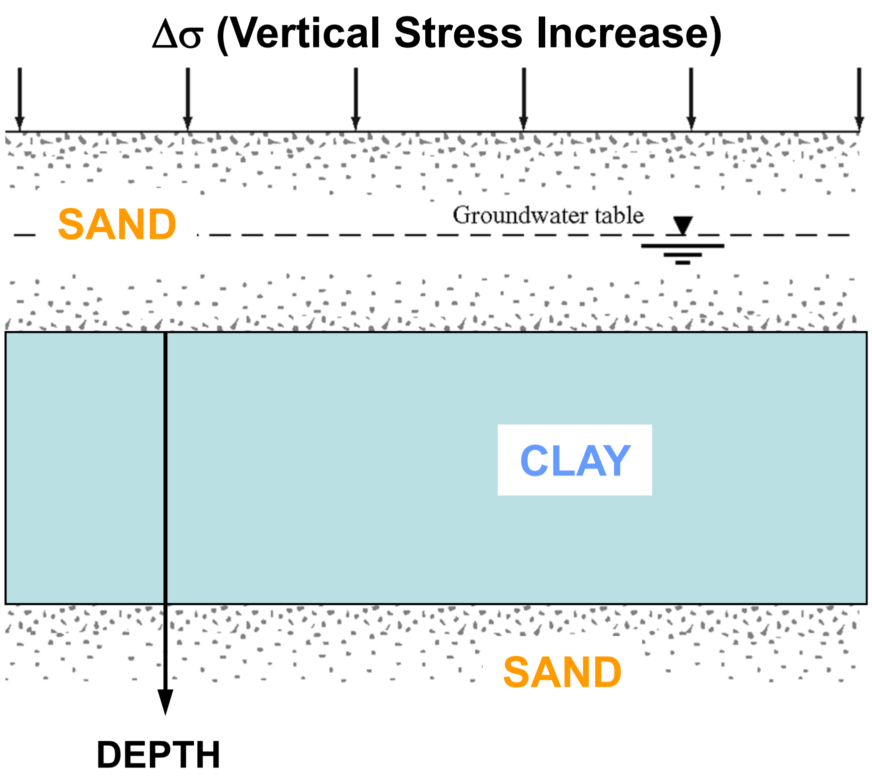
FUNDAMENTALS OF CONSOLIDATION



after Figure 7.1a. Das FGE (2005).

CONSOLIDATION:

Volume change in saturated soils caused by the expulsion of pore water from loading.

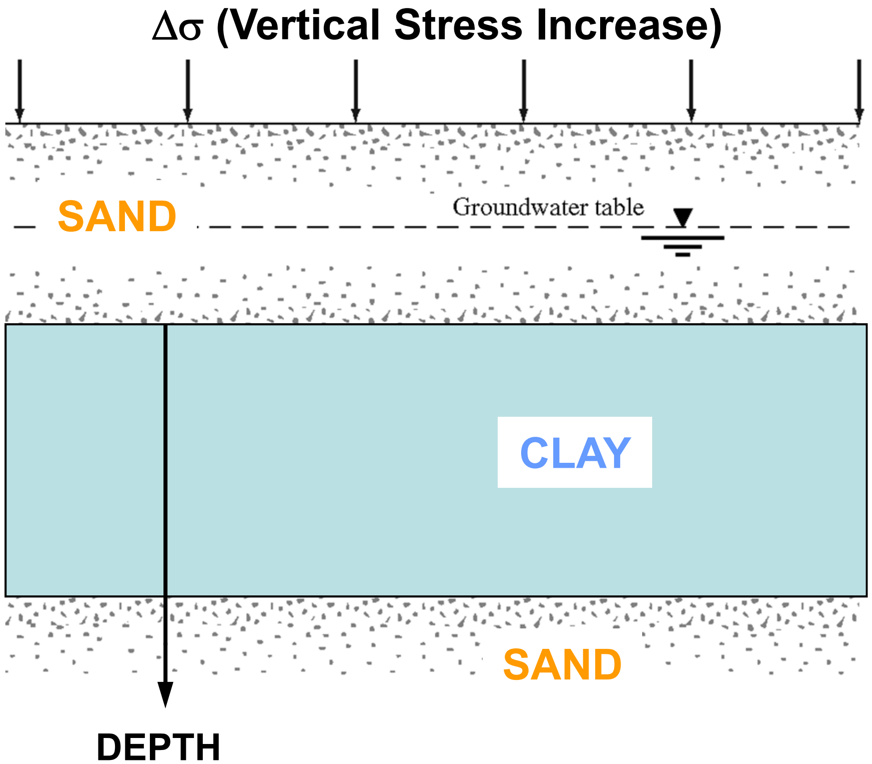
Saturated Soils:

Δσ causes *u* to increase immediately

Sands: Pore pressure increase dissipates rapidly due to high permeability.

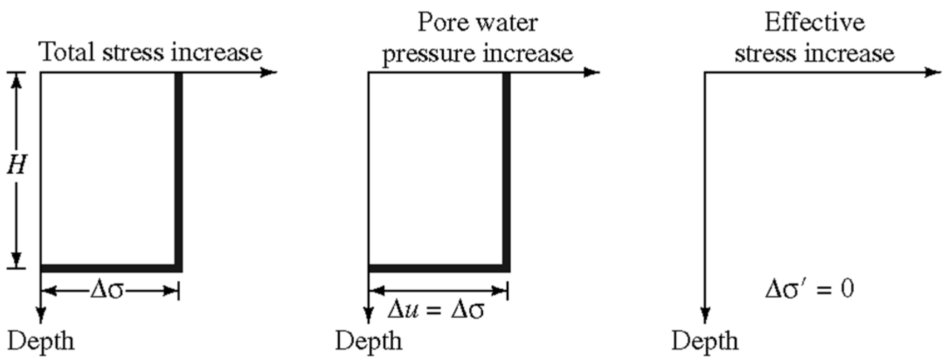
Clays: Pore Pressure dissipates slowly due to low permeability.

FUNDAMENTALS OF CONSOLIDATION



after Figure 7.1a. Das FGE (2005).

At Time of Initial Loading (t = 0)

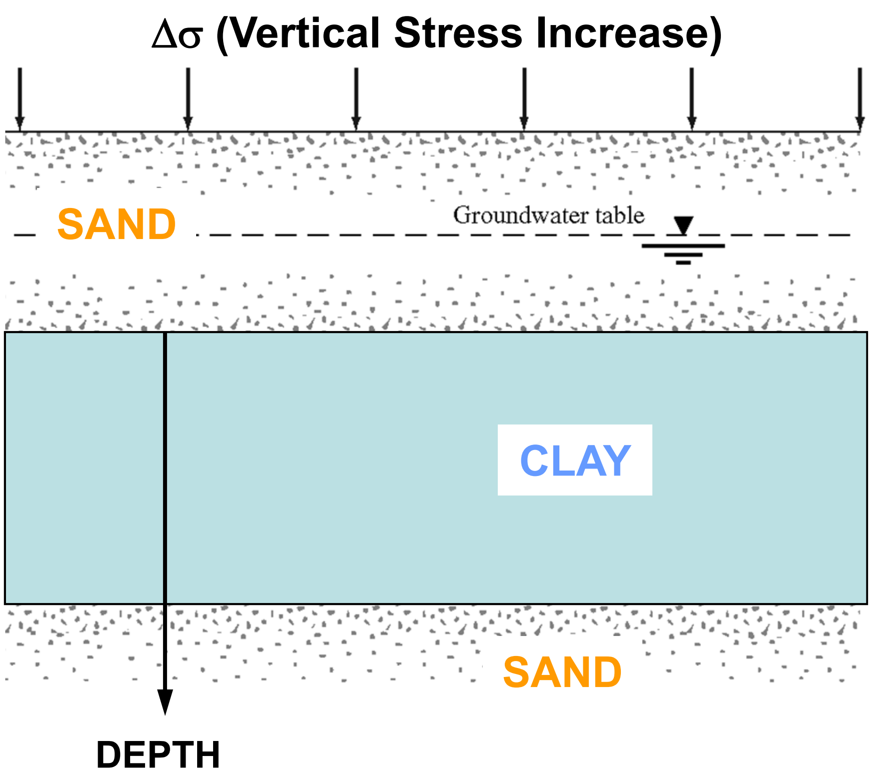


Variation in Total, Pore water, and Effective Stresses in Clay Layer
Figure 7.1b. Das FGE (2005)

Pore water takes initial change in vertical loading ($\Delta\sigma = \Delta u$) since water is incompressible

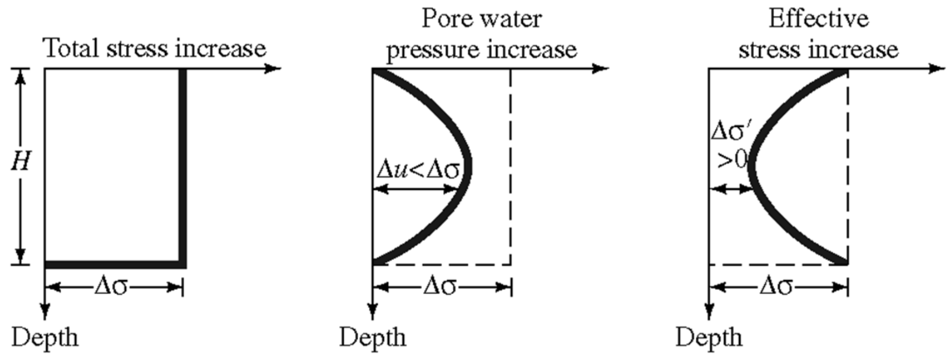
Soil skeleton does not see initial loading

FUNDAMENTALS OF CONSOLIDATION



after Figure 7.1a. Das FGE (2005).

Between time t = 0 to t = ∞

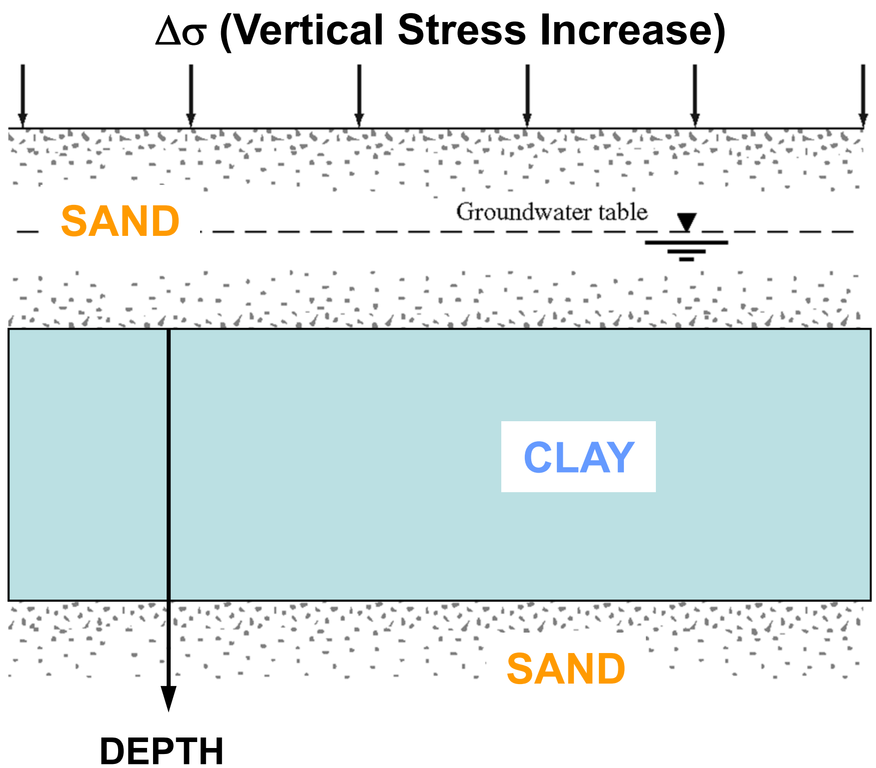


Variation in Total, Pore water, and Effective Stresses in Clay Layer
Figure 7.1c. Das FGE (2005)

Pore water increase due to initial loading dissipates

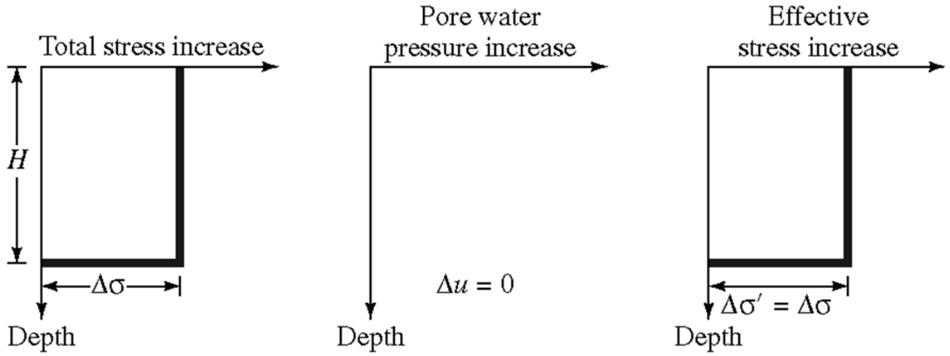
Soil skeleton takes loading as pore pressure decreases

FUNDAMENTALS OF CONSOLIDATION



after Figure 7.1a. Das FGE (2005).

