CHAPTER 6

STRESS DISTRIBUTION IN SOILS DUE TO SURFACE LOADS

6.1 INTRODUCTION

Estimation of vertical stresses at any point in a soil-mass due to external vertical loadings are of great significance in the prediction of settlements of buildings, bridges, embankments and many other structures. Equations have been developed to compute stresses at any point in a soil mass on the basis of the theory of elasticity. According to elastic theory, constant ratios exist between stresses and strains. For the theory to be applicable, the real requirement is not that the material necessarily be elastic, but there must be constant ratios between stresses and the corresponding strains. Therefore, in non-elastic soil masses, the elastic theory may be assumed to hold so long as the stresses induced in the soil mass are relatively small. Since the stresses in the subsoil of a structure having adequate factor of safety against shear failure are relatively small in comparison with the ultimate strength of the material, the soil may be assumed to behave elastically under such stresses.

When a load is applied to the soil surface, it increases the vertical stresses within the soil mass. The increased stresses are greatest directly under the loaded area, but extend indefinitely in all directions. Many formulas based on the theory of elasticity have been used to compute stresses in soils. They are all similar and differ only in the assumptions made to represent the elastic conditions of the soil mass. The formulas that are most widely used are the Boussinesq and Westergaard formulas. These formulas were first developed for point loads acting at the surface. These formulas have been integrated to give stresses below uniform strip loads and rectangular loads.

The extent of the elastic layer below the surface loadings may be any one of the following:

1. Infinite in the vertical and horizontal directions.
2. Limited thickness in the vertical direction underlain with a rough rigid base such as a rocky bed.

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The loads at the surface may act on flexible or rigid footings. The stress conditions in the elastic layer below vary according to the rigidity of the footings and the thickness of the elastic layer. All the external loads considered in this book are vertical loads only as the vertical loads are of practical importance for computing settlements of foundations.

6.2 BOUSSINESQ'S FORMULA FOR POINT LOADS

Figure 6.1 shows a load Q acting at a point O on the surface of a semi-infinite solid. A semi-infinite solid is the one bounded on one side by a horizontal surface, here the surface of the earth, and infinite in all the other directions. The problem of determining stresses at any point P at a depth z as a result of a surface point laod was solved by Boussinesq (1885) on the following assumptions.

1. The soil mass is elastic, isotropic, homogeneous and semi-infinite.
2. The soil is weightless.
3. The load is a point load acting on the surface.

The soil is said to be isotropic if there are identical elastic properties throughout the mass and in every direction through any point of it. The soil is said to be homogeneous if there are identical elastic properties at every point of the mass in identical directions.

The expression obtained by Boussinesq for computing vertical stress σ_z at point P (Fig. 6.1) due to a point load Q is

$$\sigma_z = \frac{3Q}{2\pi z^2}\frac{1}{[1+(r/z)^2]^{5/2}} = \frac{Q}{z^2}I_B \tag{6.1}$$

where, r = the horizontal distance between an arbitrary point P below the surface and the vertical axis through the point load Q.

z = the vertical depth of the point P from the surface.

$$I_B = \text{Boussinesq stress coefficient} = \frac{3}{2\pi}\frac{1}{[1+(r/z)^2]^{5/2}} \tag{6.1a}$$

The values of the Boussinesq coefficient I_B can be determined for a number of values of r/z. The variation of I_B with r/z in a graphical form is given in Fig. 6.2. It can be seen from this figure

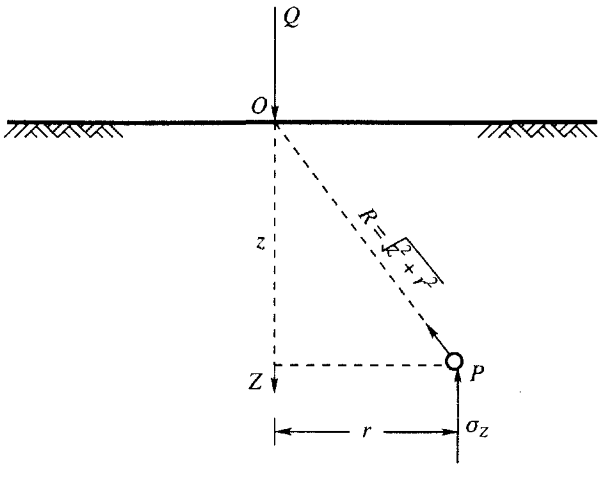

Figure 6.1 Vertical pressure within an earth mass

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that I_B has a maximum value of 0.48 at $r/z = 0$, i.e., indicating thereby that the stress is a maximum below the point load.

6.3 WESTERGAARD'S FORMULA FOR POINT LOADS

Boussinesq assumed that the soil is elastic, isotropic and homogeneous for the development of a point load formula. However, the soil is neither isotropic nor homogeneous. The most common type of soils that are met in nature are the water deposited sedimentary soils. When the soil particles are deposited in water, typical clay strata usually have their lenses of coarser materials within them. The soils of this type can be assumed as laterally reinforced by numerous, closely spaced, horizontal sheets of negligible thickness but of infinite rigidity, which prevent the mass as a whole from undergoing lateral movement of soil grains. Westergaard, a British Scientist, proposed (1938) a formula for the computation of vertical stress σ_z by a point load, Q, at the surface as

$$\sigma_z = \frac{Q}{2\pi z^2} \frac{\sqrt{(1-2\mu)/(2-2\mu)}}{[(1-2\mu)/(2-\mu)+(r/z)^2]^{3/2}} = \frac{Q}{z^2} I_w \tag{6.2}$$

in which μ is Poisson's ratio. If μ is taken as zero for all practical purposes, Eq. (6.2) simplifies to

$$\sigma_z = \frac{Q}{\pi z^2} \frac{1}{[1+2(r/z)^2]^{3/2}} = \frac{Q}{z^2} I_w \tag{6.3}$$

where $I_w = \dfrac{(1/\pi)}{[1+2(r/z)^2]^{3/2}}$ is the Westergaard stress coefficient. The variation of I_w with the ratios of (r/z) is shown graphically in Fig. 6.2 along with the Boussinesq's coefficient I_B. The value of I_w at $r/z = 0$ is 0.32 which is less than that of I_B by 33 per cent.

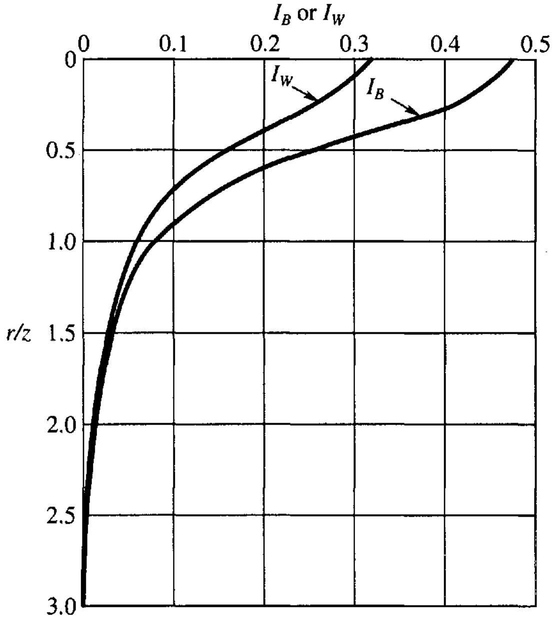

Figure 6.2 Values of I_B or I_W for use in the Boussinesq or Westergaard formula

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Geotechnical engineers prefer to use Boussinesq's solution as this gives conservative results. Further discussions are therefore limited to Boussinesq's method in this chapter.

Example 6.1

A concentrated load of 1000 kN is applied at the ground surface. Compute the vertical pressure (i) at a depth of 4 m below the load, (ii) at a distance of 3 m at the same depth. Use Boussinesq's equation.

Solution

The equation is

$$\sigma_z = \frac{Q}{z^2} I_B, \text{ where } I_B = \frac{3/2\pi}{\left[1+(r/z)^2\right]^{5/2}}$$

(i) When $r/z = 0$, $I_B = 3/2\,\pi = 0.48$, $\sigma_z = 0.48\dfrac{Q}{z^2} = 0.48 \times \dfrac{1000}{4 \times 4} = 30 \text{ kN/m}^2$

(ii) When $r/z = 3/4 = 0.75$

$$I_B = \frac{3/2\pi}{\left[1+(0.75)^2\right]^{5/2}} = 0.156, \quad \sigma_z = \frac{0.156 \times 1000}{4 \times 4} = 9.8 \text{ kN/m}^2$$

Example 6.2

A concentrated load of 45000 lb acts at foundation level at a depth of 6.56 ft below ground surface. Find the vertical stress along the axis of the load at a depth of 32.8 ft and at a radial distance of 16.4 ft at the same depth by (a) Boussinesq, and (b) Westergaard formulae for $\mu = 0$. Neglect the depth of the foundation.

Solution

(a) Boussinesq Eq. (6.1a)

$$\sigma_z = \frac{Q}{z^2} I_B, \quad I_B = \frac{3}{2\pi}\,{\frac{1}{1+(r/z)^2}}^{5/2}$$

Substituting the known values, and simplifying

$I_B = 0.2733$ for $r/z = 0.5$

$$\sigma_z = \frac{45000}{(32.8)^2} \times 0.2733 = 11.43 \text{ lb/ft}^2$$

(b) Westergaard (Eq. 6.3)

$$\sigma_z = \frac{Q}{z^2} I_w, \quad I_w = \frac{1}{\pi}\left[\frac{1}{1+2(r/z)^2}\right]^{3/2}$$

Substituting the known values and simplifying, we have,

$I_w = 0.1733$ for $r/z = 0.5$

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therefore,

$$\sigma_z = \frac{45000}{(32.8)^2} \times 0.1733 = 7.25 \text{ lb/ft}^2$$

Example 6.3

A rectangular raft of size 30 x 12 m founded at a depth of 2.5 m below the ground surface is subjected to a uniform pressure of 150 kPa. Assume the center of the area is the origin of coordinates (0, 0), and the corners have coordinates (6, 15). Calculate stresses at a depth of 20 m below the foundation level by the methods of (a) Boussinesq, and (b) Westergaard at coordinates of (0, 0), (0, 15), (6, 0) (6, 15) and (10, 25). Also determine the ratios of the stresses as obtained by the two methods. Neglect the effect of foundation depth on the stresses (Fig. Ex. 6.3).

Solution

Equations (a) Boussinesq: $\sigma_z = \frac{Q}{z^2} I_B, \; I_B = \frac{0.48}{\left[1+(r/z)^2\right]^{5/2}}$

(b) Westergaard: $\sigma_z = \frac{Q}{z^2} I_w, \; I_w = \frac{0.32}{\left[1+2(r/z)^2\right]^{3/2}}$

The ratios of r/z at the given locations for z = 20 m are as follows:

Location	r/z	Location	r/z
(0, 0)	0	(6, 15)	$\left(\sqrt{6^2+15^2}\right)/20 = 0.81$
(6, 0)	6/20 = 0.3	(10, 25)	$\left(\sqrt{10^2+25^2}\right)/20 = 1.35$
(0, 15)	15/20 = 0.75		

The stresses at the various locations at z = 20 m may be calculated by using the equations given above. The results are tabulated below for the given total load $Q = qBL$ = 150 x 12 x 30 = 54000 kN acting at (0, 0) coordinate. $Q/z^2 = 135$.

