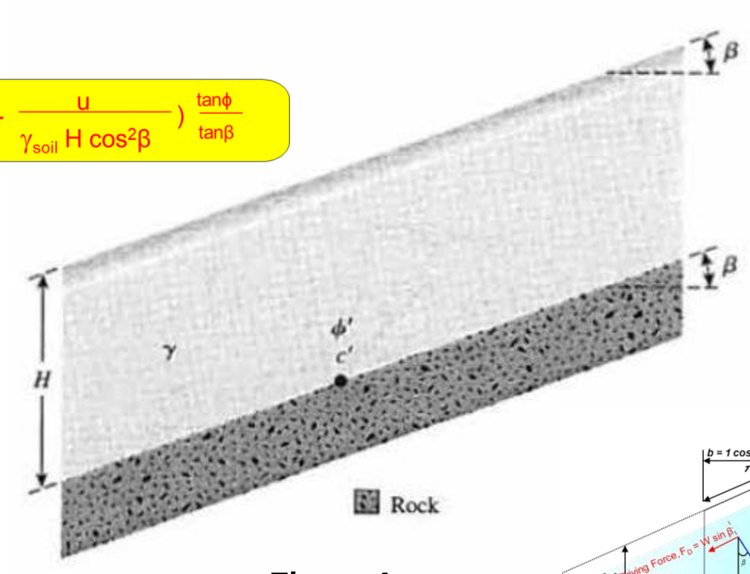
$$\frac{6.35(19)1.1 + 65 \times 18.9 \times 12.1}{13.31(19)(6.05) + 23.6(19)(4.4) + 21.93(19)(6.6)}$$

$\boxed{F.S. = 2.4}$

Problem 3

Refer to Figure 4, Given: $\beta = 20^o$, $\gamma = 18$ kN/m^3, $\phi = 25^o$, and c' = 14 kN/m^2. Find the height, H, that will have a factor of safety, F_S of 2.5 against sliding along the soil-rock interface.

$$FS = \frac{c}{\gamma_{soil}\, H \cos\beta \sin\beta} + \left(1 - \frac{u}{\gamma_{soil}\, H \cos^2\beta}\right) \frac{\tan\phi}{\tan\beta}$$



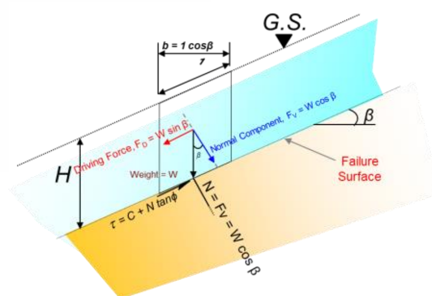
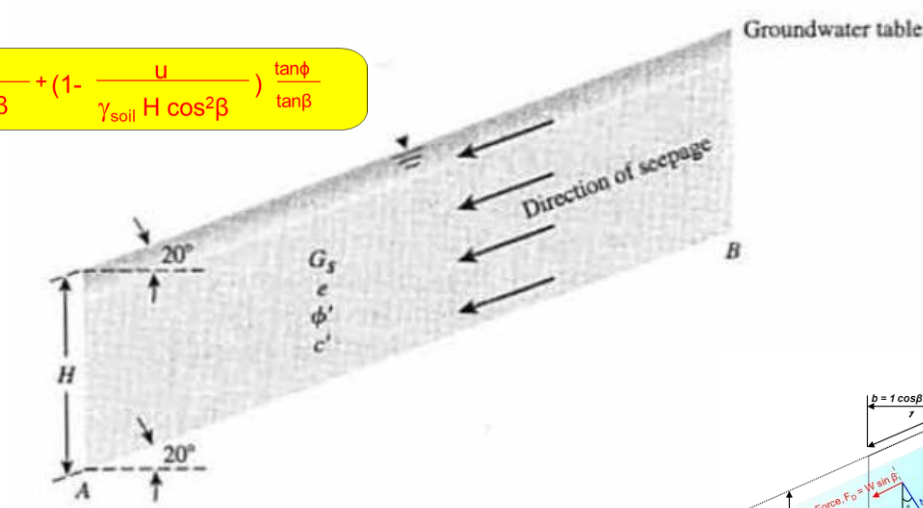


Figure 4

Problem 4

For the infinite slope shown in Figure 5, find the factor of safely against sliding along the plane *AB,* given that H = 20 ft. $\gamma = 110$ pcf, $\phi = 20^o$, and c' = 500 psf. Note that there is seepage through the soil and that the groundwater table coincides with the ground surface.