At time t = ∞



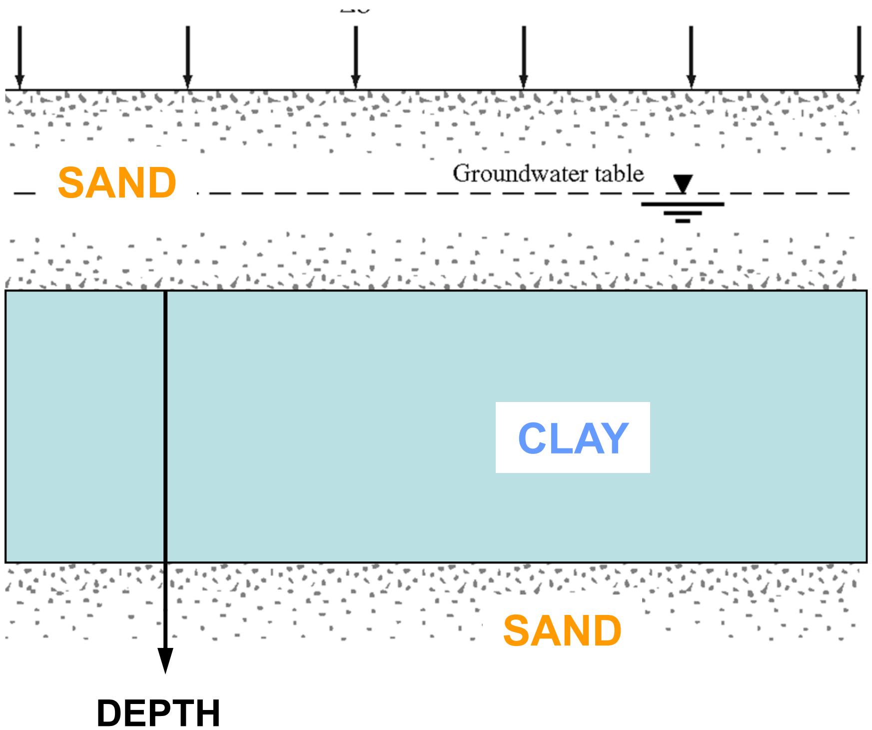
Variation in Total, Pore water, and Effective Stresses in Clay Layer
Figure 7.1e. Das FGE (2005)

Pore water increase due to initial loading completely dissipated ($\Delta u = 0$)

Soil skeleton has taken loading. Effective stress increase now equals vertical stress increase ($\Delta\sigma = \Delta'\sigma$)

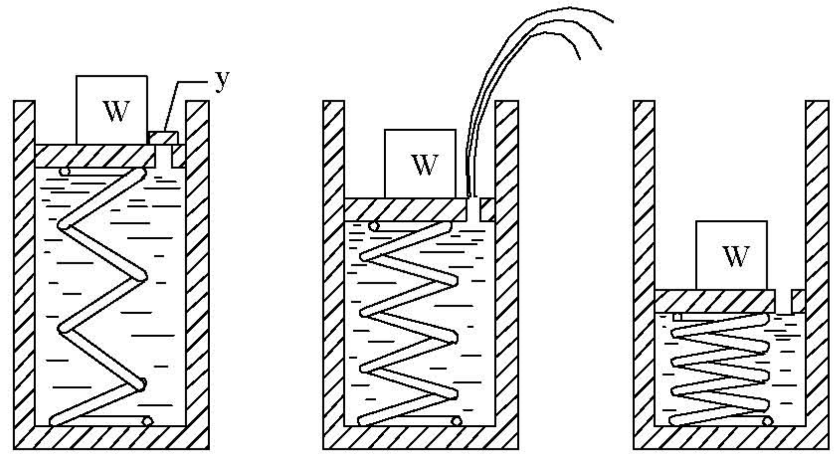
FUNDAMENTALS OF CONSOLIDATION

Δσ (Vertical Stress Increase)



after Figure 7.1a. Das FGE (2005).

THE SPRING ANALOGY



(a) Initial Loading

Water takes load

Soil (i.e. spring) has no load

(b) Dissipation of Excess Water Pressure

Water dissipating

Soil starts to

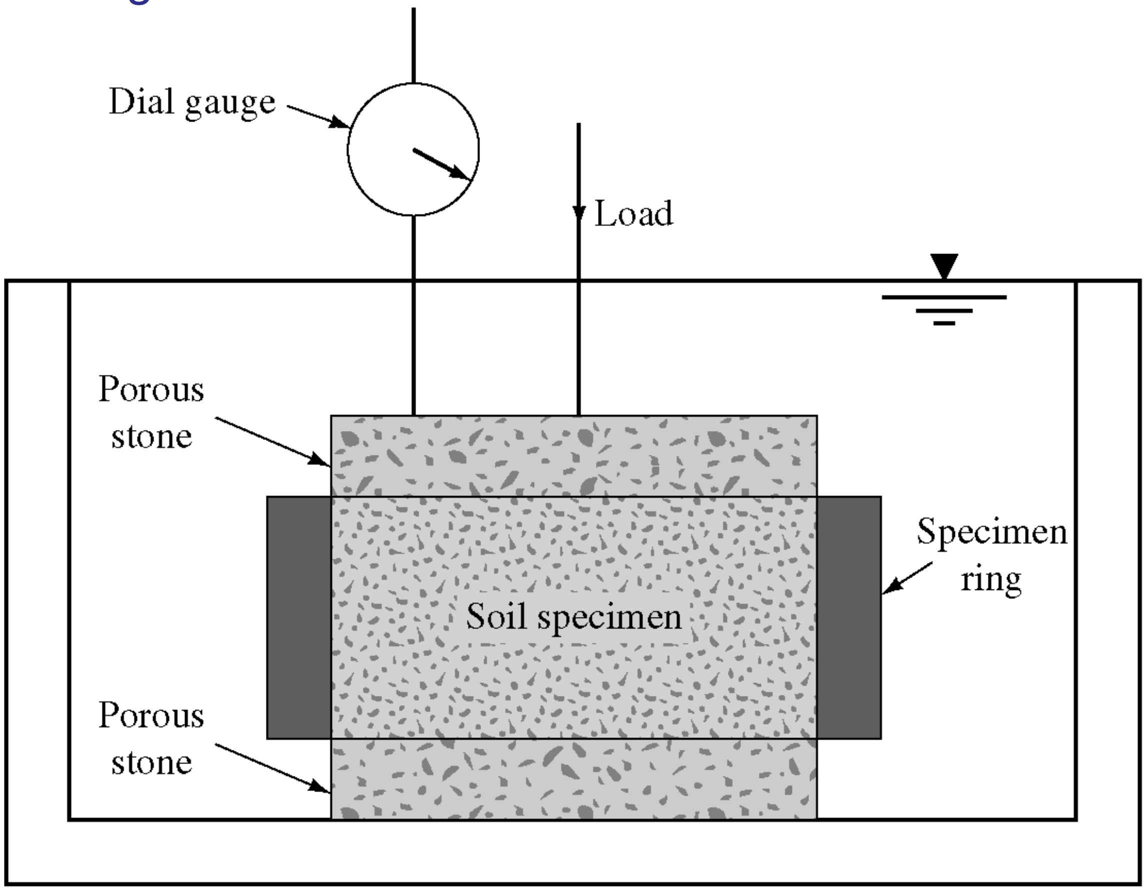
(c) Final Loading

Water dissipated

Soil has load

ONE DIMENSIONAL (1D) CONSOLIDATION TEST

D2435-11 Standard Test Methods for One-Dimensional Consolidation Properties of Soils Using Incremental Loading



Consolidometer

Figure 7.2. Das FGE (2005)

1D Consolidation Test Equipment

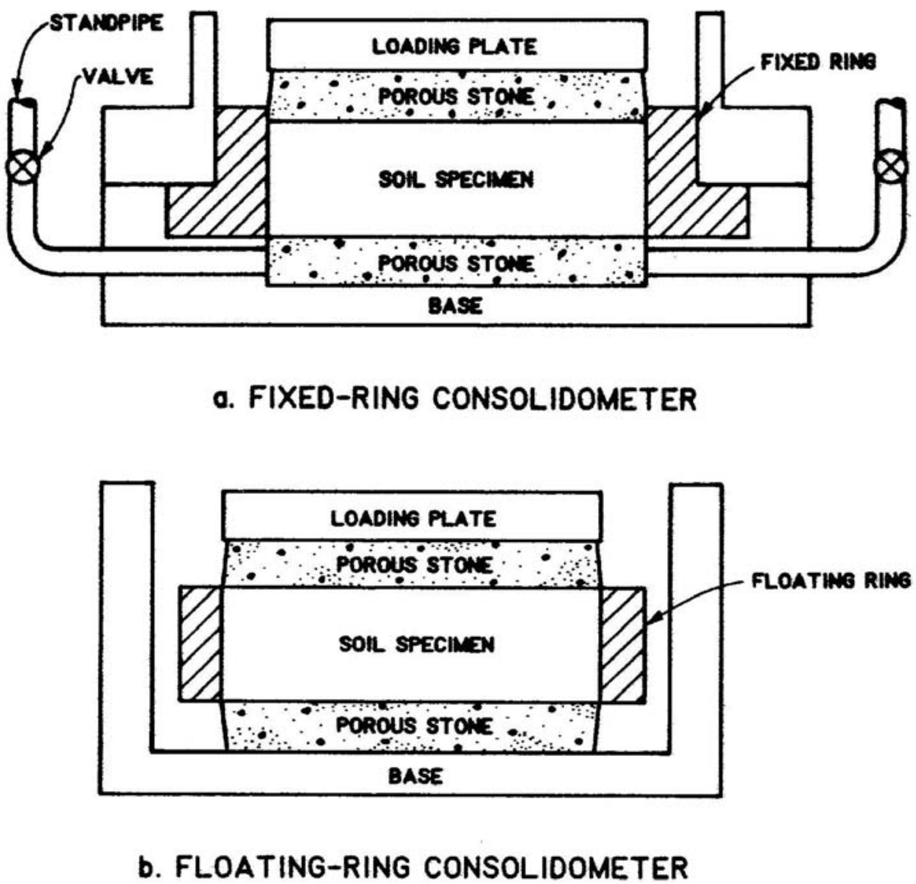


Figure E-1 USACE EM1110-1-1904.

ShearTrac II DSS Equipment
(Courtesy of Geocomp Corporation)

1D Consolidation Testing

Load Increment Data

THREE STAGES

Stage I: Initial Compression
Primarily caused by preloading.

Stage II: Primary Consolidation
Excess pore water pressure dissipation and corresponding soil volume change.

Stage III: Secondary Consolidation
Occurs after excess pore water pressure dissipation. Due to plastic deformation/ readjustment of soil particles.

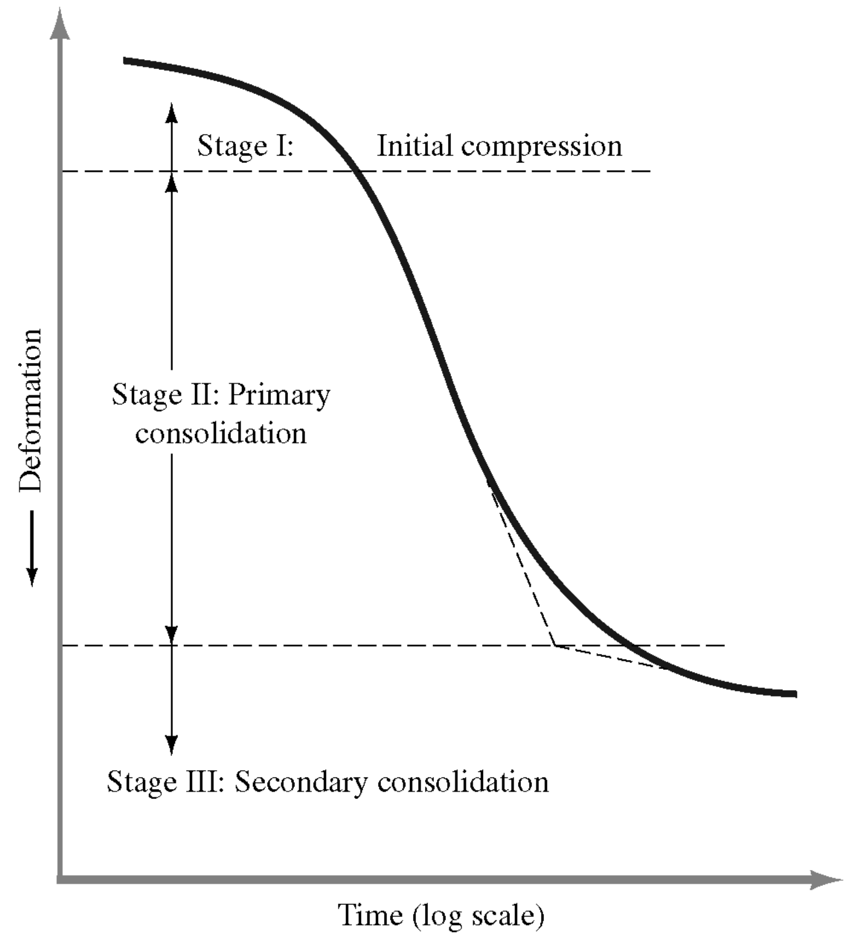


Figure 7.4. Das FGE (2005).

VOID RATIO-PRESSURE PLOTS

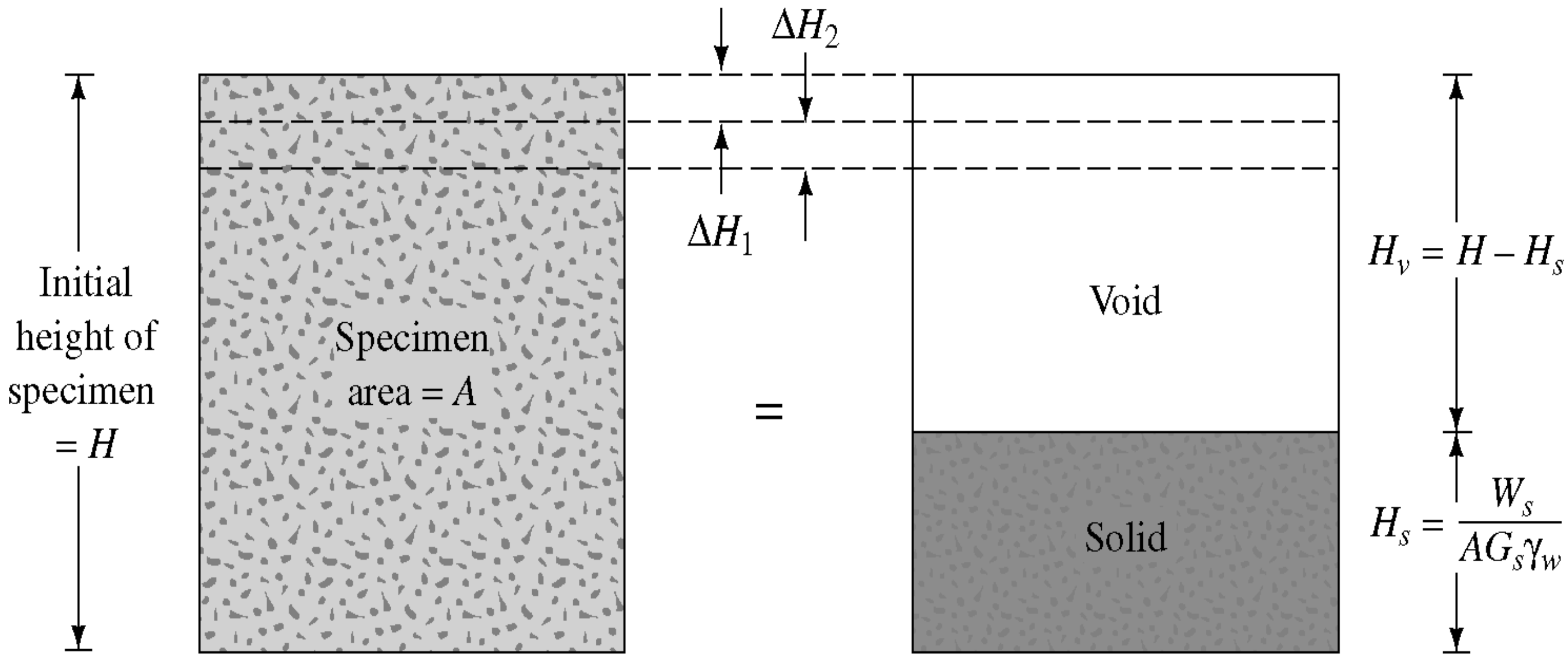


Figure 7.5. Das FGE (2005)

Initial Void Ratio (e_o): $e_o = \frac{V_v}{V_s} = \frac{H_v A}{H_s A} = \frac{H_v}{H_s}$