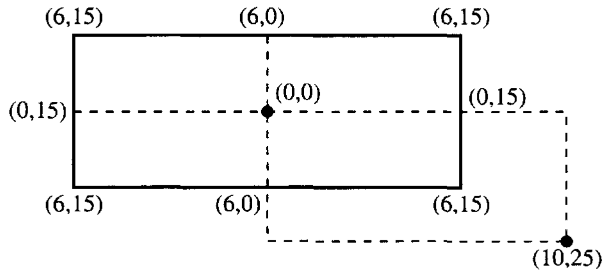

Figure Ex. 6.3

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Location	r/z	Boussinesq		Westergaard		σ_B/σ_W
		I_B	σ_B(kPa)	I_W	σ_W(kPa)	
(0, 0)	0	0.48	65	0.32	43	1.51
(6, 0)	0.3	0.39	53	0.25	34	1.56
(0, 15)	0.75	0.16	22	0.10	14	1.57
(6,15)	0.81	0.14	19	0.09	12	1.58
(10, 25)	1.35	0.036	5	0.03	4	1.25

6.4 LINE LOADS

The basic equation used for computing σ_z, at any point P in an elastic semi-infinite mass is Eq. (6.1) of Boussinesq. By applying the principle of his theory, the stresses at any point in the mass due to a line load of infinite extent acting at the surface may be obtained. The state of stress encountered in this case is that of a plane strain condition. The strain at any point P in the Y-direction parallel to the line load is assumed equal to zero. The stress σ_y normal to the XZ-plane (Fig. 6.3) is the same at all sections and the shear stresses on these sections are zero. By applying the theory of elasticity, stresses at any point P (Fig. 6.3) may be obtained either in polar coordinates or in rectangular coordinates. The vertical stress σ_z at point P may be written in rectangular coordinates as

$$\sigma_z = \frac{q}{z}\,\frac{2/\pi}{[1+(x/z)^2]^2} = \frac{q}{z} I_z \tag{6.4}$$

where, I_z is the influence factor equal to 0.637 at $x/z = 0$.

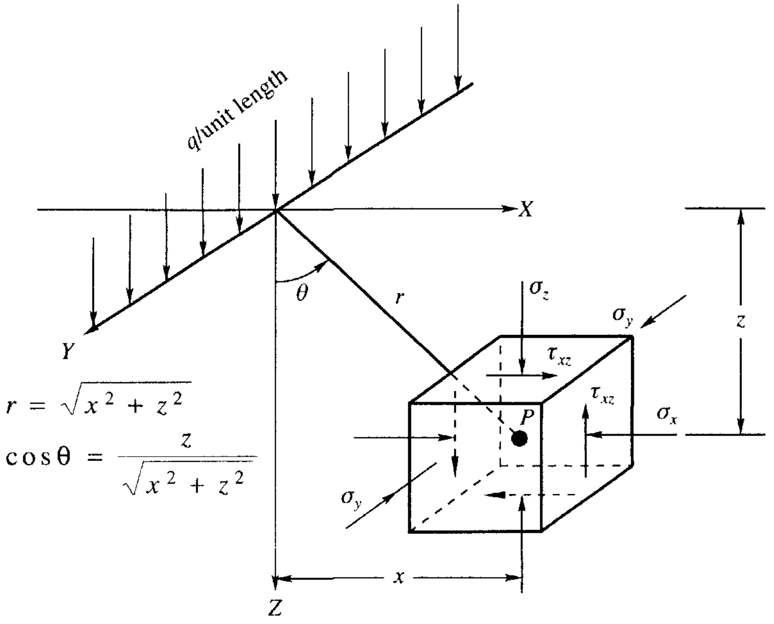

Figure 6.3 Stresses due to vertical line load in rectangular coordinates

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6.5 STRIP LOADS

The state of stress encountered in this case also is that of a plane strain condition. Such conditions are found for structures extended very much in one direction, such as strip and wall foundations, foundations of retaining walls, embankments, dams and the like. For such structures the distribution of stresses in any section (except for the end portions of 2 to 3 times the widths of the structures from its end) will be the same as in the neighboring sections, provided that the load does not change in directions perpendicular to the plane considered.

Fig. 6.4(a) shows a load q per unit area acting on a strip of infinite length and of constant width B. The vertical stress at any arbitrary point P due to a line load of $q\overline{dx}$ acting at $x = \bar{x}$ can be written from Eq. (6.4) as

$$d\sigma_z = \frac{2q}{\pi}\frac{z^3}{[(x-\bar{x})^2+z^2]^2} \tag{6.5}$$

Applying the principle of superposition, the total stress σ_z at point P due to a strip load distributed over a width $B(= 2b)$ may be written as

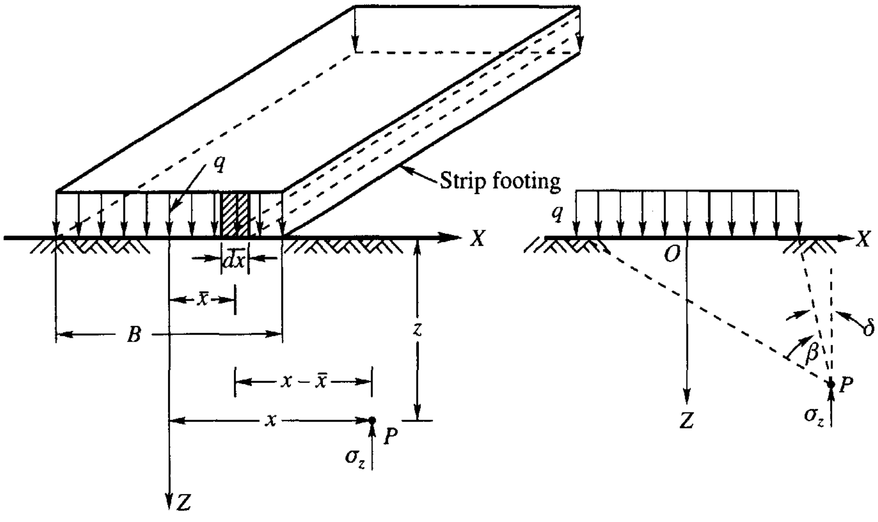
$$\sigma_z = \frac{2q}{\pi}\int_{-b}^{+b}\frac{z^3}{[(x-\bar{x})^2+z^2]^2}dx$$

or

$$\sigma_z = \frac{q}{\pi}\tan^{-1}\frac{z}{x-b} - \tan^{-1}\frac{z}{x+b} - \frac{2bz(x^2-b^2-z^2)}{(x^2-b^2+z^2)^2+4b^2z^2} \tag{6.6}$$

The non-dimensional values of σ_z/q are given graphically in Fig. 6.5. Eq. (6.6) can be expressed in a more convenient form as

$$\sigma_z = \frac{q}{\pi}[\beta + \sin\beta\cos(\beta + 2\delta)] \tag{6.7}$$



(a) (b)

Figure 6.4 Strip load

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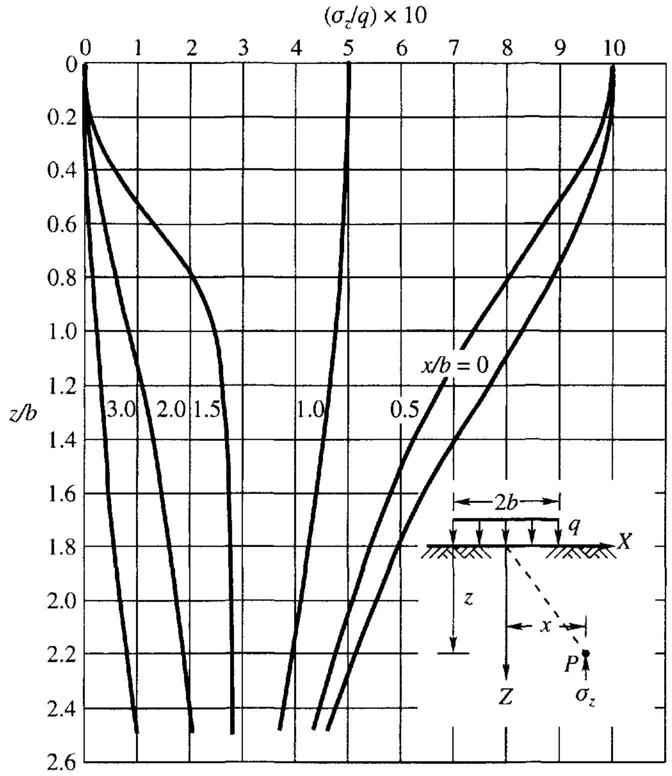

Figure 6.5 Non-dimensional values of σ_z/q for strip load

where β and δ are the angles as shown in Fig. 6.4(b). Equation (6.7) is very convenient for computing σ_z, since the angles β and δ can be obtained graphically for any point *P*. The principal stresses σ_1 and σ_3 at any point *P* may be obtained from the equations.

$$\sigma_1 = \frac{q}{\pi}(\beta + \sin\beta) \tag{6.8}$$

$$\sigma_3 = \frac{q}{\pi}(\beta - \sin\beta) \tag{6.9}$$

Example 6.4

Three parallel strip footings 3 m wide each and 5 m apart center to center transmit contact pressures of 200, 150 and 100 kN/m^2 respectively. Calculate the vertical stress due to the combined loads beneath the centers of each footing at a depth of 3 m below the base. Assume the footings are placed at a depth of 2 m below the ground surface. Use Boussinesq's method for line loads.

Solution

From Eq. (6.4), we have

$$\sigma_z = \frac{q}{z}\frac{2/\pi}{\left[1+(x/z)^2\right]^2} = \frac{q}{z}I_z$$

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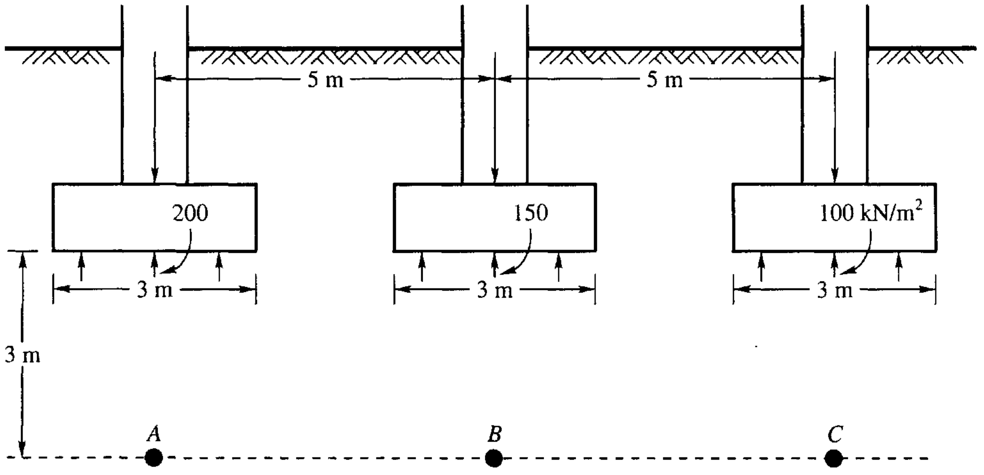

Figure Ex. 6.4 Three parallel footings

The stress at *A* (Fig. Ex. 6.4) is

$$(\sigma_z)_A = \frac{2\times 200}{3.14\times 3}\left[\frac{1}{1+(0/3)^2}\right]^2 + \frac{2\times 150}{3.14\times 3}\left[\frac{1}{1+(5/3)^2}\right]^2$$

$$+\frac{2\times 100}{3.14\times 3}\left[\frac{1}{1+(10/3)^2}\right]^2 = 45\ \text{kN}/\text{m}^2$$

The stress at *B*

$$(\sigma_z)_B = \frac{2\times 200}{3\pi}\left[\frac{1}{1+(5/3)^2}\right]^2 + \frac{2\times 150}{3\pi}\left[\frac{1}{1+(0/3)^2}\right]^2$$

$$+\frac{2\times 100}{3\pi}\left[\frac{1}{1+(5/3)^2}\right]^2 = 36.3\ \text{kN}/\text{m}^2$$

The stress at *C*

$$(\sigma_z)_C = \frac{2\times 200}{3\pi}\;{\frac{1}{1+(10/3)^2}}^{\,2} + \frac{2\times 150}{3\pi}\;{\frac{1}{1+(5/3)^2}}^{\,2} + \frac{2\times 100}{3\pi} = 23.74\ \text{kN}/\text{m}^2$$

6.6 STRESSES BENEATH THE CORNER OF A RECTANGULAR FOUNDATION

Consider an infinitely small unit of area of size *db* × *dl*, shown in Fig. 6.6. The pressure acting on the small area may be replaced by a concentrated load *dQ* applied to the center of the area.