$$FS = \frac{c}{\gamma_{soil}\, H \cos\beta \sin\beta} + \left(1 - \frac{u}{\gamma_{soil}\, H \cos^2\beta}\right) \frac{\tan\phi}{\tan\beta}$$



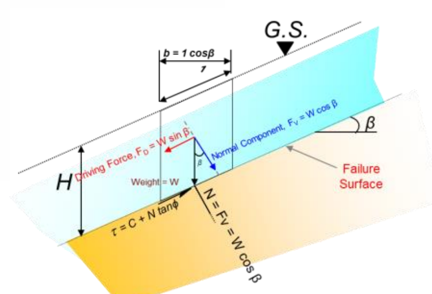
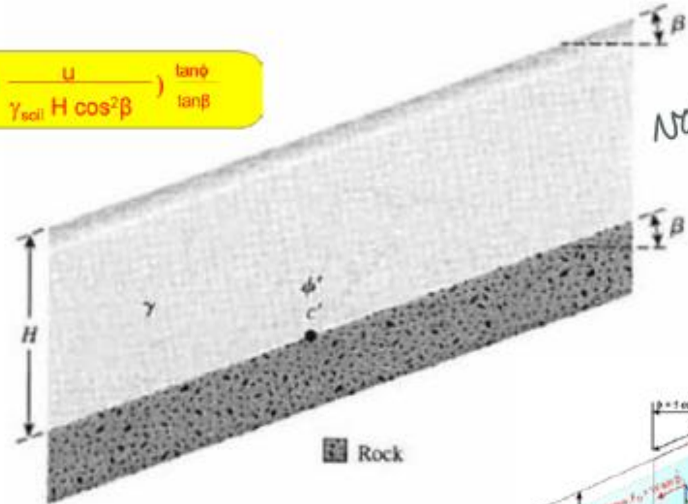


Figure 5

3.

Refer to Figure 4, Given: $\beta = 20°$, $\gamma = 18$ kN/m^3, $\phi = 25°$, and c' = 14 kN/m^2. Find the height, H, that will have a factor of safety, F_s of 2.5 against sliding along the soil-rock interface.

$$FS = \frac{c}{\gamma_{soil} H \cos\beta \sin\beta} + \left(1 - \frac{u}{\gamma_{soil} H \cos^2\beta}\right) \frac{\tan\phi}{\tan\beta}$$



No Seepage

seepage

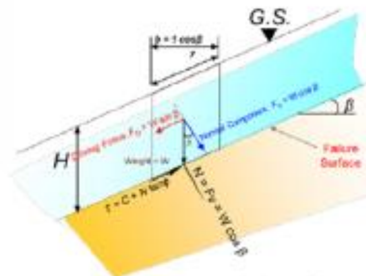


Figure 4

$\beta = 20$

$\gamma = 18$

$\phi = 25$

$c' = 14$

F.S 2.5

$$2.5 = \frac{14}{18 H \cos(20)(\sin(20))} + \left(1 - \frac{0?}{18 H \cos^2(20)}\right) \frac{\tan(25)}{\tan(20)}$$

$$2.5 = \frac{2.4200}{H} + 1.2811$$

$$1.2188 = \frac{2.4200}{H}$$

$$\boxed{H = 1.9855 \text{ m}}$$

3) * note that ϕ and c are given as max → must use c_{dev} and ϕ_{dev} for Hdesign eqtn

infinite slope in $c-\phi$ soil w/ NO seepage → $u=0$

$FS = 2.5$

$H = ?$

$$H_{cr} = \frac{c_{max}}{\gamma_{soil}\ [\sin\beta\cos\beta\ -\ \tan\emptyset_{max}\ (\cos^2\beta\ -\ r_u)]}$$

Or

$$H = \frac{c_{dev}}{\gamma_{soil}[\sin\beta\cos\beta\ -\ \tan\emptyset_{dev}(\cos^2\beta\ -\ r_u)]}$$

$$C_{dev} = \frac{C}{F.S._c} \rightarrow C_{dev} = \frac{14\ kN/m^2}{2.5} \rightarrow C_{dev} = 5.6\ kN/m^2$$

$$\tan\phi_{dev} = \frac{\tan\phi}{F.S._\phi} \rightarrow \tan\phi_{dev} = \frac{\tan 25°}{2.5} = 0.1865$$

$$H = \frac{5.6}{(18)[\sin 20°\cos 20° - (0.1865)(\cos^2 20° - 0)]}$$

$\boxed{H = 1.9855\ m}$ Answer

Check using given F.S. eqtn (don't use c_{dev} and ϕ_{dev} here b/c it already converts)