VOID RATIO-PRESSURE PLOTS

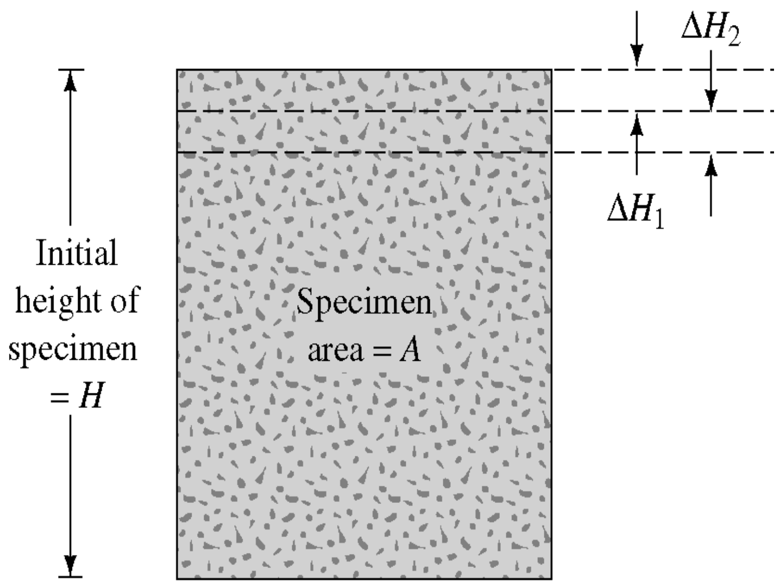


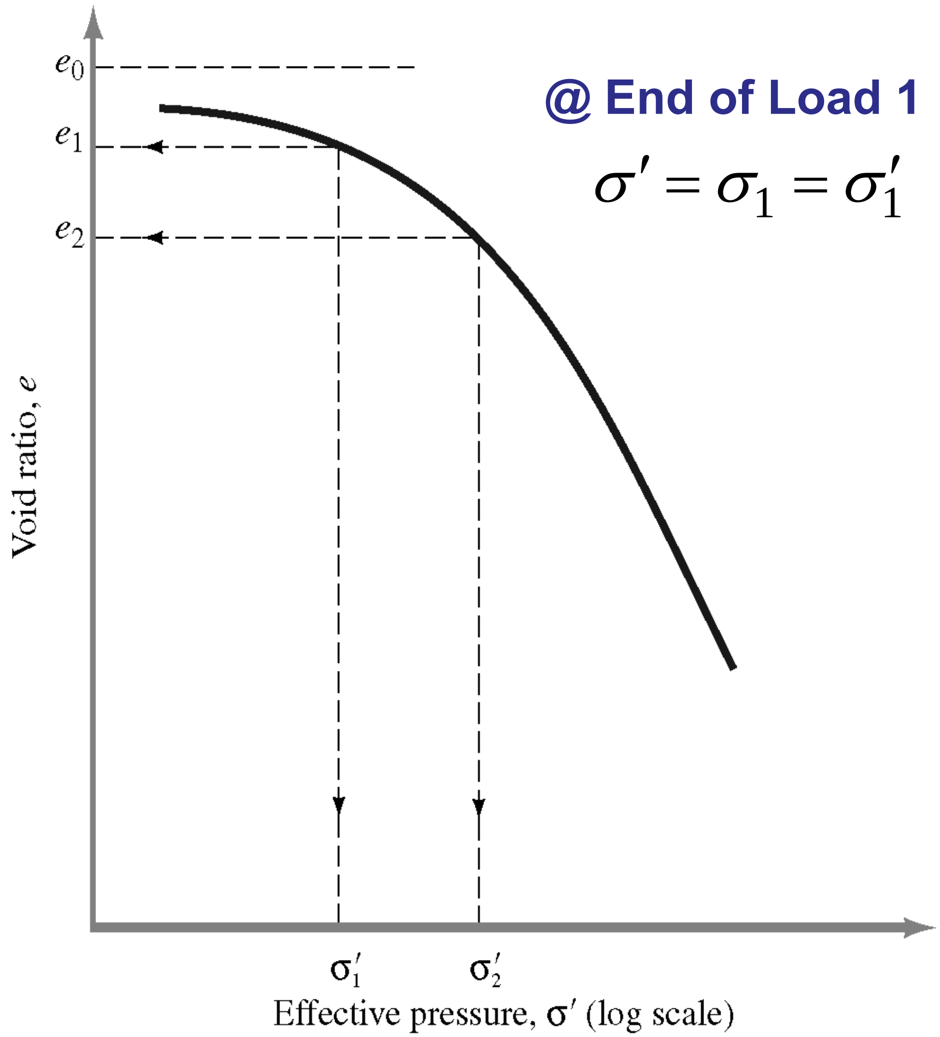
Figure 7.5. Das FGE (2005)

Change in Void Ratio due to 1st Loading (Δe_1):

$$\Delta e_1 = \frac{\Delta H_1}{H_s}$$

New Void Ratio after 1st Loading:

$$e_1 = e_o - \Delta e_1$$



@ End of Load 1

$$\sigma' = \sigma_1 = \sigma'_1$$

Figure 7.6. Das FGE (2005)

VOID RATIO-PRESSURE PLOTS

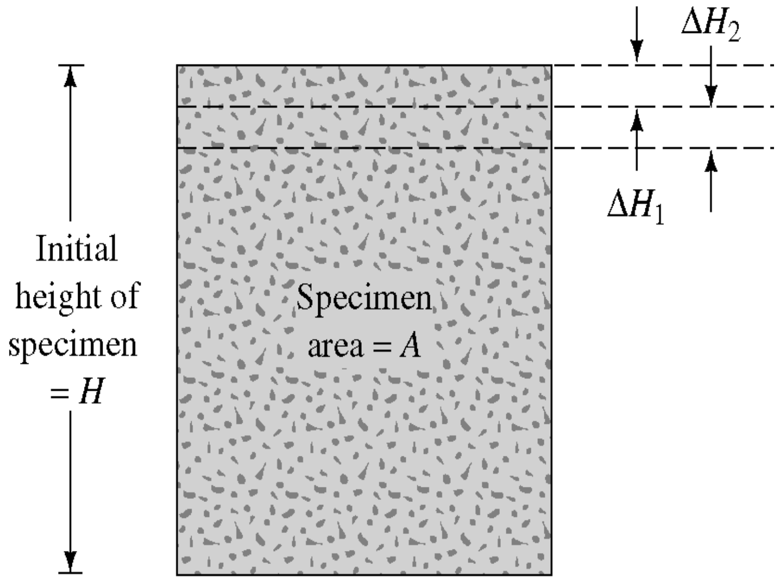


Figure 7.5. Das FGE (2005)

Change in Void Ratio due to 2nd Loading (Δe_2):

$$\Delta e_1 = \frac{\Delta H_2}{H_s}$$

New Void Ratio after 2nd Loading:

$$e_2 = e_1 - \frac{\Delta H_2}{H_s}$$

@ End of Load 2

$$\sigma' = \sigma_2 = \sigma_2'$$

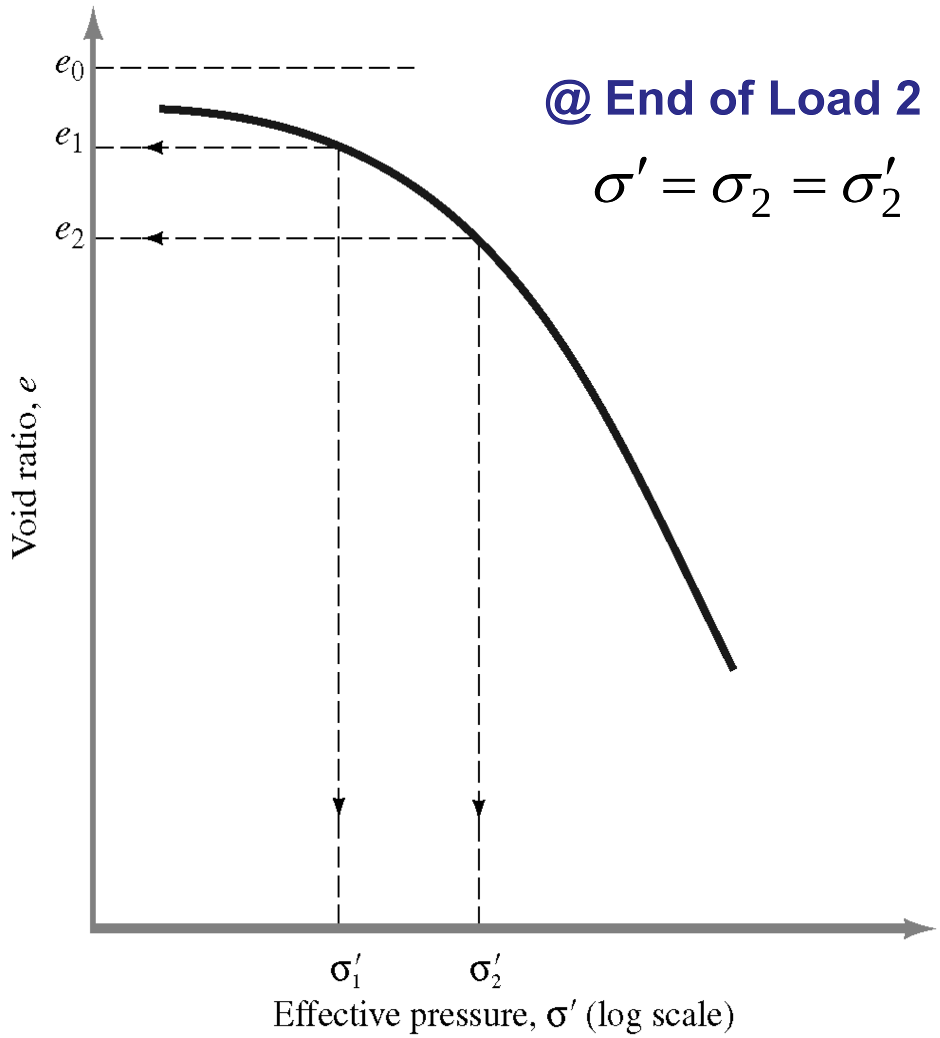


Figure 7.6. Das FGE (2005)

VOID RATIO-PRESSURE PLOTS

Final e – log σ´ plots consist of results of numerous load & unload increments

Two Definitions of Clays based on Stress History:

Normally Consolidated (NC):
The present overburden pressure (a.k.a. effective in-situ stress) is the most the soil has ever seen.

Overconsolidated Clay (OC):
The present overburden pressure is less than the soil has experienced in the past. The maximum effective past pressure is called the preconsolidation pressure (σ'_c) or ***Maximum Past Pressure (σ'_{vm})***

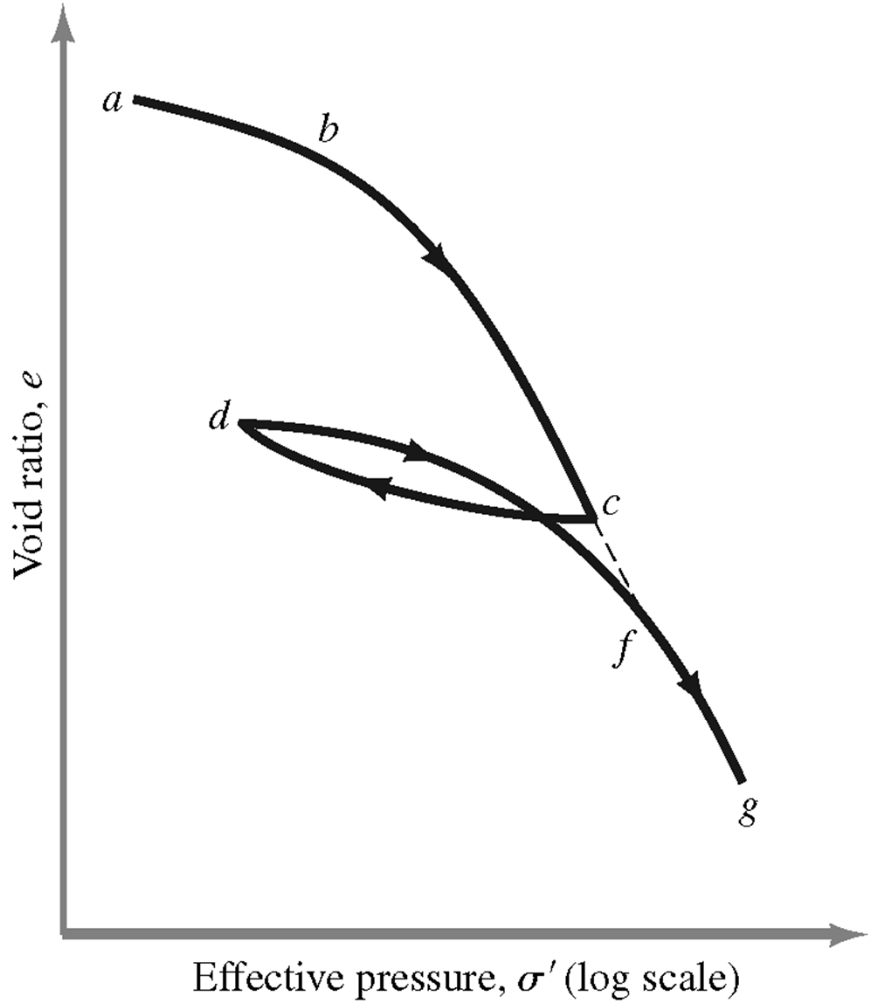


Figure 7.7. Das FGE (2005).

Determination of Maximum Past Pressure (σ'_c or σ'_{vm})

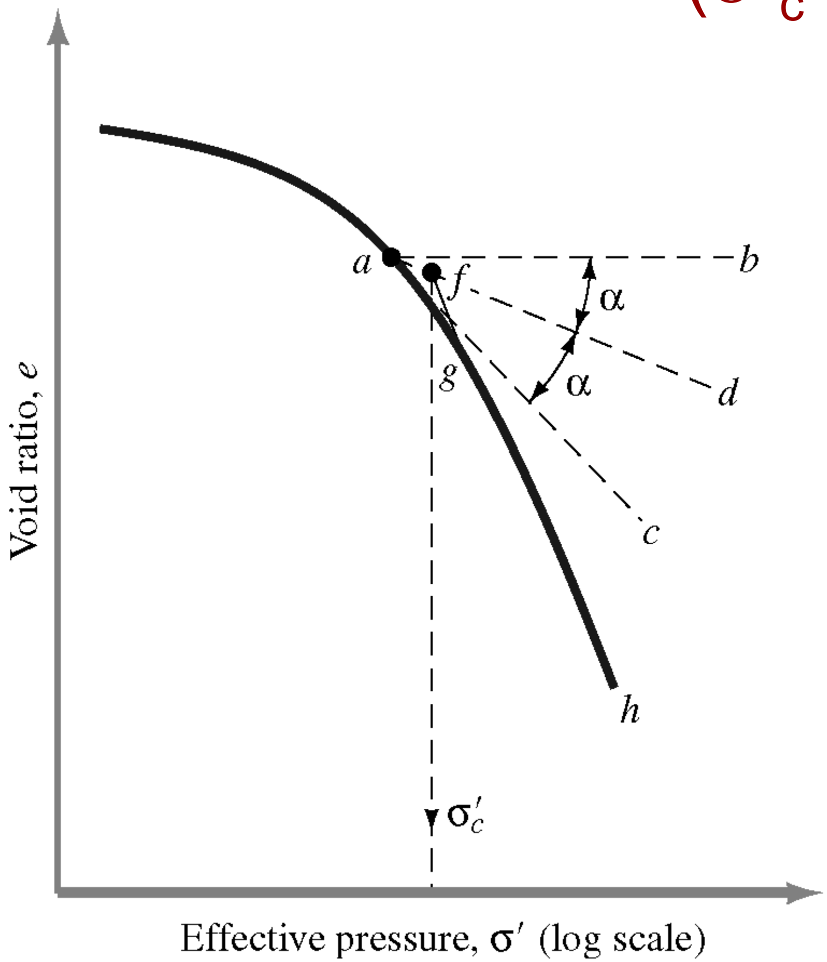


Figure 7.8. Das FGE (2005).