Hence

$$dQ = q\, db\,.\, dl \qquad (6.10)$$

The increase of the vertical stress σ_z due to the load *dQ* can be expressed per Eq. (6.11) as

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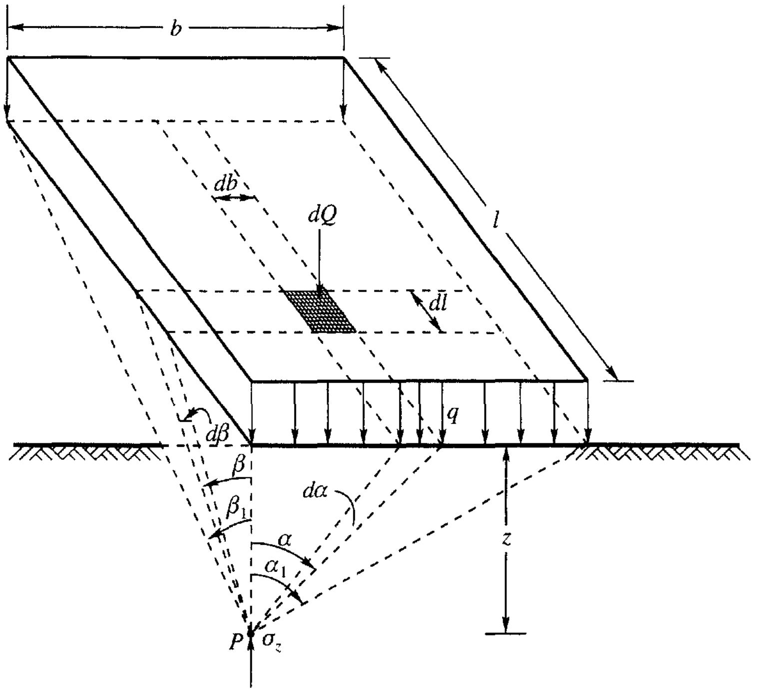

Figure 6.6 Vertical stress under the corner of a rectangular foundation

$$d\sigma_z = \frac{dQ}{2\pi} \frac{3z^3}{(z^2 + r^2)^{5/2}} \tag{6.11}$$

The stress produced by the pressure q over the entire rectangle $b \times l$ can then be obtained by expressing dl, db and r in terms of the angles α and β, and integrating

$$\sigma_z = \int_{\alpha=0}^{\alpha=\alpha_1} \int_{\beta=0}^{\beta=\beta_1} d\sigma_z \tag{6.12}$$

There are several forms of solution for Eq. (6.12). The one that is normally used is of the following form

$$\sigma_z = q \frac{1}{4\pi} \frac{2mn(m^2 + n^2 + 1)^{1/2}}{m^2 + n^2 + m^2n^2 + 1} \frac{m^2 + n^2 + 2}{m^2 + n^2 + 1} + \tan^{-1} \frac{2mn(m^2 + n^2 + 1)^{1/2}}{m^2 + n^2 - m^2n^2 + 1} \tag{6.13}$$

or $\quad \sigma_z = qI$ (6.14)

wherein, $m = b/z$, $n = l/z$, are pure numbers. I is a dimensionless factor and represents the influence of a surcharge covering a rectangular area on the vertical stress at a point located at a depth z below one of its corners.

Eq. (6.14) is presented in graphical form in Fig. 6.7. This chart helps to compute pressures beneath loaded rectangular areas. The chart also shows that the vertical pressure is not materially altered if the length of the rectangle is greater than ten times its width. Fig. 6.8 may also be used for computing the influence value I based on the values of m and n and may also be used to determine stresses below points that lie either inside or outside the loaded areas as follows.

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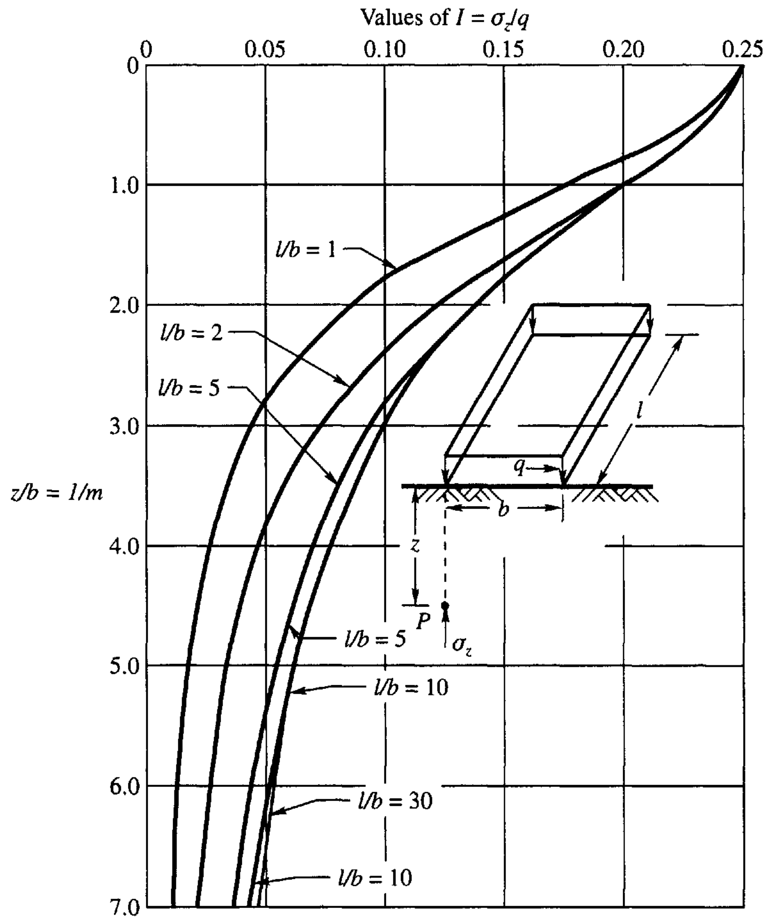

Figure 6.7 Chart for computing σ_z below the corner of a rectangular foundation (after Steinbrenner, 1934)

When the Point is Inside

Let O be an interior point of a rectangular loaded area $ABCD$ shown in Fig. 6.9(a). It is required to compute the vertical stress σ_z below this point O at a depth z from the surface. For this purpose, divide the rectangle $ABCD$ into four rectangles marked 1 to 4 in the Fig. 6.9(a) by drawing lines through O. For each of these rectangles, compute the ratios z/b. The influence value I may be obtained from Fig. 6.7 or 6.8 for each of these ratios and the total stress at P is therefore

$$\sigma_z = q\,(I_1 + I_2 + I_3 + I_4) \tag{6.15}$$

When the Point is Outside

Let O be an exterior point of loaded rectangular area $ABCD$ shown in Fig. 6.9(b). It is required to compute the vertical stress σ_z below point O at a depth z from the surface.

Construct rectangles as shown in the figure. The point O is the corner point of the rectangle OB_1CD_1. From the figure it can be seen that

$$\text{Area } ABCD = OB_1CD_1 - OB_1BD_2 - OD_1DA_1 + OA_1AD_2 \tag{6.16}$$

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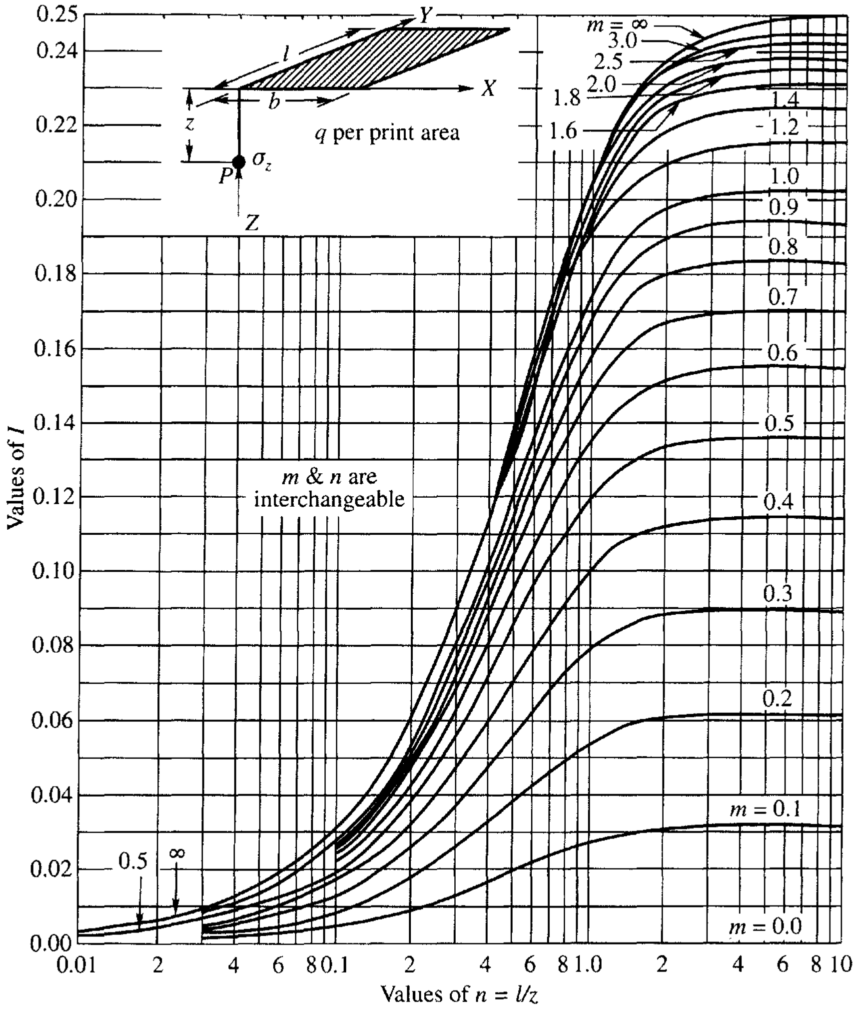
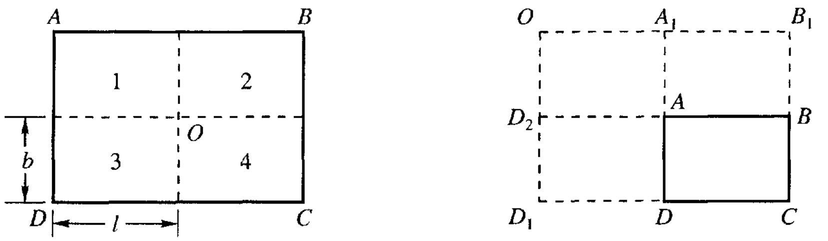


Figure 6.8 Graph for determining influence value for vertical normal stress σ_z at point P located beneath one corner of a uniformly loaded rectangular area. (After Fadum, 1948)



(a) When the point 'O' is within the rectangle (b) When the point 'O' is outside the rectangle

Figure 6.9 Computation of vertical stress below a point

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The vertical stress at point P located at a depth z below point O due to a surcharge q per unit area of $ABCD$ is equal to the algebraic sum of the vertical stresses produced by loading each one of the areas listed on the right hand side of the Eq. (6.16) with q per unit of area. If I_1 to I_4 are the influence factors of each of these areas, the total vertical stress is

$$\sigma_z = q\,(I_1 - I_2 - I_3 + I_4) \tag{6.17}$$

Example 6.5

$ABCD$ is a raft foundation of a multi-story building [Fig. 6. 9(b)] wherein AB = 65.6 ft, and BC = 39.6 ft. The uniformly distributed load q over the raft is 7310 lb/ft^2. Determine σ_z at a depth of 19.7 ft below point O [Fig. 6.9(b)] wherein AA_1 = 13.12 ft and A_1O = 19.68 ft. Use Fig. 6.8.

Solution

Rectangles are constructed as shown in [Fig. 6.9(b)].