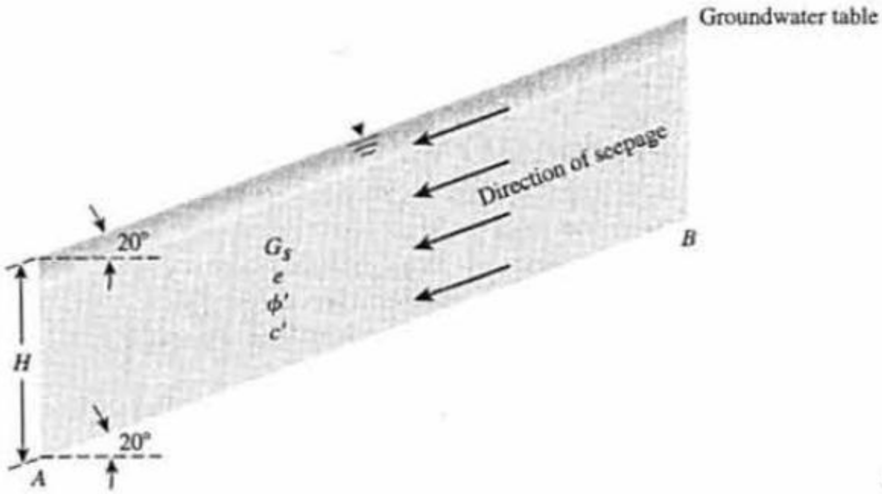
given: $$F.S. = \frac{c}{\gamma_{soil} H \cos\beta\sin\beta} + \left(1 - \frac{u}{\gamma_{soil} H \cos^2\beta}\right)\frac{\tan\phi}{\tan\beta}$$

$$2.5 = \frac{14\ kN/m^2}{(18\ kN/m^3)H\cos 20°\sin 20°} + \left(1 - \cancelto{0}{\frac{0}{(18)H\cos^2 20°}}\right)\left(\frac{\tan 25°}{\tan 20°}\right)$$

$$1.2188 = \frac{14\ kN/m^2}{(18\ kN/m^3)H\cos 20°\sin 20°}$$

$H = 1.9855\ m$ checked ✓

4



$$H = \frac{c_{dev}}{\gamma_{soil}[sin\beta\ cos\beta\ -\ tan\emptyset_{dev}(cos^2\beta\ -\ r_u)]}$$

Pore water pressure = u = $Z\gamma_w = (d_w cos\alpha\ cos\alpha)\gamma_w = (20\ cos20\ cos20)x62.4 = 1102\ psf$

$$r_u = \frac{u}{\gamma_{soil}\ H} = \frac{1102}{110x20} = 0.5$$

$Assume\ FS_\emptyset = 1\ \to\to\ \ \emptyset_d = \emptyset = 20^o$

$Thus,$

$c_{dev} = H\gamma_{soil}[sin\beta\ cos\beta - tan\emptyset_{dev}(cos^2\beta\ -\ r_u)] = 20x110[0.342x0.94 - 0.364(0.883\text{-}0.5)$= 400.549 psf

$$FS_c = \frac{c}{c_{dev}} = \frac{500}{400.546} = 1.25$$

$Assume\ FS_\emptyset = 1.2\ \to\to\ \emptyset_{dev} = tan^{-1}\left(\frac{tan\emptyset}{F.S._\emptyset}\right) = tan^{-1}\left(\frac{tan20}{1.2}\right) = 16.87^o$

$c_{dev} = H\gamma_{soil}[sin\beta\ cos\beta - tan\emptyset_{dev}(cos^2\beta\ -\ r_u)] = 20x110[0.342x0.94 - 0.303(0.883\text{-}0.5)$= 451.95 psf

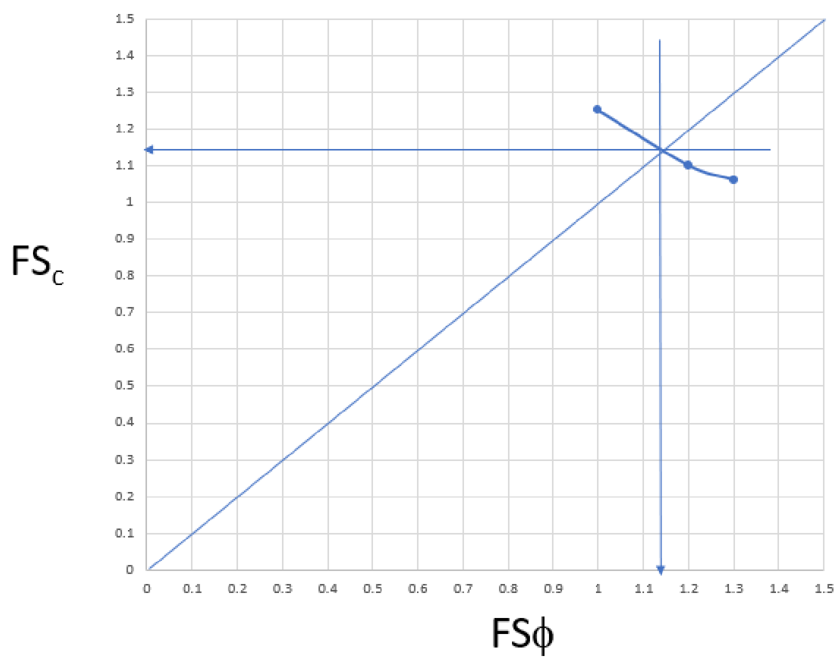
$$FS_c = \frac{c}{c_{dev}} = \frac{500}{451.95} = 1.1$$