Graphical Method (Casagrande, 1936)

1. Visually identify point of minimum radius of curvature on *e*-log σ´ curve (i.e. Point *a*).
2. Draw horizontal line from Point *a* (i.e. Line *ab*).
3. Draw Line *ac* tangent to Point *a*.
4. Draw Line ad bisecting Angle *bac*.
5. Project the straight line portion of *gh* on *e*-log σ´ curve to intersect Line *ad*. This intersection (Point *f*) is the maximum past pressure (a.k.a. preconsolidation pressure).

OVERCONSOLIDAITON RATIO (OCR)

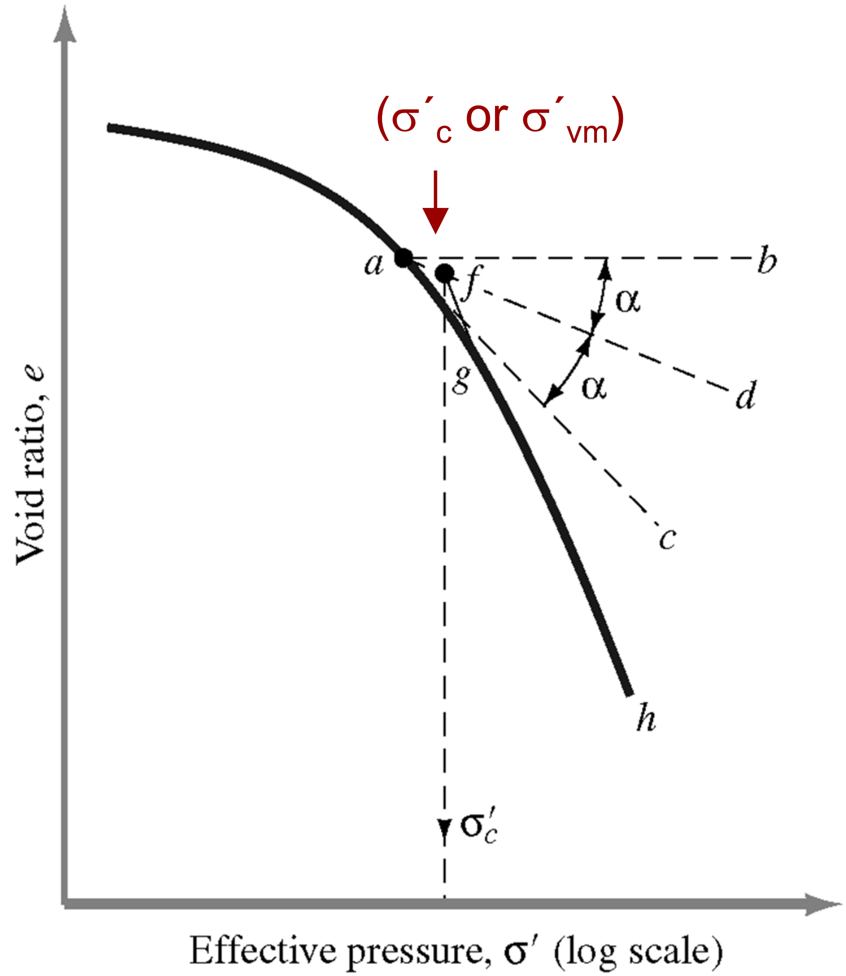


Figure 7.8. Das FGE (2005).

$$OCR = \frac{\sigma'_c}{\sigma'}$$

Where:

σ'_c (a.k.a. σ'_{vm}) = Preconsolidation Pressure (a.k.a Maximum Past Pressure).

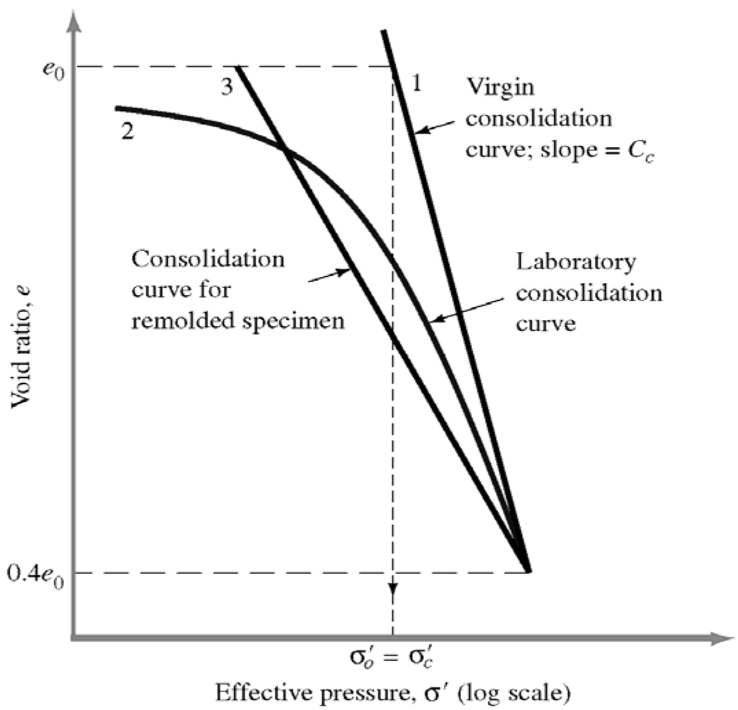
σ' = Present Effective Vertical Stress

General Guidelines:
NC Soils: $1 \leq OCR \leq 2$
OC Soils : $OCR > 2$

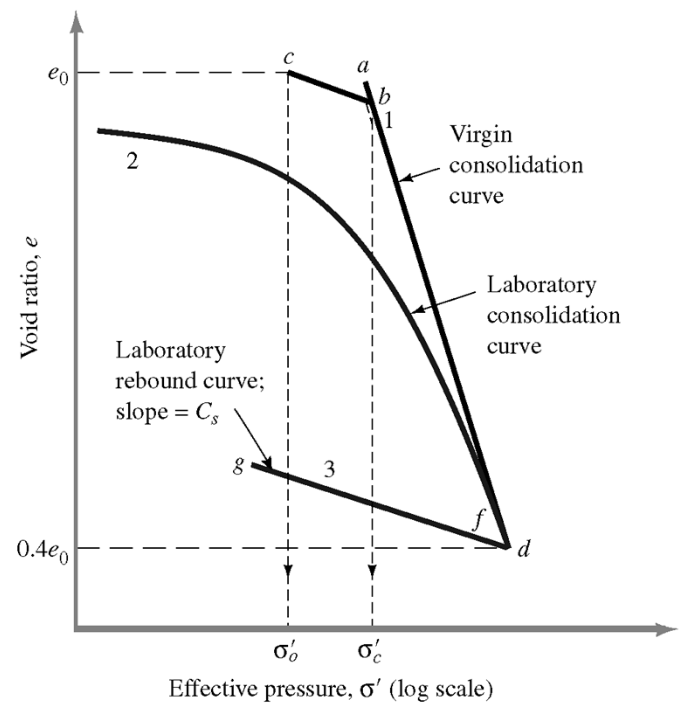
Possible Causes of OC Soils:
Preloading (thick sediments, glacial ice); fluctuations of GWT, underdraining, light ice/snow loads, desiccation above GWT, secondary compression.

EFFECTS OF SAMPLE DISTURBANCE

NC and OC soils of low to medium sensitivity will experience disturbance due to remolding. This changes the consolidation characteristics of the 1D consolidation tests.



NC Clays - Figure 7.9. Das FGE (2005)



OC Clays - Figure 7.10. Das FGE (2005)

Virgin Compression Curve – Consolidation Curve Insitu (i.e. w/o disturbance)

Sensitivity (S_t) $$S_t = \frac{q_{u(undisturbed)}}{q_{u(remolded)}}$$

Where q_u = Unconfined Compressive Strength

EFFECTS OF SAMPLE DISTURBANCE

Reconstruction of Virgin Consolidation Curves (EM 1110-1-1904)

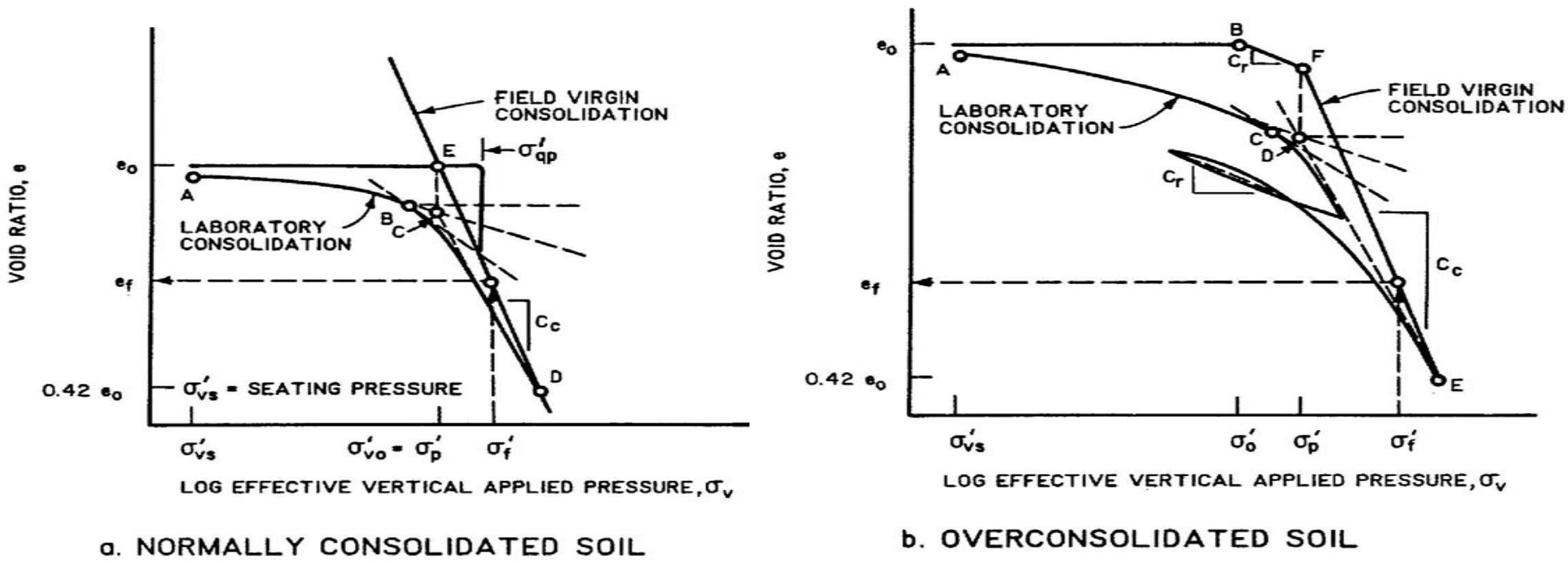


Figure 3-12. EM 1110-1-1904 Settlement Analysis.

EFFECTS OF SAMPLE DISTURBANCE

Reconstruction of Virgin Consolidation Curves (EM 1110-1-1904)

Table 3-6. EM 1110-1-1904 Settlement Analysis.

a. Normally Consolidated Soil (Figure 3-12a)

Step	Description
1	Plot point B at the point of maximum radius of curvature of the laboratory consolidation curve.
2	Plot point C by the Casagrande construction procedure: (1) Draw a horizontal line from B ; (2) Draw a line tangent to the laboratory consolidation curve through B ; and (3) Draw the bisector between horizontal and tangent lines. Point C is the intersection of the straight portion of the laboratory curve with the bisector. Point C indicates the maximum past pressure σ'_p .
3	Plot point E at the intersection e_o and σ'_p . e_o is given as the initial void ratio prior to testing in the consolidometer and σ'_p is found from step 2.
4	Plot point D at the intersection of the laboratory virgin consolidaation curve with void ratio $e = 0.42e_o$.
5	The field virgin consolidation curve is the straight line determined by points E and D.

EFFECTS OF SAMPLE DISTURBANCE

Reconstruction of Virgin Consolidation Curves (EM 1110-1-1904)

Table 3-6. EM 1110-1-1904 Settlement Analysis.

b. Overconsolidated Soil (Figure 3-12b)

Step	Description
1	Plot point B at the intersection of the given e_o and the initial estimated in situ effective overburden pressure σ'_o .
2	Draw a line through B parallel to the mean slope C_r of the rebound laboratory curve.
3	Plot point D using step 2 in Table 3-6a above for normally consolidated soil.
4	Plot point F by extending a vertical line through D up through the intersection of the line of slope C_r extending through B.
5	Plot point E at the intersection of the laboratory virgin consolidation curve with void ratio $e = 0.42e_o$.
6	The field virgin consolidation curve is the straight line through points F and E.