Area $ABCD = OB_1CD_1 - OB_1BD_2 - OD_1DA_1 + OA_1AD_2$

Rectangle	l (ft)	b (ft)	m	n	I
OB_1CD_1	85.28	52.72	2.67	4.33	0.245
OB_1BD_2	85.28	13.12	0.67	4.33	0.168
OD_1DA_1	52.72	19.68	1.00	2.67	0.194
OA_1AD_2	19.68	13.12	0.67	1.00	0.145

Per Eq. (6.17)

$$\sigma_z = q\,(I_1 - I_2 - I_3 + I_4) = 7310\,(0.245 - 0.168 - 0.194 + 0.145) = 204.67 \text{ lb/ft}^2$$

The same value can be obtained using Fig. 6.7.

Example 6.6

A rectangular raft of size 30 × 12 m founded on the ground surface is subjected to a uniform pressure of 150 kN/m^2. Assume the center of the area as the origin of coordinates (0, 0), and corners with coordinates (6, 15). Calculate the induced stress at a depth of 20 m by the exact method at location (0, 0).

Solution

Divide the rectangle 12 × 30 m into four equal parts of size 6 × 15 m.

The stress below the corner of each footing may be calculated by using charts given in Fig. 6.7 or Fig. 6.8. Here Fig. 6.7 is used.

For a rectangle 6 × 15 m, $z/b = 20/6 = 3.34$, $l/b = 15/6 = 2.5$.

For $z/b = 3.34$, $l/b = 2.5$, $\bar{\sigma}_z/q = 0.07$

Therefore, $\sigma_z = 4\bar{\sigma}_z = 4 \times 0.07q = 4 \times 0.07 \times 150 = 42$ kN/m^2.

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6.7 STRESSES UNDER UNIFORMLY LOADED CIRCULAR FOOTING

Stresses Along the Vertical Axis of Symmetry

Figure 6.10 shows a plan and section of the loaded circular footing. The stress required to be determined at any point P along the axis is the vertical stress σ_z.

Let dA be an elementary area considered as shown in Fig. 6.10. dQ may be considered as the point load acting on this area which is equal to $q\,dA$. We may write

$$dQ = q\,dA = qr\,d\theta\,dr \tag{6.18}$$

The vertical stress $d\sigma$ at point P due to point load dQ may be expressed [Eq. (6.1a)] as

$$d\sigma_z = \frac{3q}{2\pi}\frac{z^3 r\,d\theta\,dr}{(r^2+z^2)^{5/2}} \tag{6.19}$$

The integral form of the equation for the entire circular area may be written as

$$\sigma_z = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=R_0} d\sigma_z = \frac{3qz^3}{2\pi}\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=R_0}\frac{r\,d\theta\,dr}{(r^2+z^2)^{5/2}}$$

On integration we have, $$\sigma_z = q\;1-\frac{z^3}{(R_0^2+z^2)^{3/2}} \tag{6.20}$$

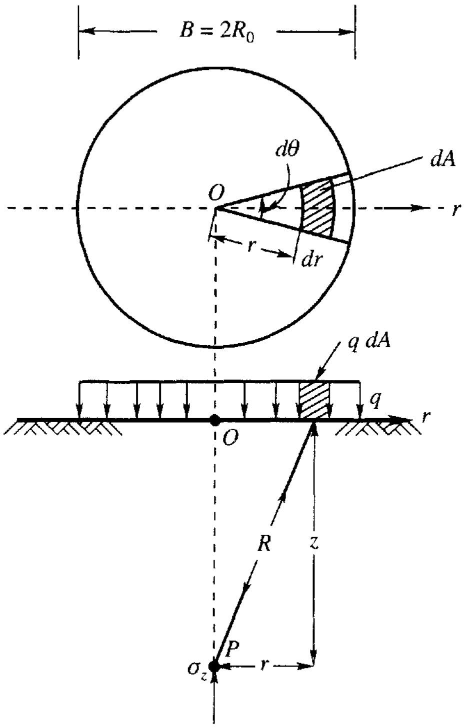

Figure 6.10 Vertical stress under uniformly loaded circular footing

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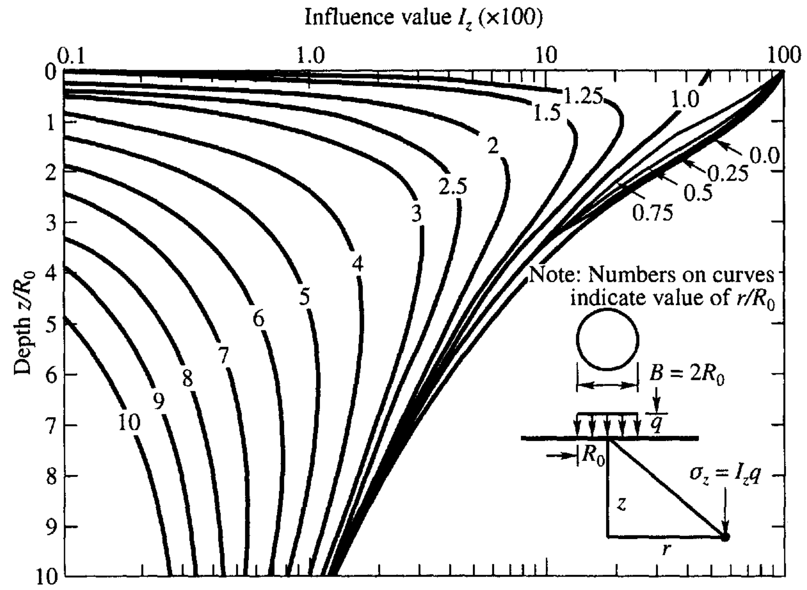

Figure 6.11 Influence diagram for vertical normal stress at various points within an elastic half-space under a uniformly loaded circular area. (After Foster and Ahlvin, 1954)

or

$$\sigma_z/q = 1 - \left[\frac{1}{1+(R_0/z)^2}\right]^{3/2} = I_z \tag{6.21}$$

where, I_z *is the Influence coefficient.* The stress at any point P on the axis of symmetry of a circular loaded area may be calculated by the use of Eq. (6.21) Vertical stresses σ_z may be calculated by using the influence coefficient diagram given in Fig. 6.11.

Example 6.7

A water tank is required to be constructed with a circular foundation having a diameter of 16 m founded at a depth of 2 m below the ground surface. The estimated distributed load on the foundation is 325 kN/m^2. Assuming that the subsoil extends to a great depth and is isotropic and homogeneous, determine the stresses σ_z at points (i) $z = 8$ m, $r = 0$, (ii) $z = 8$ m, $r = 8$ m, (iii) $z = 16$ m, $r = 0$ and (iv) $z = 16$ m, $r = 8$ m, where r is the radial distance from the central axis. Neglect the effect of the depth of the foundation on the stresses. (Use Fig. 6.11)

Solution

$q = 325$ kN/m^2, $R_0 = 8$ m. The results are given in a tabular form as follows:

	Point	z/R_0	r/R_0	I	σ_z kN/m^2
(i)	(8, 0)	1	0	0.7	227.5
(ii)	(8, 8)	1	1.0	0.33	107.25
(iii)	(16, 0)	2	0	0.3	97.5
(iv)	(16, 8)	2	1.0	0.2	65

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Example 6.8

For a raft of size 98.4 x 39.36 ft, compute the stress at 65.6 ft depth below the center of the raft by assuming that the rectangle can be represented by an equivalent circle. The load intensity on the raft is 3133 lb/ft^2.

Solution

The radius of a fictitious circular footing of area equal to the rectangular footing of size 98.4 x 39.36 ft is

$$\pi R_0^2 = 98.4 \times 39.36 = 3873 \text{ sq. ft} \text{ or } R_0 = \sqrt{\frac{3873}{\pi}} = 35.12 \text{ ft}$$

Use Eq. (6.21) for computing σ_z at 35.6 ft depth

Now, $z / R_0 = \dfrac{65.6}{35.12} = 1.9$, and $r/R_0 = 0$. From Fig. 6.11, $I_z = 0.3$

Therefore, $\sigma_z = 0.3\, q = 0.3 \times 3133 = 940$ lb/ft^2.

6.8 VERTICAL STRESS BENEATH LOADED AREAS OF IRREGULAR SHAPE

Newmark's Influence Chart

When the foundation consists of a large number of footings or when the loaded mats or rafts are not regular in shape, a chart developed by Newmark (1942) is more practical than the methods explained before. It is based on the following procedure. The vertical stress σ_z below the center of a circular area of radius R which carries uniformly distributed load q is determined per Eq. (6.21).

It may be seen from Eq. (6.21) that when $R/z = \infty$, $\sigma_z/q = 1$, that is $\sigma_z = q$. This indicates that if the loaded area extends to infinity, the vertical stress in the semi-infinite solid at any depth z is the same as unit load q at the surface. If the loaded area is limited to any given radius R_1 it is possible to determine from Eq. (6.21) the ratios R/z for which the ratio of σ_z/q may have any specified value, say 0.8 or 0.6. Table 6.1 gives the ratios of R/z for different values of σ_z/q.

Table 6.1 may be used for the computation of vertical stress σ_z at any depth z below the center of a circular loaded area of radius R. For example, at any depth z, the vertical stress $\sigma_z = 0.8\, q$ if the radius of the loaded area at the surface is $R = 1.387\, z$. At the same depth, the vertical stress is $\sigma_z = 0.7\, q$ if $R = 1.110\, z$. If instead of loading the whole area, if only the annular space between the circles of radii $1.387\, z$ and $1.110\, z$ are loaded, the vertical stress at z at the center of the circle is $\Delta\sigma_z = 0.8\, q - 0.7\, q = 0.1\, q$. Similarly if the annular space between circles of radii $1.110\, z$ and $0.917\, z$ are loaded, the vertical stress at the same depth z is $\Delta\sigma_z = 0.7\, q - 0.6\, q = 0.1\, q$. We may therefore draw a series of concentric circles on the surface of the ground in such a way that when the annular space between any two consecutive circles is loaded with a load q per unit area, the vertical stress $\Delta\sigma_z$ produced at any depth z below the center remains a constant fraction of q. We may write, therefore,

$$\Delta\sigma_z = Cq \tag{6.22}$$

where C is constant. If an annular space between any two consecutive concentric circles is divided into n equal blocks and if any one such block is loaded with a distributed load q, the vertical stress produced at the center is, therefore,

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Table 6.1 Values of R/z for different values of σ_z/q

σ_z/q	R/z	σ_z/q	R/z
0.00	0.000	0.80	1.387
0.10	0.270	0.90	1.908
0.20	0.401	0.92	2.094
0.30	0.518	0.94	2.351
0.40	0.637	0.96	2.748
0.50	0.766	0.98	3.546
0.60	0.917	1.00	∞
0.70	1.110	–	–

$$\frac{\Delta\sigma_z}{n} = \frac{C}{n}q = C_i q \tag{6.23}$$

$$\frac{\Delta\sigma_z}{n} = C_i \text{ when } q = 1.$$

A load $q = 1$ covering one of the blocks will produce a vertical stress C_i. In other words, the '*influence value*' of each loaded block is C_i. If the number of loaded blocks is N, and if the intensity of load is q per unit area, the total vertical stress at depth z below the center of the circle is

$$\sigma_z = C_i N q \tag{6.24}$$

The graphical procedure for computing the vertical stress σ_z due to any surface loading is as follows.

Select some definite scale to represent depth z. For instance a suitable length AB in cm as shown in Fig. 6.12 to represent depth z in meters. In such a case, the scale is 1 cm = z/AB meters. The length of the radius $R_{0.8}$ which corresponds to $\sigma_z/q = 0.8$ is then equal to $1.387 \times AB$ cm, and a circle of that radius may be drawn. This procedure may be repeated for other ratios of σ_z/q, for instance, for $\sigma_z/q = 0.7$, 0. 5 etc. shown in Fig. 6.12.

The annular space between the circles may be divided into n equal blocks, and in this case $n = 20$. The influence value C_i is therefore equal to $0.1/20 = 0.005$. A plan of the foundation is drawn on a tracing paper to a scale such that the distance AB on the chart corresponds to the depth z at which the stress σ_z is to be computed. For example, if the vertical stress at a depth of 9 m is required, and if the length AB chosen is 3 cm, the foundation plan is drawn to a scale of 1 cm = 9/3 = 3 m. In case the vertical stress at a depth 12 m is required, a new foundation plan on a separate tracing paper is required. The scale for this plan is 1 cm = 12/AB = 12/3 = 4 m.