$Assume\ FS_\emptyset = 1.3\ \to\to\ \emptyset_{dev} = tan^{-1}\left(\frac{tan\emptyset}{F.S._\emptyset}\right) = tan^{-1}\left(\frac{tan20}{1.2}\right) = 15.64^o$

$c_{dev} = H\gamma_{soil}[sin\beta\ cos\beta - tan\emptyset_{dev}(cos^2\beta\ -\ r_u)] = 20x110[0.342x0.94 - 0.279(0.883\text{-}0.5)$= 471.35 psf

$$FS_c = \frac{c}{c_{dev}} = \frac{500}{471.34} = 1.061$$

The actual Factor of Safety = 1.14



The actual Factor of Safety = 1.14

Problem 5

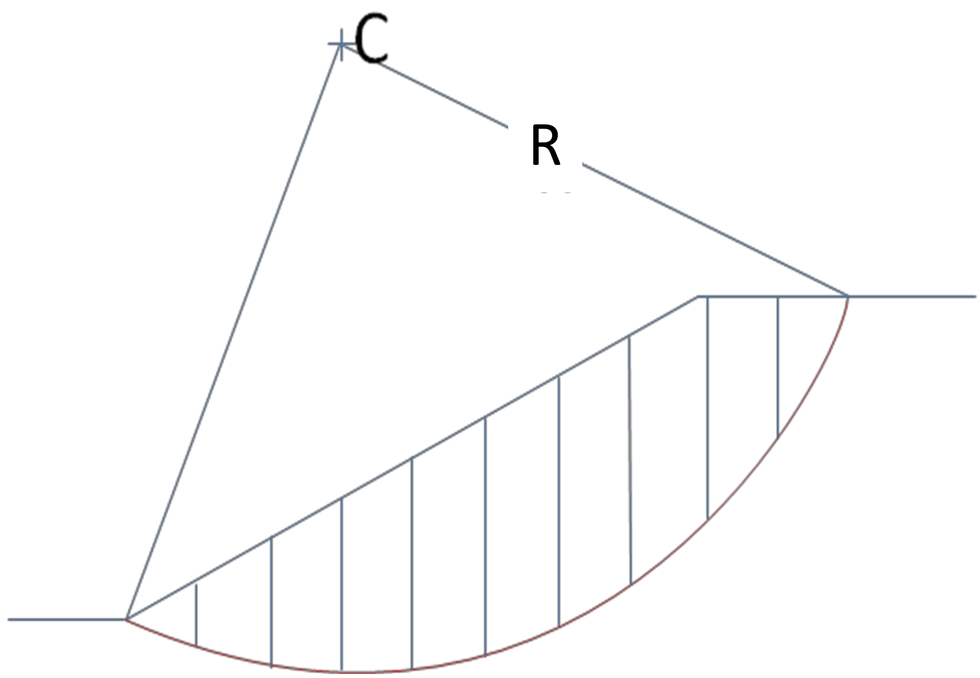
Find the factor of safety for a 20 meter high 2H - I V slope shown in the following figure using ordinary method of slices. Each slice has a width of 5 meters.