SETTLEMENT FROM 1D PRIMARY CONSOLIDATION

At End of Primary Consolidation Δσ = Δσ´

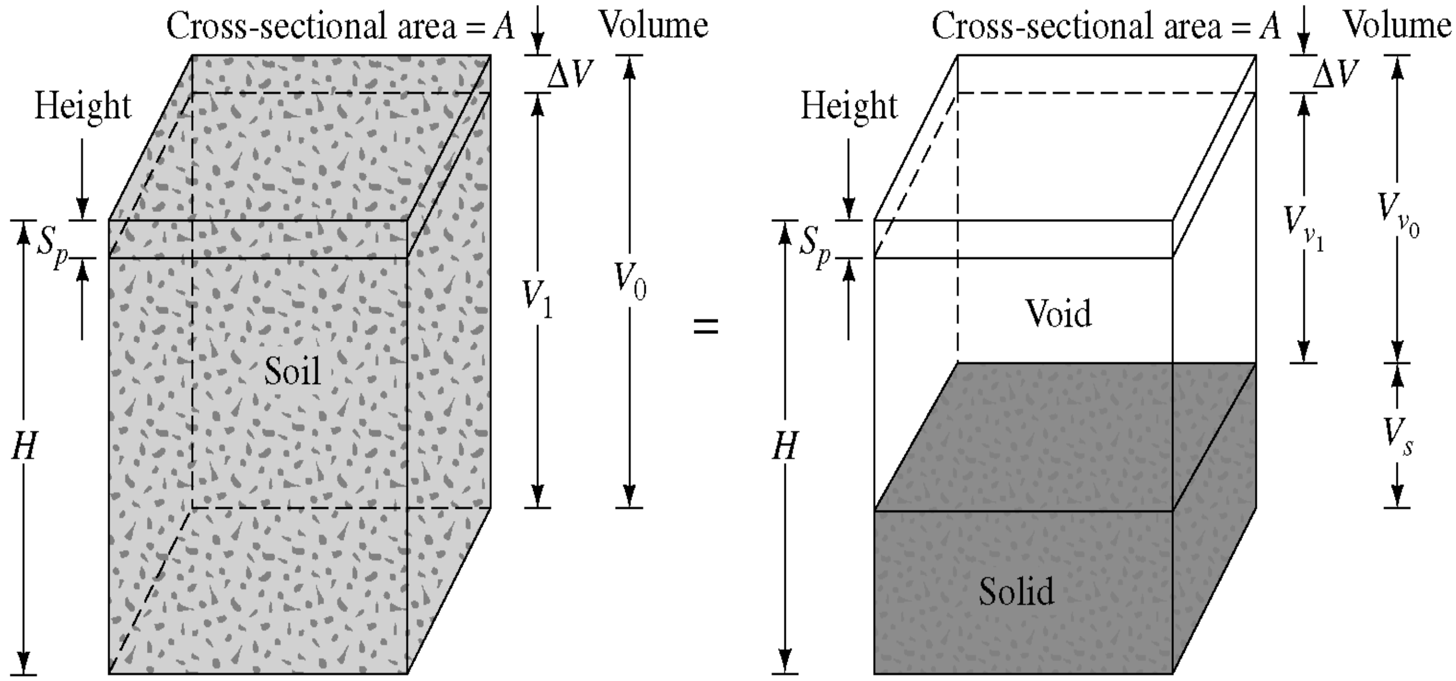


Figure 7.11. Das FGE (2005)

$$\Delta V = V_o - V_1 = HA - (H - S_p)A = S_p A$$

Where:

V = Volume, V_o = Initial Volume, V_1 = Final Volume, S_p = Primary Settlement

Settlement from 1D Primary Consolidation

At End of Primary Consolidation Δσ = Δσ ´

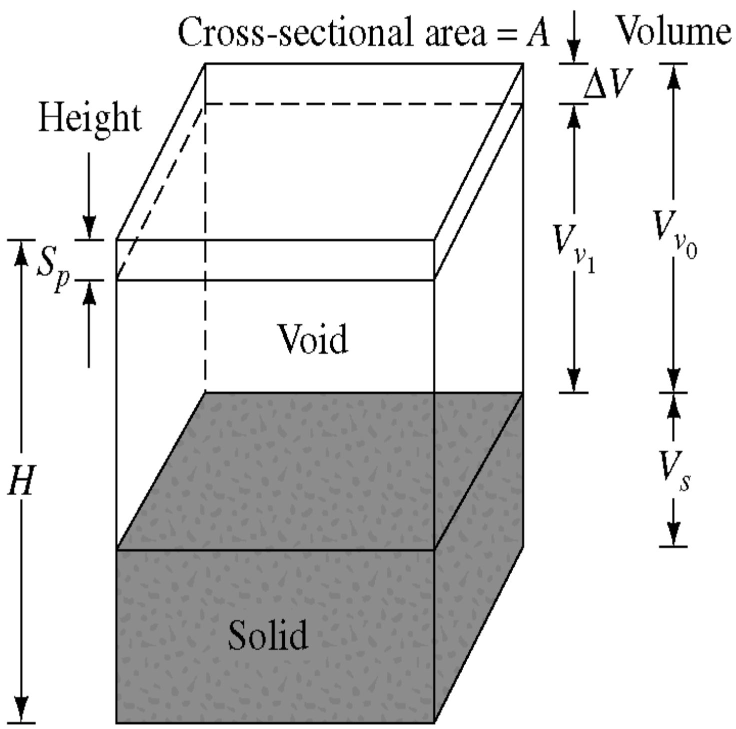


Figure 7.11. Das FGE (2005).

$$\Delta V = S_p A = V_{vo} - V_{v1} = \Delta V_v$$

Where:

V_{vo} = Initial Void Volume, V_{v1} = Final Void Volume

$$\Delta V_v = \Delta e V_s$$

Where:

Δe = Change in Void Ratio

$$V_s = \frac{V_o}{1+e_o} = \frac{AH}{1+e_o}$$

Where:

e_0 = Initial Void Ratio

SETTLEMENT FROM 1D PRIMARY CONSOLIDAITON

At End of Primary Consolidation $\Delta\sigma = \Delta\sigma'$

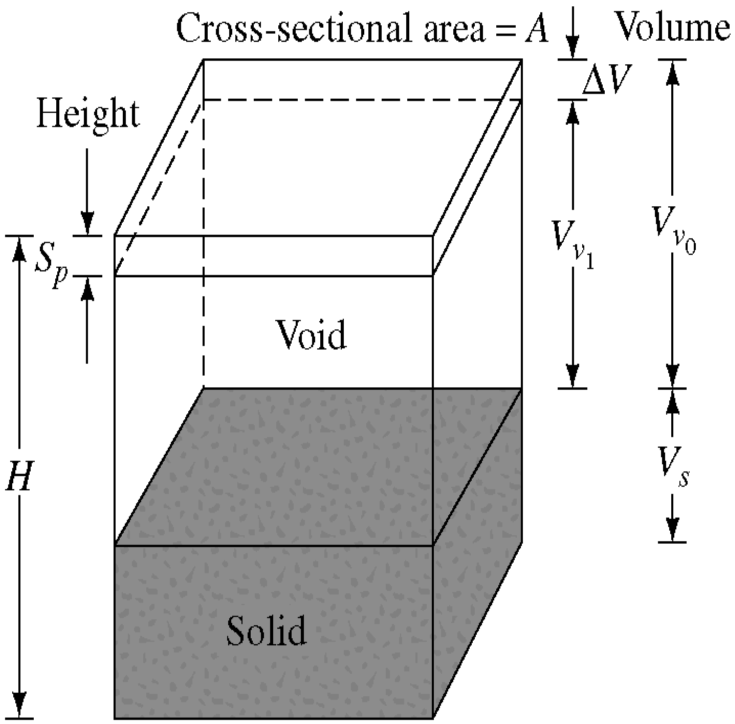


Figure 7.11. Das FGE (2005).

Therefore:

$$\Delta V = S_p A = \Delta e V_s = \frac{AH}{1+e_o}\Delta e$$

or

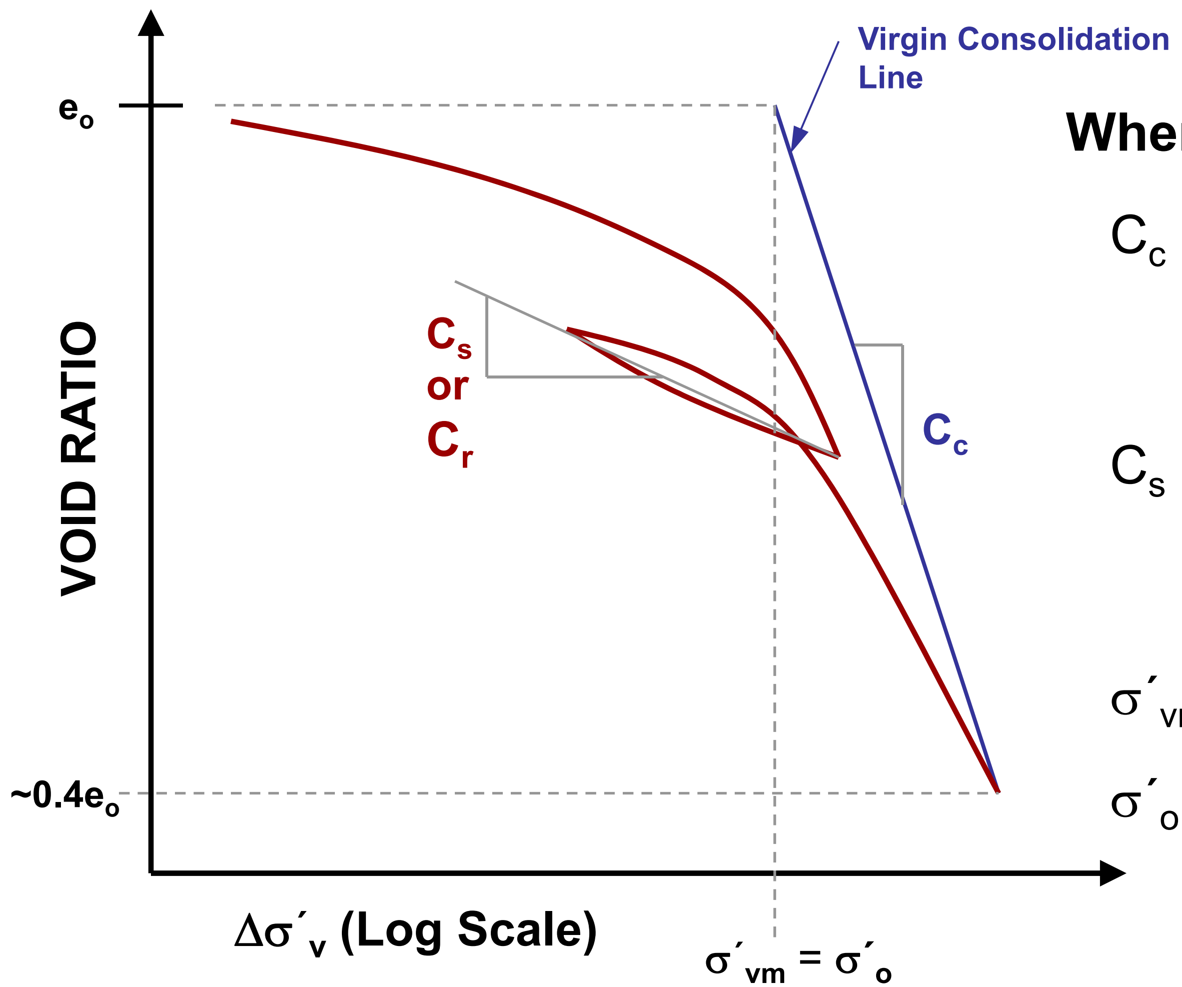
$$S_p = H\frac{\Delta e}{1+e_o}$$

$$\frac{S_p}{H} = \frac{\Delta e}{1+e_o} = \varepsilon_v$$

Where:

ε_v = Vertical Strain

Settlement from 1D Primary Consolidation



NC Clay

Where:

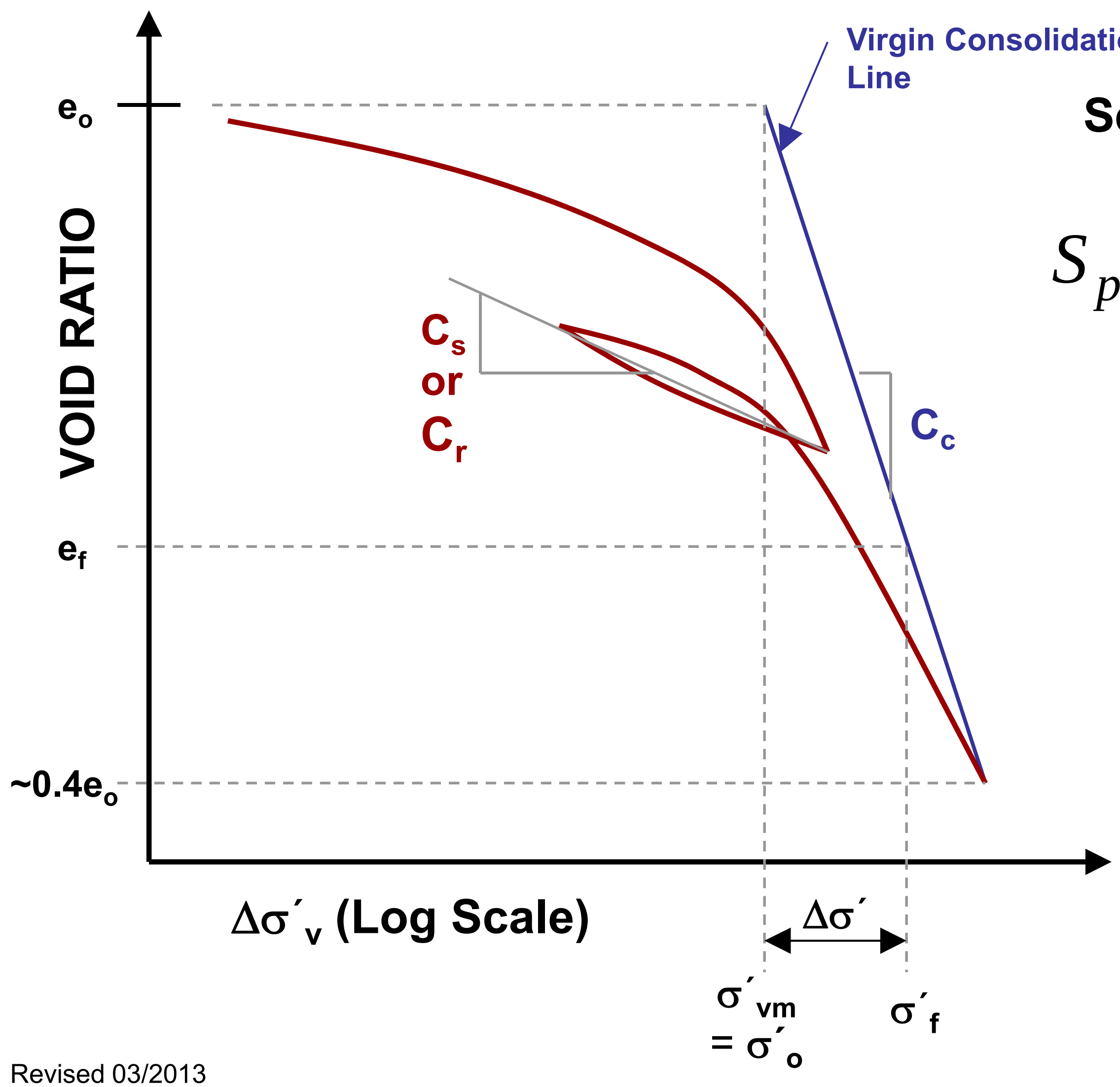
C_c = Slope of Field Virgin Consolidation Curve = **Compression Index**