This means that a different tracing has to be made for each different depth whereas the chart remains the same for all. Fig. 6.12(b) gives a foundation plan, which is loaded with a uniformly distributed load q per unit area. It is now required to determine the vertical stress σ_z at depth vertically below point O shown in the figure. In order to determine σ_z, the foundation plan is laid over the chart in such a way that the surface point O coincides with the center O' of the chart as shown in Fig. 6.12. The number of small blocks covered by the foundation plan is then counted. Let this number be N. Then the value of σ_z at depth z below O is

$\sigma_z = C_i Nq$, which is the same as Eq. (6.24).

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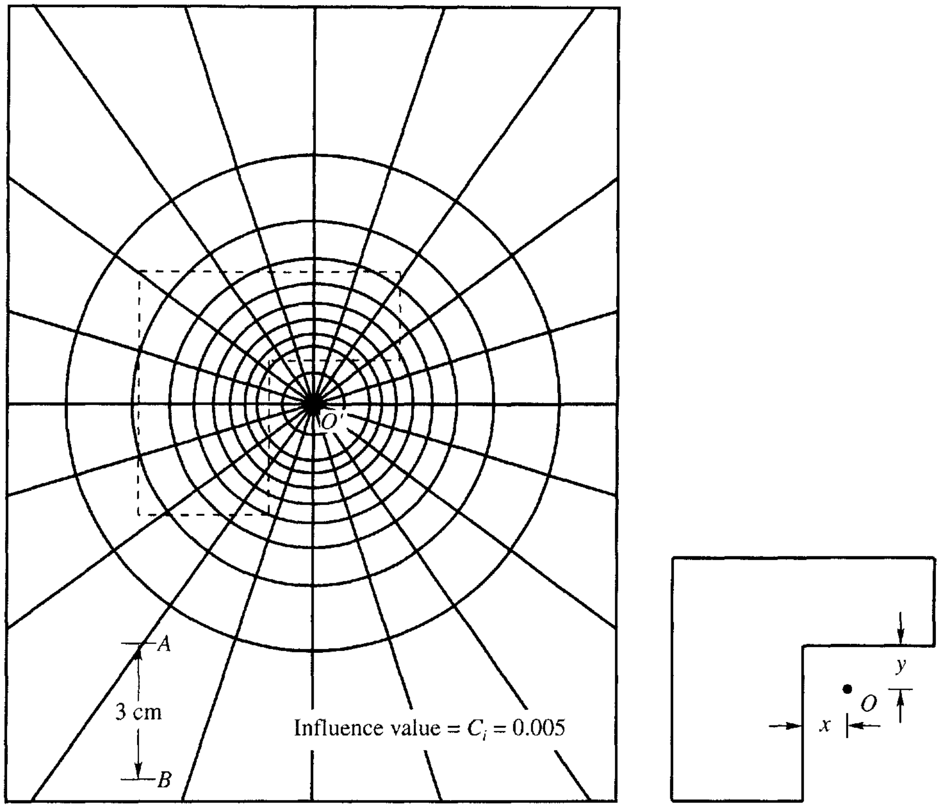

(a) (b)

Figure 6.12 Newmark's influence chart

Example 6.9

A ring footing of external diameter 8 m and internal diameter 4 m rests at a depth 2 m below the ground surface. It carries a load intensity of 150 kN/m^2. Find the vertical stress at depths of 2, 4 and 8 m along the axis of the footing below the footing base. Neglect the effect of the excavation on the stress.

Solution

From Eq. (6.21) we have,

$$\frac{\sigma_z}{q} = 1 - \left[\frac{1}{1+(R_0/z)^2}\right]^{3/2} = I_z$$

where q = contact pressure 150 kN/m^2, I_z = Influence coefficient.

The stress σ_z at any depth z on the axis of the ring is expressed as

$$\sigma_z = \sigma_{z_1} - \sigma_{z_2} = q(I_{z_1} - I_{z_2})$$

where σ_{z_1} = stress due to the circular footing of diameter 8 m, and $I_z = I_{z_1}$ and $R_0/z = (R_1/z)$

σ_{z_2} = stress due to the footing of diameter 4 m, $I_z = I_{z_2}$ and $R_0/z = (R_2/z)$.

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The values of I_z may be obtained from Table 6.1 for various values of R_0/z. The stress σ_z at depths 2, 4 and 8 m are given below:

Depth (m)	R_1/z	I_{z_1}	R_2/z	I_{z_2}	$(I_{z_1} - I_{z_2})q = \sigma_z$ kN/m^2
2	2	0.911	1.0	0.697	39.6
4	1.0	0.647	0.5	0.285	54.3
8	0.5	0.285	0.25	0.087	29.7

Example 6.10

A raft foundation of the size given in Fig. Ex. 6.10 carries a uniformly distributed load of 300 kN/m^2. Estimate the vertical pressure at a depth 9 m below the point O marked in the figure.

Solution

The depth at which σ_z required is 9 m.

Using Fig. 6.12, the scale of the foundation plan is AB = 3 cm = 9 m or 1 cm = 3 m. The foundation plan is required to be made to a scale of 1 cm = 3 m on tracing paper. This plan is superimposed on Fig. 6.12 with O coinciding with the center of the chart. The plan is shown in dotted lines in Fig. 6.12.

Number of loaded blocks occupied by the plan, $N = 62$

Influence value, $C_i = 0.005$, $q = 300$ kN/m^2

The vertical stress, $\sigma_z = C_i Nq = 0.005 \times 62 \times 300 = 93$ kN/m^2.

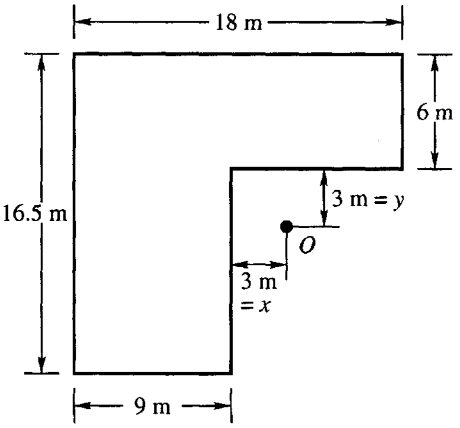

Figure Ex. 6.10

6.9 EMBANKMENT LOADINGS

Long earth embankments with sloping sides represent trapezoidal loads. When the top width of the embankment reduces to zero, the load becomes a triangular strip load. The basic problem is to determine stresses due to a linearly increasing vertical loading on the surface.

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Linearly Increasing Vertical Loading

Fig. 6.13(a) shows a linearly increasing vertical loading starting from zero at A to a finite value q per unit length at B. Consider an elementary strip of width db at a distance b from A. The load per unit length may be written as

$$dq = (q/a)\, b\, db$$

If dq is considered as a line load on the surface, the vertical stress $d\sigma_z$ at P [Fig. 6.13(a)] due to dq may be written from Eq. (6.4) as

$$d\sigma_z = \left(\frac{1}{a}\right)\left(\frac{2q}{\pi}\right)\frac{z^3 b\, db}{\left[(x-b)^2+z^2\right]^2}$$

Therefore,

$$\sigma_z = \int d\sigma_z = \left(\frac{1}{a}\right)\left(\frac{2q}{\pi}\right)\int_{b=0}^{b=a}\frac{z^3 b\, db}{\left[(x-b)^2+z^2\right]^2}$$

on integration, $$\sigma_z = \frac{q}{2\pi}\left(\frac{2x}{a}\alpha - \sin 2\beta\right) = qI_z \tag{6.25}$$

where I_z is non-dimensional coefficient whose values for various values of x/a and z/a are given in Table 6.2.

If the point P lies in the plane BC [Fig. 6.13(a)], then $\beta = 0$ at $x = a$. Eq. (6.25) reduces to

$$\sigma_z = \frac{q}{\pi}(\alpha) \tag{6.26}$$

Figs. 6.13(b) and (c) show the distribution of stress σ_z on vertical and horizontal sections under the action of a triangular loading as a function of q. The maximum vertical stress occurs below the center of gravity of the triangular load as shown in Fig. 6.13(c).

Vertical Stress Due to Embankment Loading

Many times it may be necessary to determine the vertical stress σ_z beneath road and railway embankments, and also beneath earth dams. The vertical stress beneath embankments may be

Table 6.2 I_z for triangular load (Eq. 6.25)

x/a	*z/a*						
	0.00	0.5	1.0	1.5	2	4	6
−1.500	0.00	0.002	0.014	0.020	0.033	0.051	0.041
−1.00	0.00	0.003	0.025	0.048	0.061	0.060	0.041
0.00	0.00	0.127	0.159	0.145	0.127	0.075	0.051
0.50	0.50	0.410	0.275	0.200	0.155	0.085	0.053
0.75	0.75	0.477	0.279	0.202	0.163	0.082	0.053
1.00	0.50	0.353	0.241	0.185	0.153	0.075	0.053
1.50	0.00	0.056	0.129	0.124	0.108	0.073	0.050
2.00	0.00	0.017	0.045	0.062	0.069	0.060	0.050
2.50	0.00	0.003	0.013	0.041	0.050	0.049	0.045

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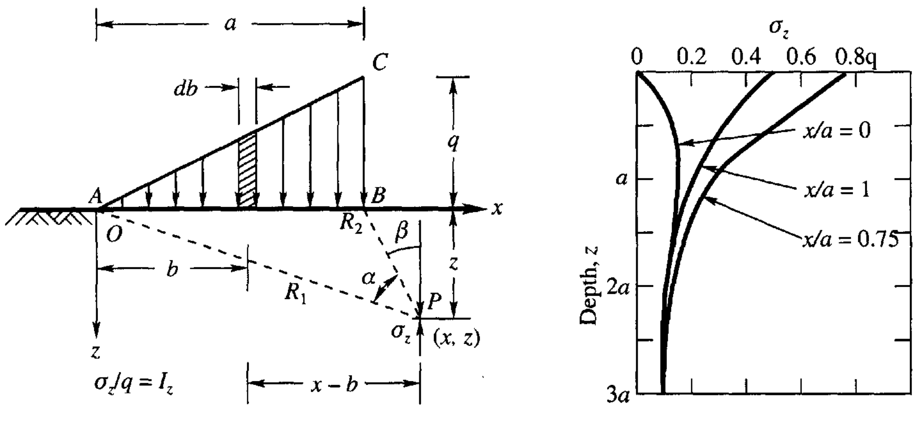
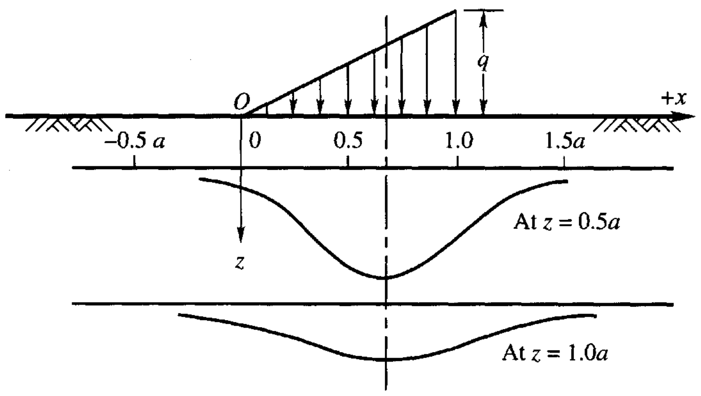


(a) Triangular loading (b) Vertical stress on vertical sections



(c) Vertical stress on horizontal sections

Figure 6.13 Stresses in a semi-infinite mass due to triangular loading on the surface

determined either by the method of superposition by making use of Eq. (6.26) or by making use of a single formula which can be developed from first principles.