γ = 16 kN/m^3

c =20 *kPa*

ϕ = *20O*

R =38.1 *m*



Slice	Width Δx (ft)	Ave Height (ft)	Weight (Kips)	θ_i	$W_i \sin\theta_i$	$W_i \cos\theta_i$	u_i	Δl_i	$U_i = u_i \Delta l_i$	$N_i = W_i \cos\theta_i - U_i$
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

$$\text{F.S.} = \frac{cL + \tan\varphi \sum_{i=1}^{i=n}(W_i cos\theta_i - u_i \Delta l_i)}{\sum_{i=1}^{i=n} W_i sin\theta_i}$$

γ = 16 kN/m^3

c =20 kPa kN/m^2

ϕ = 20O

R =38.1 m

Scale: 1 cm = 5 m

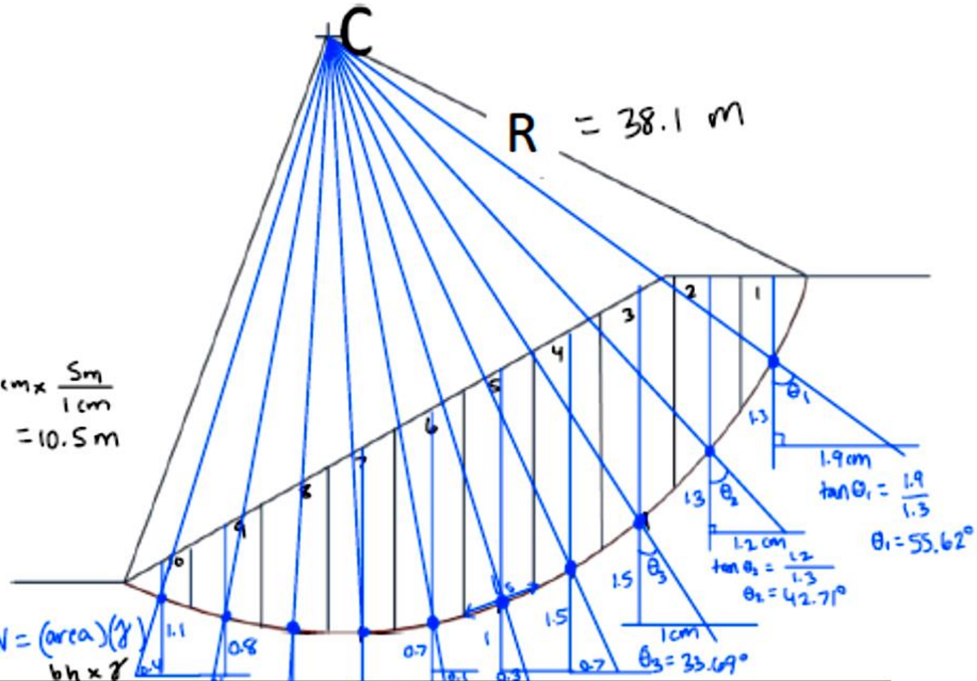
① 1.4 cm × $\frac{5m}{1cm}$ = 7 m

② 2.4 cm × $\frac{5m}{1cm}$ = 12 m

③ 3.5 cm × $\frac{5m}{1cm}$ = 17.5 m

④ 3.6 cm × 5m = 18 m

2.1 cm × $\frac{5m}{1cm}$ = 10.5 m



W = (area)(γ)
bh × γ

Slice	Width Δx m	Ave Height m	Weight kN/m	θ_i	$W_i \sin\theta_i$	$W_i \cos\theta_i$	u_i	Δl_i [m]	$U_i = u_i \Delta l_i$	$N_i = W_i \cos\theta_i - U_i$
1	5	7	560	55.62°	462.17	316.22	0	10.5	0	316.22
2	5	12	960	42.71°	651.16	705.40	0	7.5	0	705.40
3	5	17.5	1400	33.69°	776.58	1164.87	0	7	0	1164.87
4	5	18	1440	25.02°	609.03	1304.87	0	5.5	0	1304.87
5	5	17.5	1400	16.70°	402.3	1340.95	0	5	0	1340.95
6	5	16.5	1320	8.13°	186.67	1306.73	0	5	0	1306.73
7	5	14	1120	0°	0	1120	0	4.5	0	1120
8	5	11	880	−4.0°	−61.39	877.86	0	4.5	0	877.86
9	5	6.5	520	−7.13°	−64.54	515.98	0	5	0	515.98
10	5	3	240	−19.98°	−82.01	225.55	0	5	0	225.55
				Σ=	2879.97			59.5		8878.43

$$\text{F.S.} = \frac{cL + \tan\varphi \sum_{i=1}^{i=n}(W_i cos\theta_i - u_i \Delta l_i)}{\sum_{i=1}^{i=n} W_i sin\theta_i}$$

cont.

$$F.S. = \frac{cL + \tan\phi \, \Sigma(W_i \cos\theta_i - u_i \Delta l_i)}{\Sigma W_i \sin\theta_i}$$

$$= \frac{(20 \frac{kN}{m^2})(59.5) + (\tan 20°)(8878.43)}{2879.97}$$

$$\boxed{F.S. = 1.535}$$