C_s (or C_r) = Slope of Rebound Curve = **Swell Index**

σ'_{vm} = Maximum Past Pressure

σ'_o = Initial Vertical Effective Stress

SETTLEMENT FROM 1D PRIMARY CONSOLIDATION



NC Clay

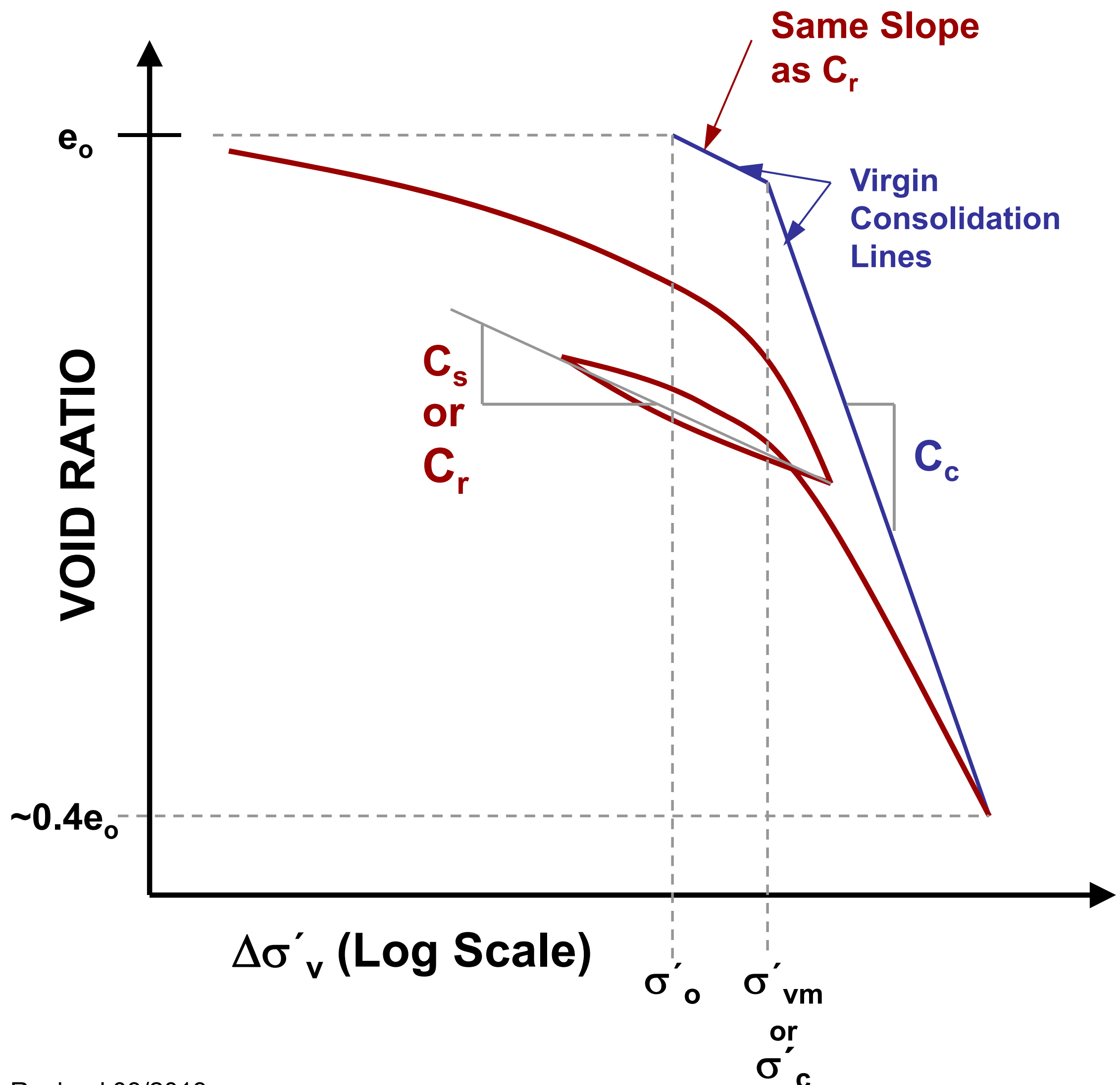
Settlement (S_p) using Void Ratio

$$S_p = \frac{C_c H}{1+e_0} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

Where:

- S_p = Settlement
- H = Height of Soil Layer
- σ'_{vm} = Final Vertical Effective Stress = σ_o' - Current Vertical Effective Stress
- $\Delta\sigma'$ = Change in Vertical Effective Stress
- σ_f' = Final Vertical Effective Stress
- e_f = Final Void Ratio

Settlement from 1D Primary Consolidation



OC Clay

Where:

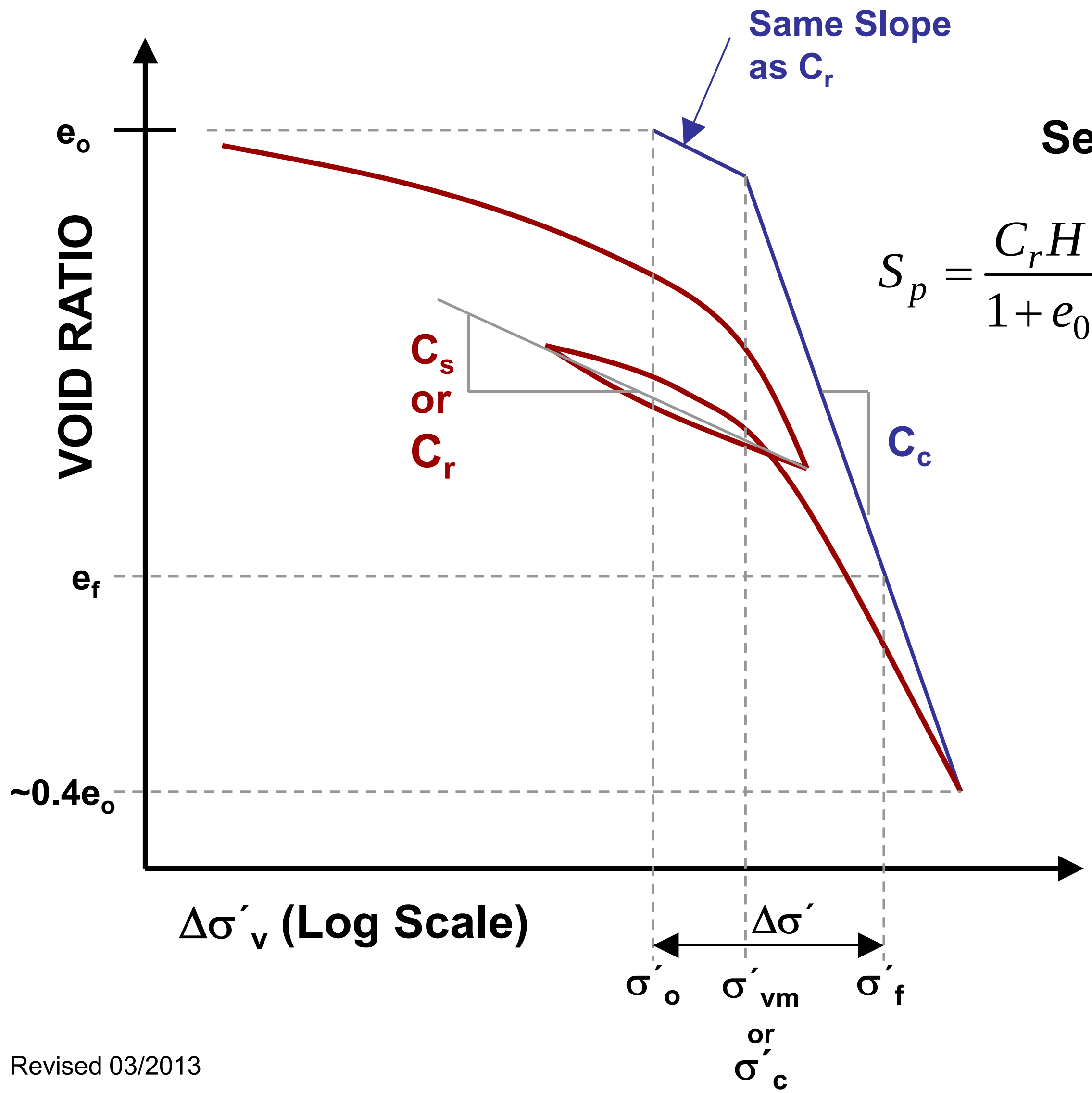
C_c = Slope of Field Virgin Consolidation Curve = **Compression Index**

C_s (or C_r) = Slope of Rebound Curve = **Swell Index**

σ'_{vm} = Maximum Past Pressure

σ'_o = Initial Vertical Effective Stress

SETTLEMENT FROM 1D PRIMARY CONSOLIDATION



OC Clay

Settlement (S_p) using Void Ratio

$$S_p = \frac{C_r H}{1+e_0}\log\left(\frac{\sigma'_{vm}}{\sigma'_o}\right) + \frac{C_c H}{1+e_0}\log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

Where:

S_p = Settlement
H = Height of Soil Layer
$\Delta\sigma'$ = Change in Vertical Effective Stress
σ_o' = Initial Vertical Effective Stress
σ_f' = Final Vertical Effective Stress
e_f = Final Void Ratio

SETTLEMENT FROM 1D PRIMARY CONSOLIDATION

Compression Index (C_c) Estimates from Other Laboratory Tests

Soil	C_c Equation	Reference
Undisturbed Clays	$C_c = 0.009(LL - 10)$	Terzaghi & Peck (1967)
Disturbed Clays	$C_c = 0.007(LL - 10)$	
Organic Soils, Peat	$C_c = 0.0115W_n$	EM 1110-1-1904
Clays	$C_c = 1.15(e_o - 0.35)$	
	$C_c = 0.012W_n$	
	$C_c = 0.01(LL - 13)$	
Varved Clays	$C_c = (1 + e_o) - [0.1 + 0.006(W_n - 25)]$	
Uniform Silts	$C_c = 0.20$	

SETTLEMENT FROM 1D PRIMARY CONSOLIDATION

Compression Index (C_c) Estimates from Other Laboratory Tests

Soil	C_c Equation	Reference
Clays	$C_c = 0.141 G_s^{1.2} \left(\frac{1+e_o}{G_s} \right)^{2.38}$	Rendon-Herrero (1983)
Clays	$C_c = 0.2343 \left[\frac{LL}{100} \right] G_s$	Nagaraj & Murty (1985)

Where:

G_s = Specific Gravity of Solids
LL = Liquid Limit (in %)
W_n = Natural Water Content
e_o = Initial Void Ratio

SETTLEMENT FROM 1D PRIMARY CONSOLIDATION

Compression Index (C_c) Estimates from Other Laboratory Tests

Soil	C_c Equation	Reference
Clays	$C_c = 0.141 G_s^{1.2} \left(\frac{1+e_o}{G_s} \right)^{2.38}$	Rendon-Herrero (1983)
Clays	$C_c = 0.2343 \left[\frac{LL}{100} \right] G_s$	Nagaraj & Murty (1985)

EXAMPLE: SETTLEMENT FROM VIRGIN CONSOLIDATION CURVES

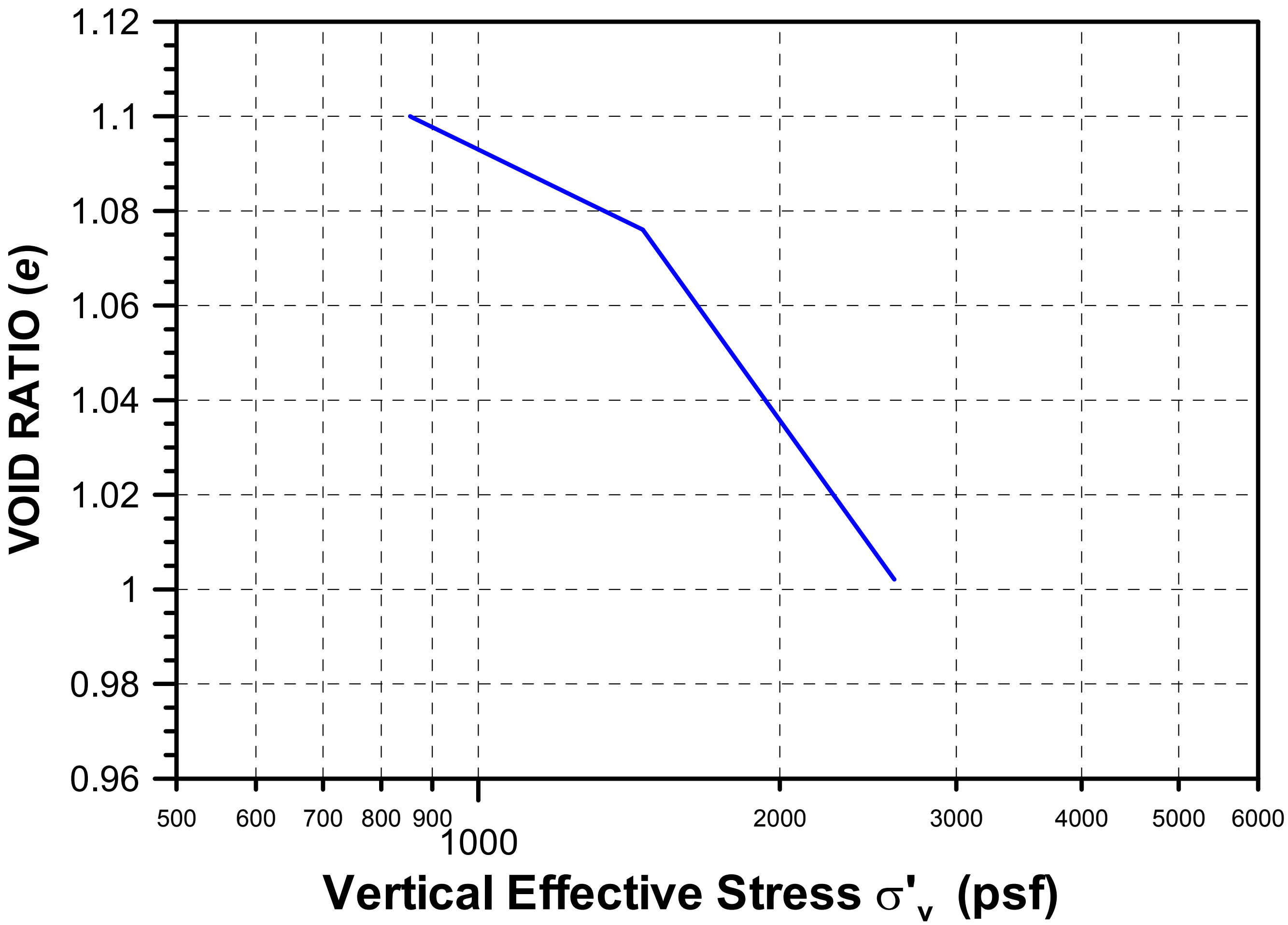


Figure 1. Example of Virgin Consolidation Curves.