σ_z by Method of Superposition

Consider an embankment given in Fig. 6.14. σ_z at P may be calculated as follows:

The trapezoidal section of embankment *ABCD*, may be divided into triangular sections by drawing a vertical line through point P as shown in Fig. 6.14. We may write

$$ABCD = AGE + FGB - EDJ - FJC \tag{6.27}$$

If σ_{z1}, σ_{z2}, σ_{z3}, and σ_{z4} are the vertical stresses at point P due to the loadings of figures *AGE*, *FGB*, *EDJ* and *FJC* respectively, the vertical stress σ_z due to the loading of figure *ABCD* may be written as

$$\sigma_z = \sigma_{z_1} + \sigma_{z_2} - \sigma_{z_3} - \sigma_{z_4} \tag{6.28}$$

By applying the principle of superposition for each of the triangles by making use of Eq. (6.26), we obtain

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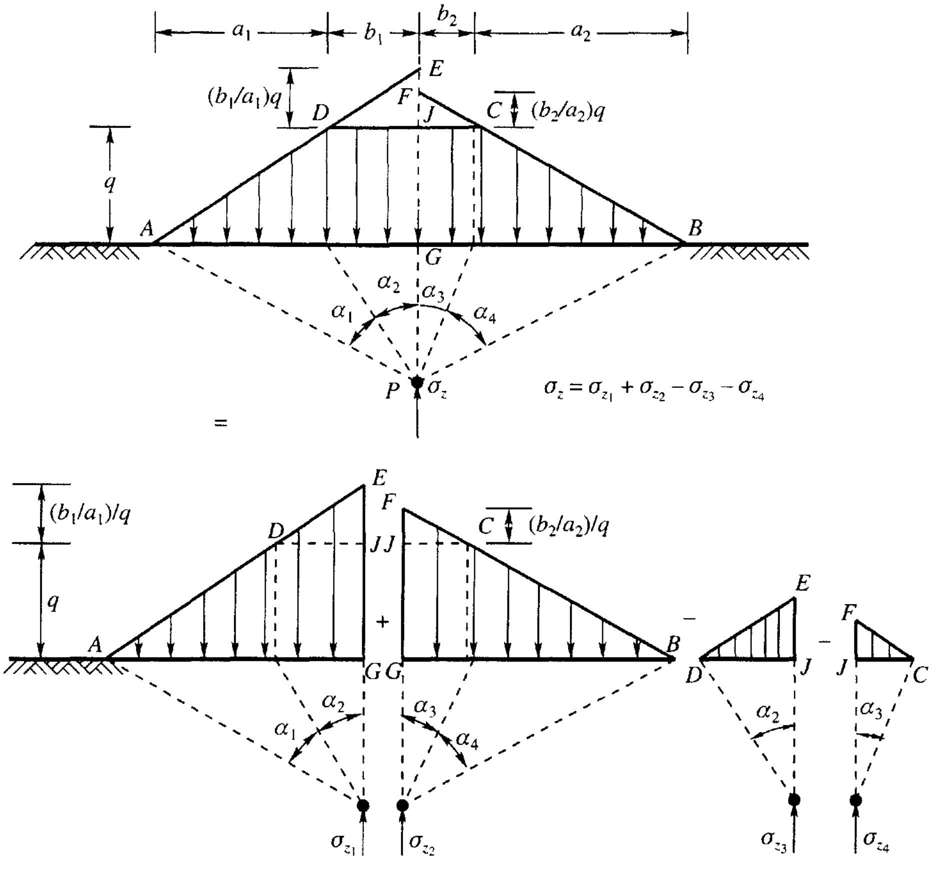

Figure 6.14 Vertical stress due to embankment

$$\sigma_z = \frac{q}{\pi}\left[(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) + (b_1 / a_1)\alpha_1 + (b_2 / a_2)\alpha_4\right] \tag{6.29}$$

$$\sigma_z = qI_z = \frac{q}{\pi} f(a / z, b / z) \tag{6.30}$$

where I_z is the influence factor for a trapezoidal load which is a function of a/z and b/z.

The values of I_z for various values of a/z and b/z are given in Fig. 6.15. (After Osterberg, 1957)

σ_z from a Single Formula for Asymmetrical Trapezoidal Loading

A single formula can be developed for trapezoidal loading for computing σ_z at a point P (Fig. 6.16) by applying Eq. (6.26). The origin of coordinates is as shown in the figure. The final equation may be expressed as

$$\sigma_z = \frac{q}{\pi}\left[(\alpha_1 + \alpha_2 + \alpha_3) + \frac{b}{a_1}(\alpha_1 + R\alpha_3) + \frac{x}{a_1}(\alpha_1 - R\alpha_3)\right] \tag{6.31}$$

where α_1, α_2, and α_3 are the angles subtended at the point P in the supporting medium by the loading and $R = a_1/a_2$. When $R = 1$, the stresses are due to that of a symmetrical trapezoidal loading.

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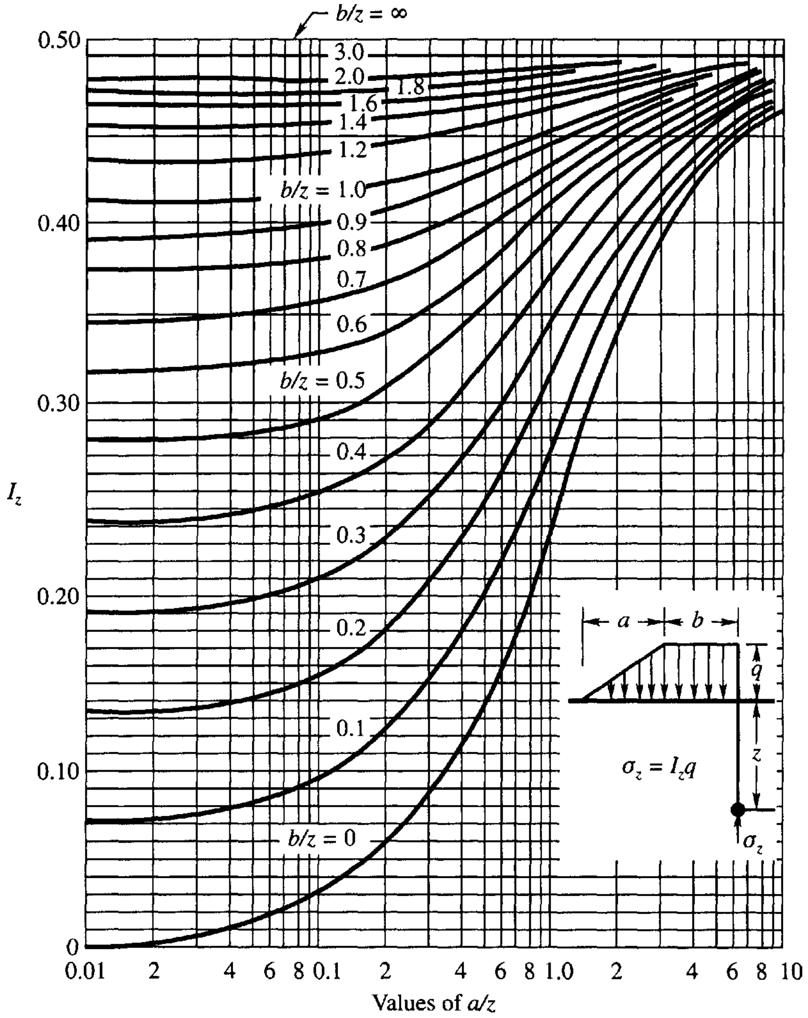

Figure 6.15 A graph to determine compressive stresses from a load varying by straight line law (After Osterberg, 1957)

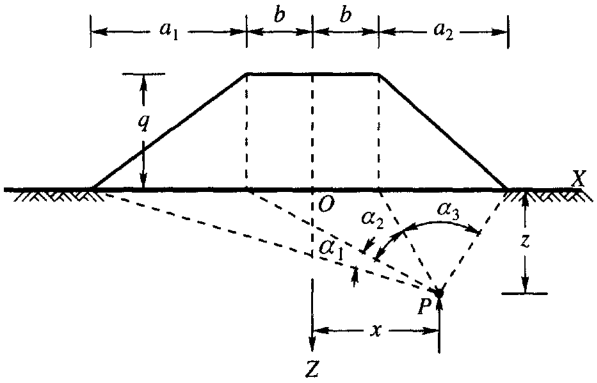

Figure 6.16 Trapezoidal loads

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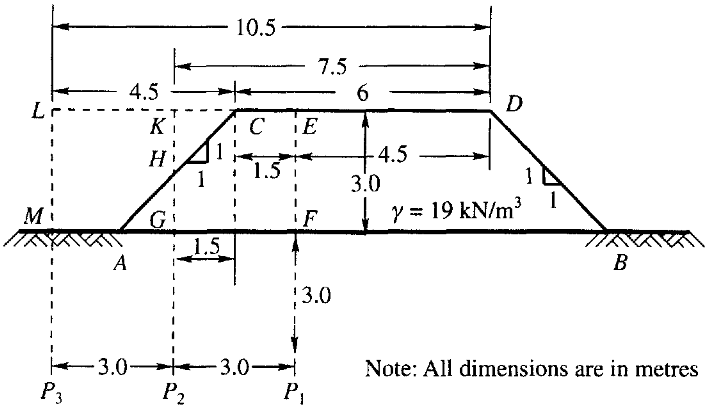
When the top width is zero, i.e, when $b = 0$, $\alpha_2 = 0$, the vertical stress σ_z will be due to a triangular loading. The expression for triangular loading is

$$\sigma_z = \frac{q}{\pi}\left[(\alpha_1 + \alpha_3) + \frac{x}{a_1}(\alpha_1 - R\alpha_3)\right] \tag{6.32}$$

Eq. (6.31) and Eq. (6.32) can be used to compute σ_z at any point in the supporting medium. The angles α_1, α_2, and α_3 may conveniently be obtained by a graphical procedure where these angles are expressed as radians in the equations.

Example 6.11

A 3 m high embankment is to be constructed as shown in Fig. Ex. 6. 11. If the unit weight of soil used in the embankment is 19. 0 kN/m^3, calculate the vertical stress due to the embankment loading at points P_1, P_2, and P_3.



Note: All dimensions are in metres

Figure Ex. 6.11 Vertical stresses at P_1, P_2 & P_3

Solution

$q = \gamma H = 19 \times 3 = 57$ kN/m^2, $z = 3$ m

The embankment is divided into blocks as shown in Fig. Ex. 6.11 for making use of the graph given in Fig. 6. 15. The calculations are arranged as follows:

Point	Block	b (m)	a (m)	b/z	a/z	I
P_1	*ACEF*	1.5	3	0.5	1	0.39
	EDBF	4.5	3	1.5	1	0.477
P_2	*AGH*	0	1.5	0	0.5	0.15
	GKDB	7.5	3	2.5	1.0	0.493
	HKC	0	1.5	0	0.5	0.15
P_3	*MLDB*	10.5	3.0	3.5	1.0	0.498
	MACL	1.5	3.0	0.5	1.0	0.39

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Vertical stress σ_z

At point P_1, σ_z = $(0.39 + 0.477) \times 57 = 49.4$ kN/m^2

At point P_2, σ_z = $0.15 \times (57/2) + 0.493 \times 57 - 0.15 \times (57/2) = 28.1$ kN/m^2

At point P_3, σ_z = $(0.498 - 0.39)\, 57 = 6.2$ kN/m^2

6.10 APPROXIMATE METHODS FOR COMPUTING σ_z

Two approximate methods are generally used for computing stresses in a soil mass below loaded areas. They are

1. Use of the point load formulas such as Boussinesq's equation.
2. 2 : 1 method which gives an average vertical stress σ_z at any depth z. This method assumes that the stresses distribute from the loaded edge points at an angle of 2 (vertical) to 1 (horizontal)

The first method if properly applied gives the point stress at any depth which compares fairly well with exact methods, whereas the second does not give any point stress but only gives an average stress σ_z at any depth. The average stress computed by the second method has been found to be in error depending upon the depth at which the stress is required.

Point Load Method

Eq. (6.1) may be used for the computation of stresses in a soil mass due to point loads acting at the surface. Since loads occupy finite areas, the point load formula may still be used if the footings are divided into smaller rectangles or squares and a series of concentrated loads of value $q\,dA$ are assumed to act at the center of each square or rectangle. Here dA is the area of the smaller blocks and q the pressure per unit area. The only principle to be followed in dividing a bigger area into smaller blocks is that the width of the smaller block should be less than one-third the depth z of the point at which the stress is required to be computed. The loads acting at the centers of each smaller area may be considered as point loads and Boussinesq's formula may then be applied. The difference between the point load method and the exact method explained earlier is clear from

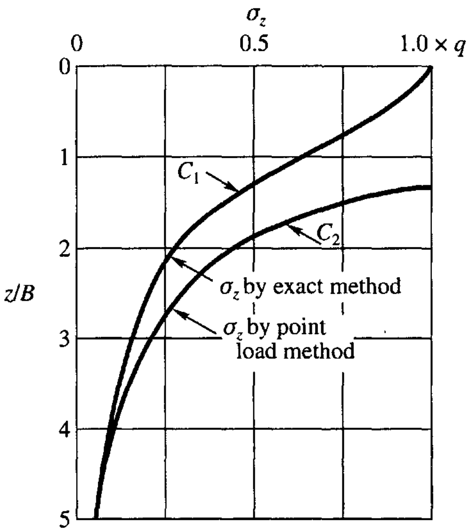

Figure 6.17 σ_z by point load method

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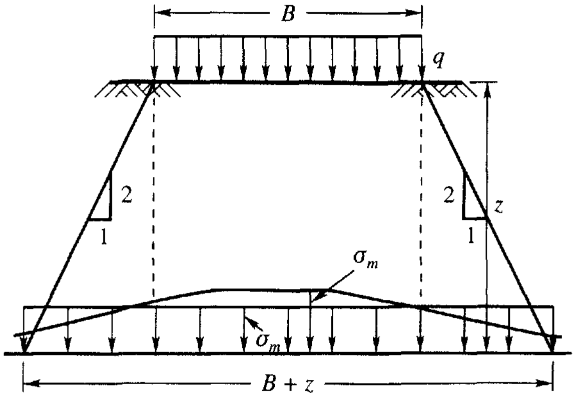

Figure 6.18 σ_m 2 : 1 method

Fig. 6.17. In this figure the abscissa of the curve C_1 represents the vertical stress σ_z at different depths z below the center of a square area $B \times B$ which carries a surcharge q per unit area or a total surcharge load of B^2q. This curve is obtained by the exact method explained under Sect. 6.6. The abscissa of the curve C_2 represents the corresponding stresses due to a concentrated load $Q = B^2q$ acting at the center of the square area. The figure shows that the difference between the two curves becomes very small for values of z/B in excess of three. Hence in a computation of the vertical stress σ_z at a depth z below an area, the area should be divided into convenient squares or rectangles such that the least width of any block is not greater than $z/3$.

2 : 1 Method

In this method, the stress is assumed to be distributed uniformly over areas lying below the foundation. The size of the area at any depth is obtained by assuming that the stresses spread out at an angle of 2 (vertical) to 1 (horizontal) from the edges of the loaded areas shown in Fig. 6.18. The average stress at any depth z is

$$\sigma_a = \frac{Q}{(B+z)(L+z)} \tag{6.33}$$

The maximum stress σ_m by an exact method below the loaded area is different from the average stress σ_a at the same depth. The value of σ_m/σ_a reaches a maximum of about 1.6 at z/b = 0-5, where b = half width.

6.11 PRESSURE ISOBARS

Definition

An *isobar* is a line which connects all points of equal stress below the ground surface. In other words, an isobar is a stress contour. We may draw any number of isobars as shown in Fig. 6.19 for any given load system. Each isobar represents a fraction of the load applied at the surface. Since these isobars form closed figures and resemble the form of a bulb, they are also termed *bulb of pressure* or simply the *pressure bulb*. Normally isobars are drawn for vertical, horizontal and shear stresses. The one that is most important in the calculation of settlements of footings is the vertical pressure isobar.

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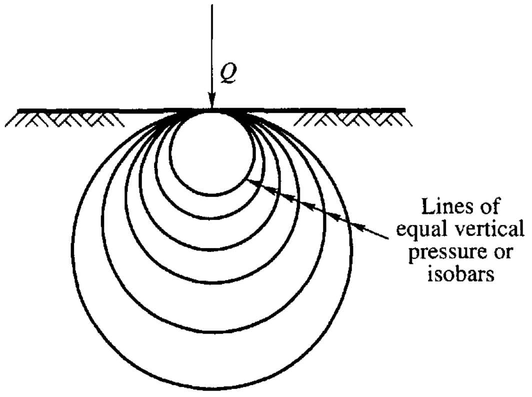
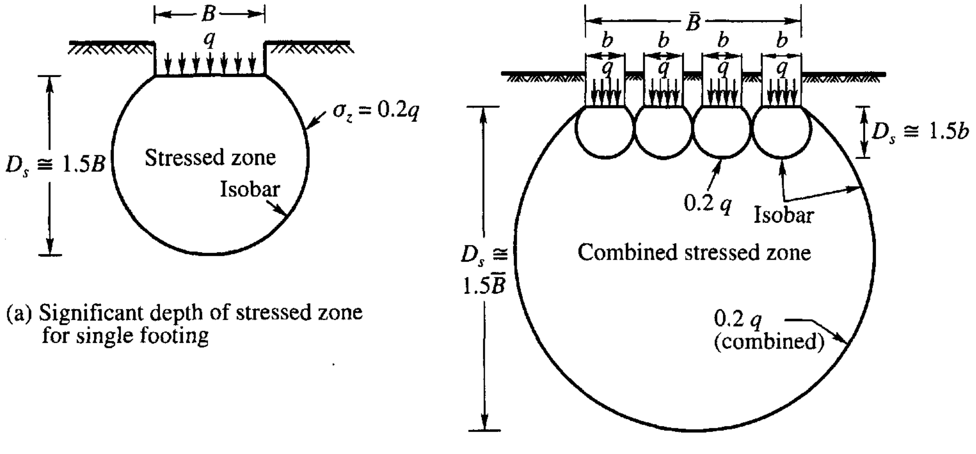


Figure 6.19 Bulb of pressure

Significant Depth

In his opening discussion on settlement of structures at the First International Conference on Soil Mechanics and Foundation Engineering (held in 1936 at Harvard University in Cambridge, Mass, USA), Terzaghi stressed the importance of the bulb of pressure and its relationship with the seat of settlement. As stated earlier we may draw any number of isobars for any given load system, but the one that is of practical significance is the one which encloses a soil mass which is responsible for the settlement of the structure. The depth of this stressed zone may be termed as the *significant depth* D_s which is responsible for the settlement of the structure. Terzaghi recommended that for all practical purposes one can take a *stress contour* which represents 20 per cent of the foundation contact pressure q, i.e, equal to $0.2q$. The depth of such an isobar can be taken as the *significant depth* D_s which represents the seat of settlement for the foundation. Terzaghi's recommendation was based on his observation that direct stresses are considered of negligible magnitude when they are smaller than 20 per cent of the intensity of the applied stress from structural loading, and that most of the settlement, approximately 80 per cent of the total, takes place at a depth less than D_s. The depth D_s is approximately equal to 1.5 times the width of square or circular footings [Fig. 6.20(a)].

If several loaded footings are spaced closely enough, the individual isobars of each footing in question would combine and merge into one large isobar of the intensity as shown in [Fig. 6.20(b)]. The combined significant depth D_s is equal to about 1.5 $\bar{B}$.



(a) Significant depth of stressed zone for single footing

(b) Effect of closely placed footings

Figure 6.20 Significant depth of stressed zone

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Pressure Isobars for Footings

Pressure isobars of square, rectangular and circular footings may conveniently be used for determining vertical pressure, σ_z, at any depth, z, below the base of the footings. The depths z from the ground surface, and the distance r (or x) from the center of the footing are expressed as a function of the width of the footing B. In the case of circular footing B represents the diameter.

The following pressure isobars are given based on either Boussinesq or Westergaard's equations

1. Boussinesq isobars for square and continuous footings, Fig. 6.21.
2. Boussinesq isobar for circular footings, Fig. 6.22.
3. Westergaard isobars for square and continuous footings, Fig. 6.23.

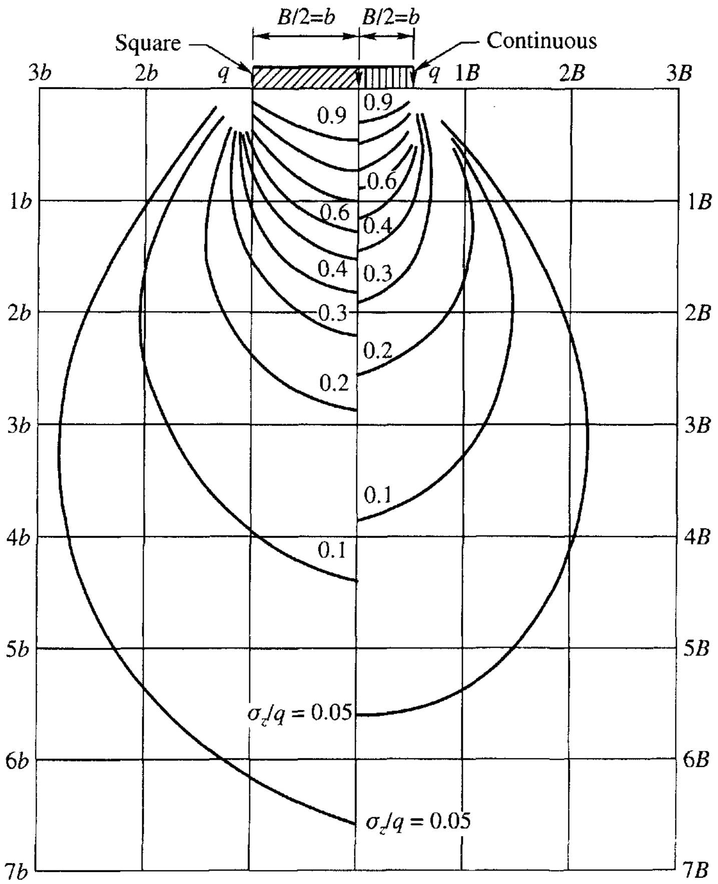

Figure 6.21 Pressure isobars based on Boussinesq equation for square and continuous footings

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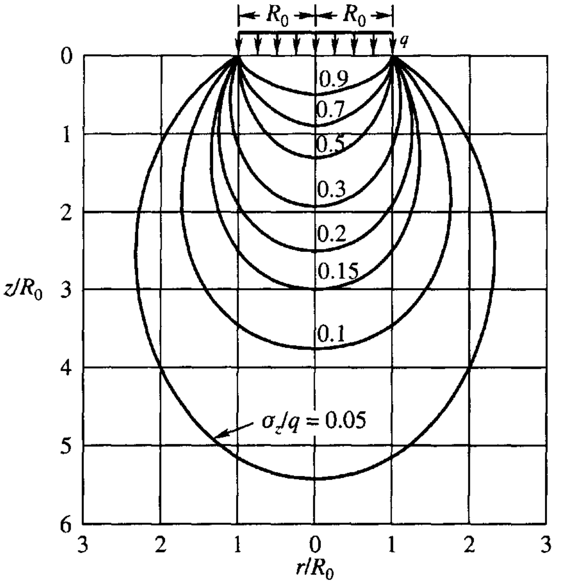

Figure 6.22 Pressure isobars based on Boussinesq equation for uniformly loaded circular footings

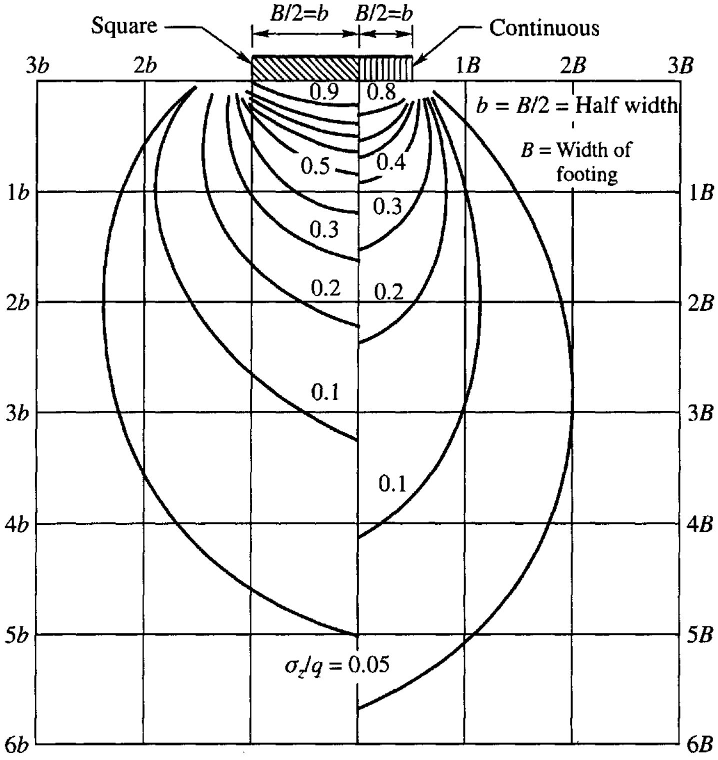

Figure 6.23 Pressure isobars based on Westergaard equation for square and continuous footing

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Example 6.12

A single concentrated load of 1000 kN acts at the ground surface. Construct an isobar for $\sigma_z = 40$ kN/m^2 by making use of the Boussinesq equation.