GIVEN:

OC CH layer

$\sigma_o' = 855$ psf

$\sigma_{vm}' = 1460$ psf

$\Delta\sigma' = 1005$ psf

$e_o = 1.1$

Height of CH Layer = 10 ft

Example: Settlement from Virgin Consolidation Curves

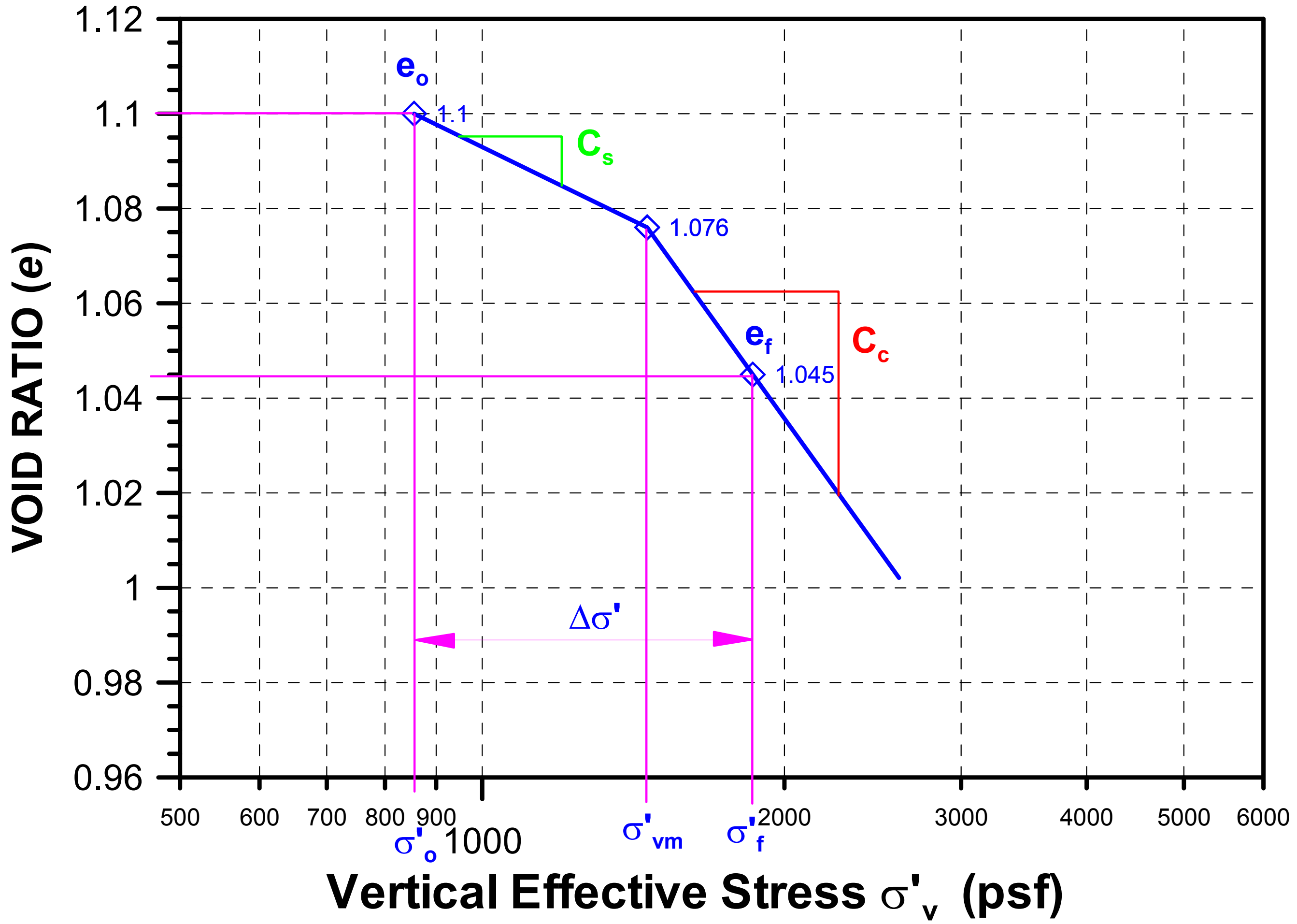


Figure 1. Example of Virgin Consolidation Curves.