Solution

From Eq. (6.1a) we have

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

We may now write by rearranging an equation for the radial distance r as

$$r = \sqrt{z}\sqrt{\left(\frac{3Q}{2\pi z^2 \sigma_z}\right)^{2/5} - 1}$$

Now for $Q = 1000$ kN, $\sigma_z = 40$ kN/m^2, we obtain the values of r_1, r_2, r_3, etc. for different depths z_1, z_2, z_3, etc. The values so obtained are

z (m)	r (m)
0.25	1.34
0.50	1.36
1.0	1.30
2.0	1.04
3.0	0.60
3.455	0.00

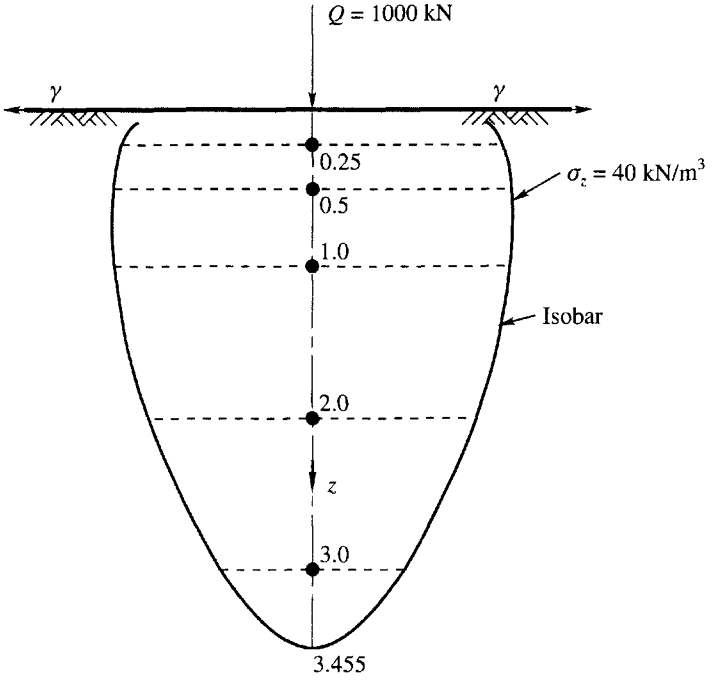

Figure Ex. 6.12

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The isobar for σ_z = 40 kN/m^2 may be obtained by plotting z against r as shown in Fig. Ex. 6.12.

6.12 PROBLEMS

6.1 A column of a building transfers a concentrated load of 225 kips to the soil in contact with the footing. Estimate the vertical pressure at the following points by making use of the Boussinesq and Westergaard equations.

(i) Vertically below the column load at depths of 5, 10, and 15 ft.

(ii) At radial distances of 5, 10 and 20 ft and at a depth of 10 ft.

6.2 Three footings are placed at locations forming an equilateral triangle of 13 ft sides. Each of the footings carries a vertical load of 112.4 kips. Estimate the vertical pressures by means of the Boussinesq equation at a depth of 9 ft at the following locations :

(i) Vertically below the centers of the footings.

(ii) Below the center of the triangle.

6.3 A reinforced concrete water tank of size 25 ft x 25 ft and resting on the ground surface carries a uniformly distributed load of 5.25 kips/ft^2. Estimate the maximum vertical pressures at depths of 37.5 and 60 ft by point load approximation below the center of the tank.

6.4 Two footings of sizes 13 x 13 ft and 10 x 10 ft are placed 30 ft center to center apart at the same level and carry concentrated loads of 337 and 281 kips respectively. Compute the vertical pressure at depth 13 ft below point C midway between the centers of the footings.

6.5 A and B are two footings of size 1.5 x 1.5 m each placed in position as shown in Fig. Prob. 6.5. Each of the footings carries a column load of 400 kN. Determine by the Boussinesq method, the excess load footing B carries due to the effect of the load on A. Assume the loads at the centers of footings act as point loads.

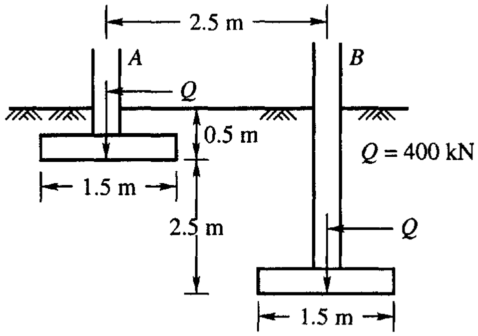

Figure Prob. 6.5

6.6 If both footings A and B in Fig. Prob. 6.5 are at the same level at a depth of 0.5 m below the ground surface, compute the stress σ_z midway between the footings at a depth of 3 m from the ground surface. Neglect the effect of the size for point load method.

6.7 Three concentrated loads Q_1 = 255 kips, Q_2 = 450 kips and Q_3 = 675 kips act in one vertical plane and they are placed in the order Q_1–Q_2–Q_3. Their spacings are 13 ft–10 ft. Determine

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the vertical pressure at a depth of 5 ft along the center line of footings using Boussinesq's point load formula.

6.8 A square footing of 13 × 13 ft is founded at a depth of 5 ft below the ground level. The imposed pressure at the base is 8732 lb/ft^2. Determine the vertical pressure at a depth of 24 ft below the ground surface on the center line of the footing.

6.9 A long masonry wall footing carries a uniformly distributed load of 200 kN/m^2. If the width of the footing is 4 m, determine the vertical pressures at a depth of 3 m below the (i) center, and (ii) edge of the footing.

6.10 A long foundation 0.6 m wide carries a line load of 100 kN/m. Calculate the vertical stress σ_z at a point *P*, the coordinates of which are $x = 2.75$ m, and $z = 1.5$ m, where the *x*-coordinate is normal to the line load from the central line of the footing.

6.11 A strip footing 10 ft wide is loaded on the ground surface with a pressure equal to 4177 lb/ft^2. Calculate vertical stresses at depths of 3, 6, and 12 ft under the center of the footing.

6.12 A rectangular footing of size 25 × 40 ft carries a uniformly distributed load of 5200 lb/ft^2. Determine the vertical pressure 20 ft below a point *O* which is located at a distance of 35 ft from the center of the footing on its longitudinal axis by making use of the curves in Fig. 6.8.

6.13 The center of a rectangular area at the ground surface has cartesian coordinate (0,0) and the corners have coordinates (6,15). All dimensions are in foot units. The area carries a uniform pressure of 3000 lb/ft^2. Estimate the stresses at a depth of 30 ft below ground surface at each of the following locations: (0,0), (0,15), (6,0).

6.14 Calculate the vertical stress at a depth of 50 ft below a point 10 ft oubide the corner (along the longer side) of a rectangular loaded area 30 × 80 ft carrying a uniform load of 2500 lb/ft^2.

6.15 A rectangular footing 6 × 3 m carries a uniform pressure of 300 kN/m^2 on the surface of a soil mass. Determine the vertical stress at a depth of 4.5 m below the surface on the center line 1.0 m inside the long edge of the foundation.

6.16 A circular ring foundation for an overhead tank transmits a contact pressure of 300 kN/m^2. Its internal diameter is 6 m and external diameter 10 m. Compute the vertical stress on the center line of the footing due to the imposed load at a depth of 6.5 m below the ground level. The footing is founded at a depth of 2.5 m.

6.17 In Prob. 6.16, if the foundation for the tank is a raft of diameter 10 m, determine the vertical stress at 6.5 m depth on the center line of the footing. All the other data remain the same.

6.18 How far apart must two 20 m diameter tanks be placed such that their combined stress overlap is not greater than 10% of the surface contact stress at a depth of 10 m?

6.19 A water tower is founded on a circular ring type foundation. The width of the ring is 4 m and its internal radius is 8 m. Assuming the distributed load per unit area as 300 kN/m^2, determine the vertical pressure at a depth of 6 m below the center of the foundation.

6.20 An embankment for road traffic is required to be constructed with the following dimensions :

Top width = 8 m, height = 4 m, side slopes= 1 V : 1.5 Hor

The unit weight of soil under the worst condition is 21 kN/m^3. The surcharge load on the road surface may be taken as 50 kN/m^2. Compute the vertical pressure at a depth of 6 m below the ground surface at the following locations:

(i) On the central longitudinal plane of the embankment.

(ii) Below the toes of the embankment.

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6.21 If the top width of the road given in Prob. 6.20 is reduced to zero, what would be the change in the vertical pressure at the same points?

6.22 A square footing of size 13 × 13 ft founded on the surface carries a distributed load of 2089 $\mathrm{lb/ft^2}$. Determine the increase in pressure at a depth of 10 ft by the 2:1 method

6.23 A load of 337 kips is imposed on a foundation 10 ft square at a shallow depth in a soil mass. Determine the vertical stress at a point 16 ft below the center of the foundation (a) assuming the load is uniformly distributed over the foundation, and (b) assuming the load acts as a point load at the center of the foundation.

6.24 A total load of 900 kN is uniformly distributed over a rectangular footing of size 3 × 2 m. Determine the vertical stress at a depth of 2.5 m below the footing at point *C* (Fig. Prob. 6.24), under one corner and *D* under the center. If another footing of size 3 × 1 m with a total load of 450 kN is constructed adjoining the previous footing, what is the additional stress at the point *C* at the same depth due to the construction of the second footing?

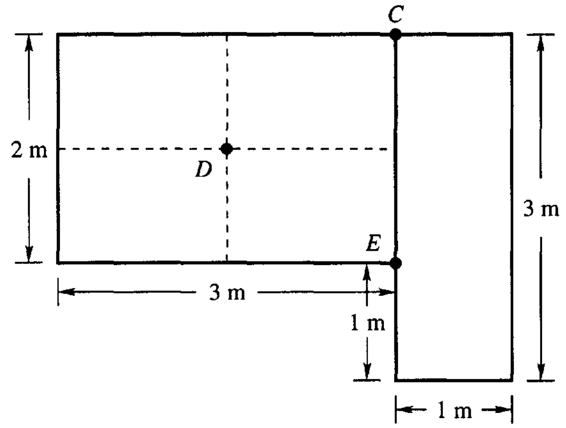

Figure Prob. 6.24

6.25 Refer to Prob. 6.24. Determine the vertical stress at a depth of 2.5 m below point *E* in Fig. Prob. 6.24. All the other data given in Prob. 6.24 remain the same.

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