$$S_p = H\frac{\Delta e}{1+e_o}$$

$$\Delta e = 1.1 - 1.045 = 0.055$$

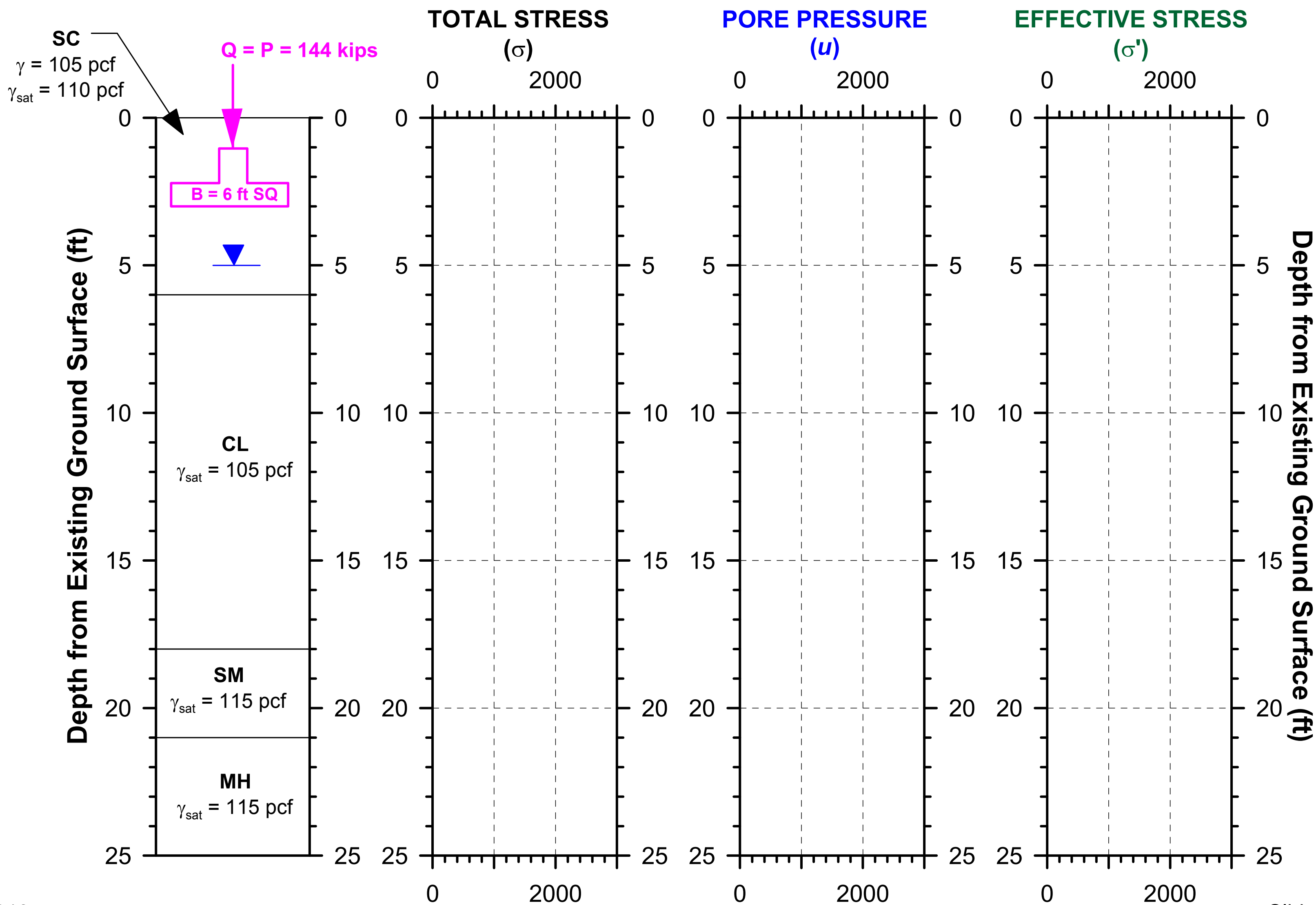
$$e_o = 1.1$$

$$S_p = (10\,ft)\left(\frac{0.055}{1+1.1}\right)$$

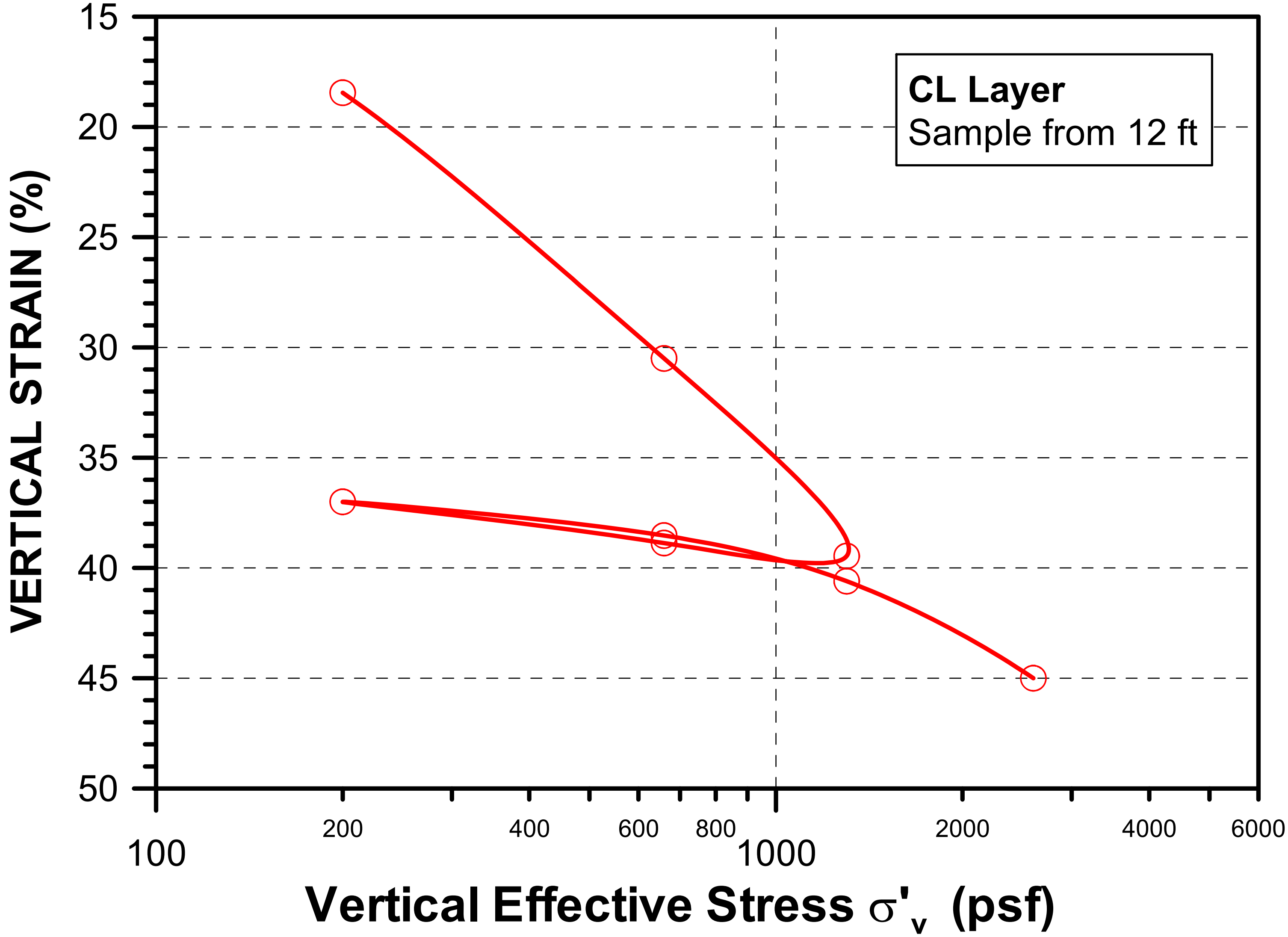
$$S_p = 0.262\,ft$$

$$S_p = 3.14in = 3\frac{1}{4}in$$

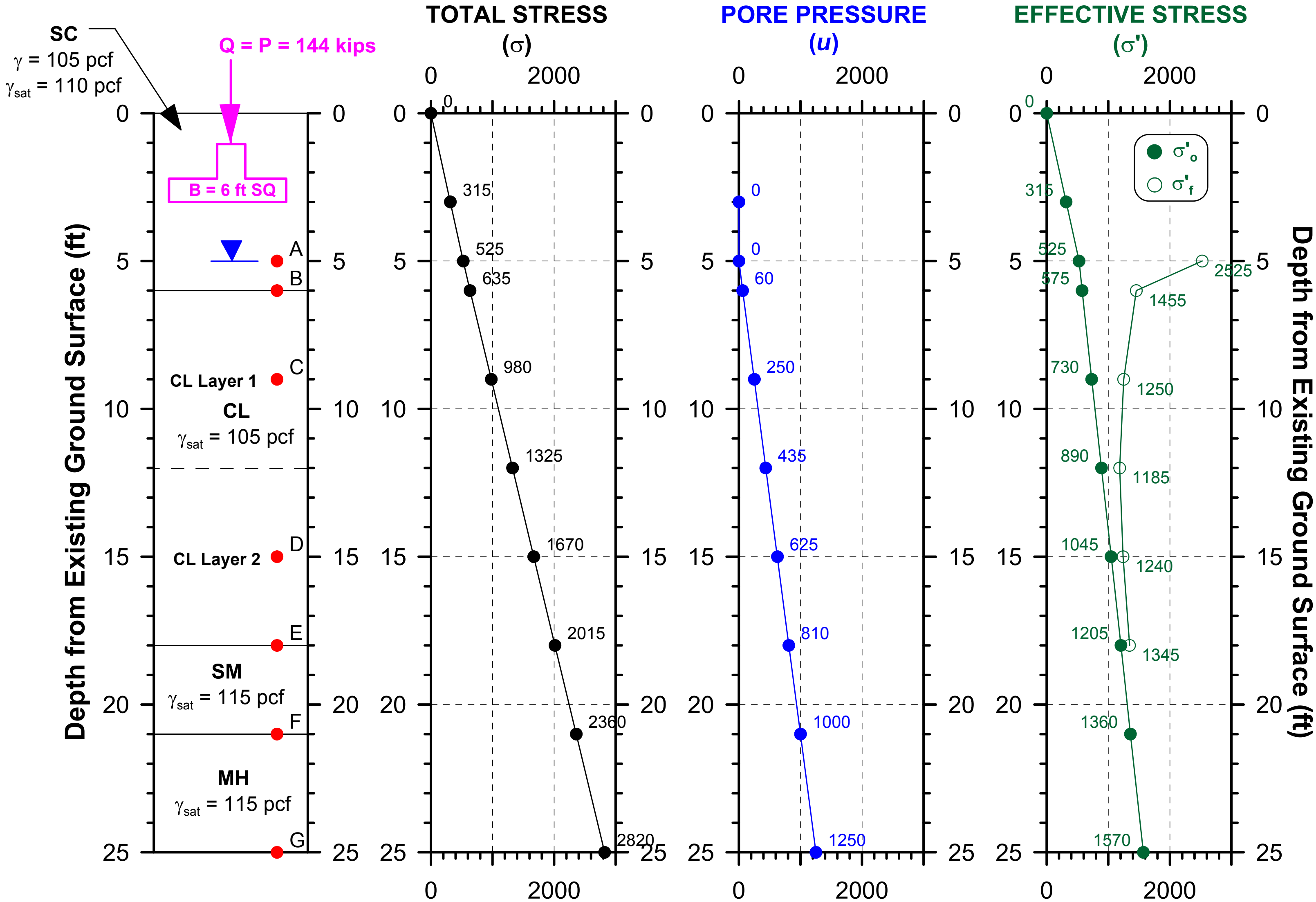
EXAMPLE: SETTLEMENT FROM 1D TEST STRAIN RESULTS



EXAMPLE: SETTLEMENT FROM 1D TEST STRAIN RESULTS



EXAMPLE: SETTLEMENT FROM 1D TEST STRAIN RESULTS



EXAMPLE: SETTLEMENT FROM 1D TEST STRAIN RESULTS

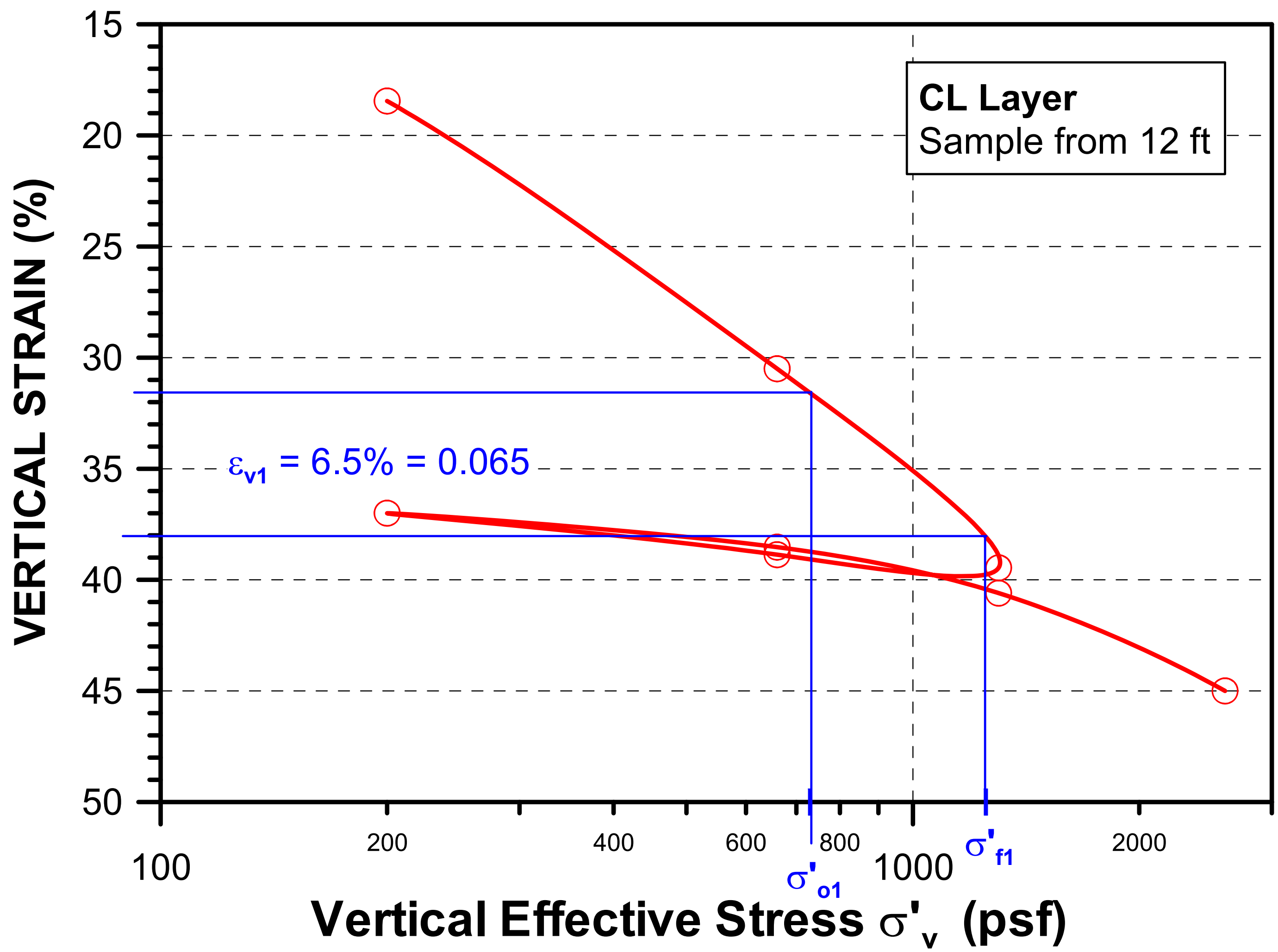
CL Layer 1

$$S_{p_1} = H_1 \varepsilon_{v1}$$

$$S_{p_1} = (6\,ft)(0.065)$$

$$S_{p_1} = 0.39\,ft$$

$$S_{p_1} = 4.7in$$



EXAMPLE: SETTLEMENT FROM 1D TEST STRAIN RESULTS

CL Layer 2

$$S_{p2} = H_2 \varepsilon_{v2}$$

$$S_{p2} = (6\,ft)(0.024)$$

$$S_{p2} = 0.14\,ft$$

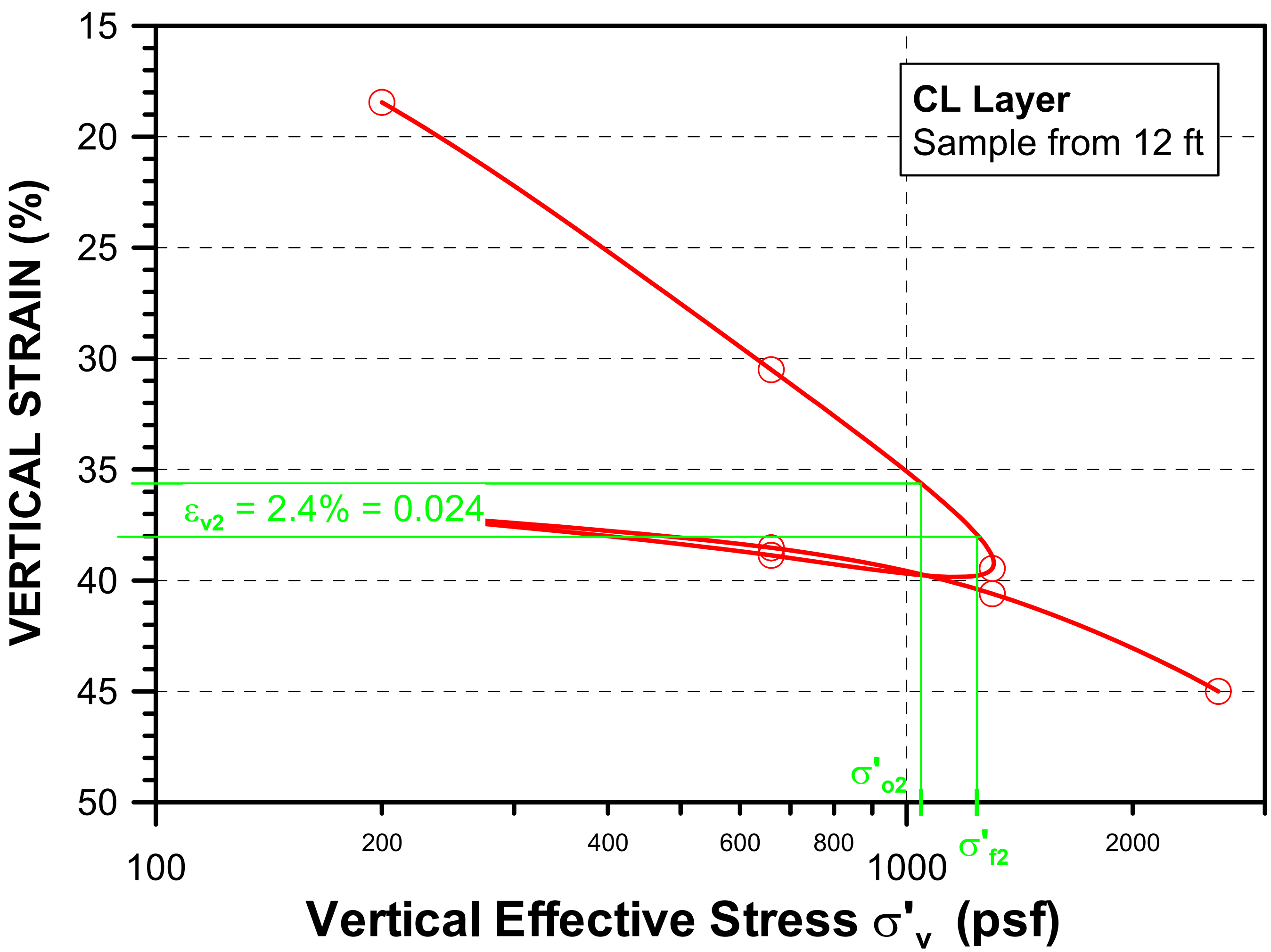
$$S_{p2} = 1.7 in$$

Total Settlement

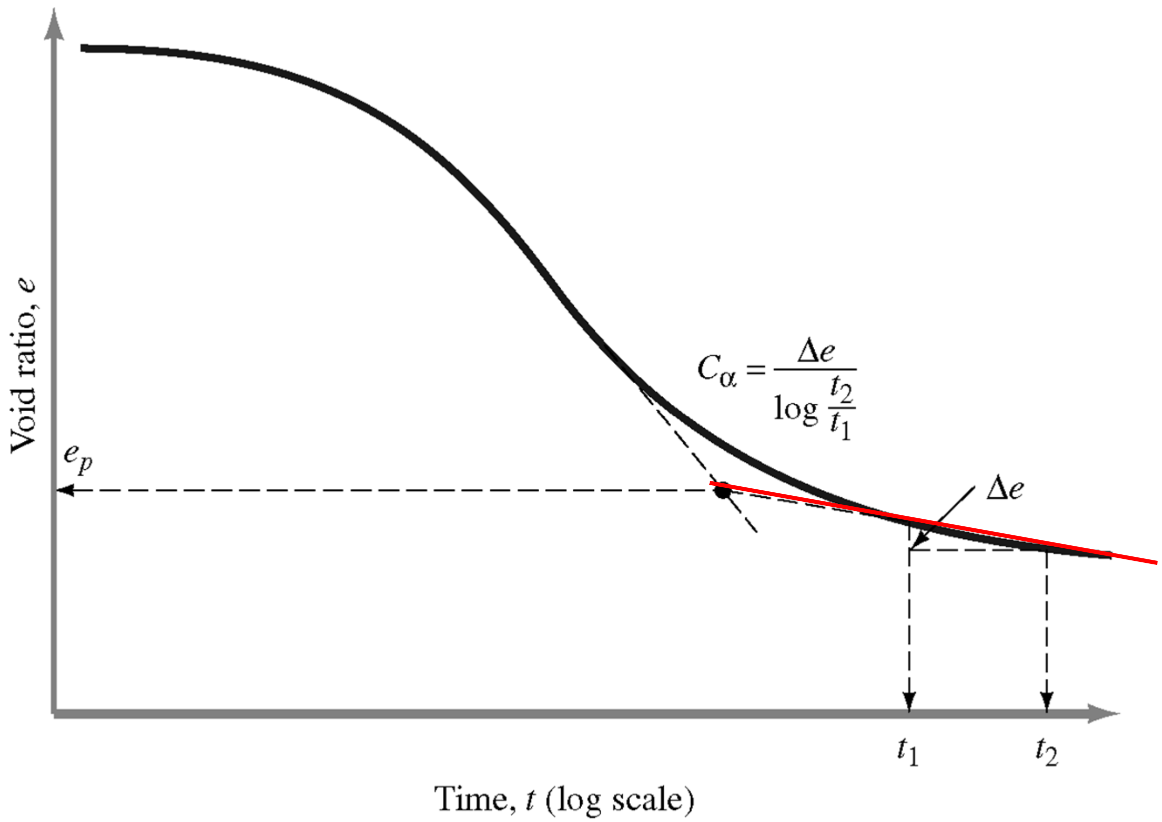
$$S_{Ptotal} = S_{p1} + S_{p2}$$

$$S_{Ptotal} = 6.4 in$$

$$S_{Ptotal} = 6\frac{1}{2} in$$



SETTLEMENT FROM SECONDARY CONSOLIDATION



Results of 1D Consolidation Test @ One Load Increment

Figure 7.15. Das FGE (2005).

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1}$$

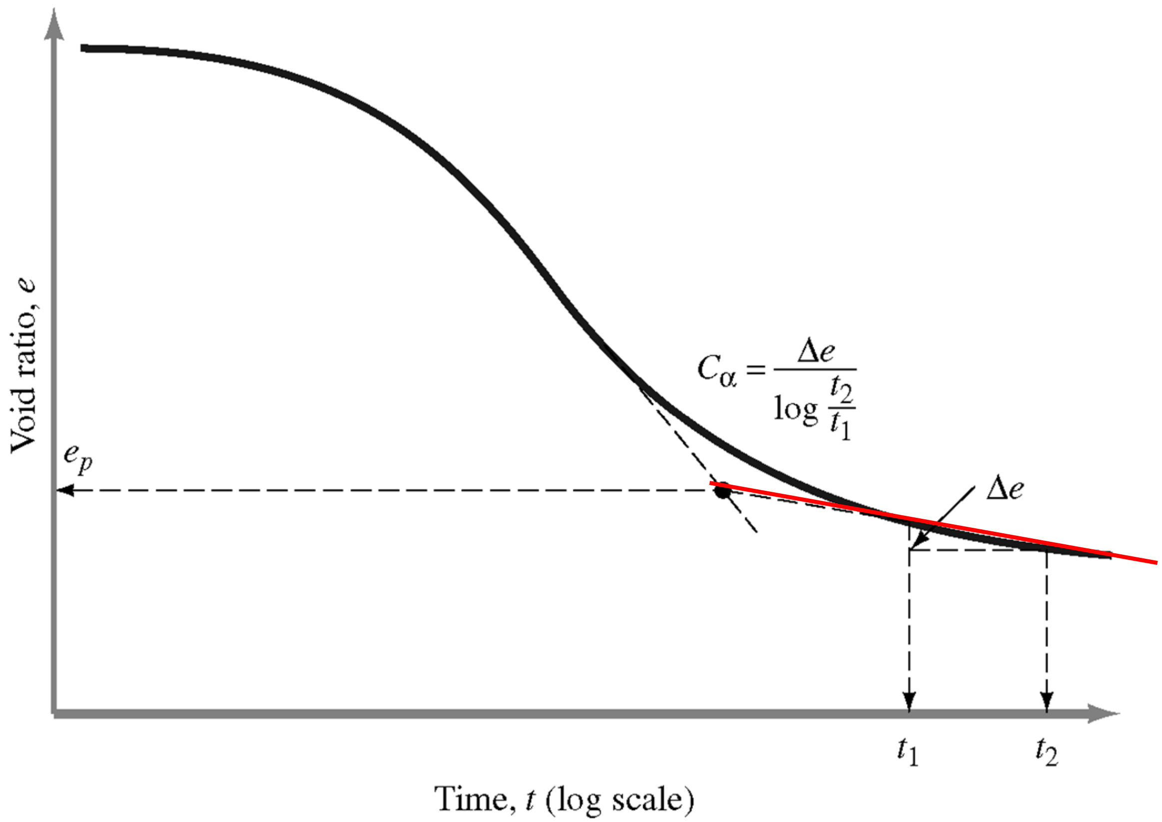
Where:

C_α = Secondary Compression Index

Δe = Change in Void Ratio

t = Time

SETTLEMENT FROM SECONDARY CONSOLIDATION



Results of 1D Consolidation Test @ One Load Increment

Figure 7.15. Das FGE (2005).

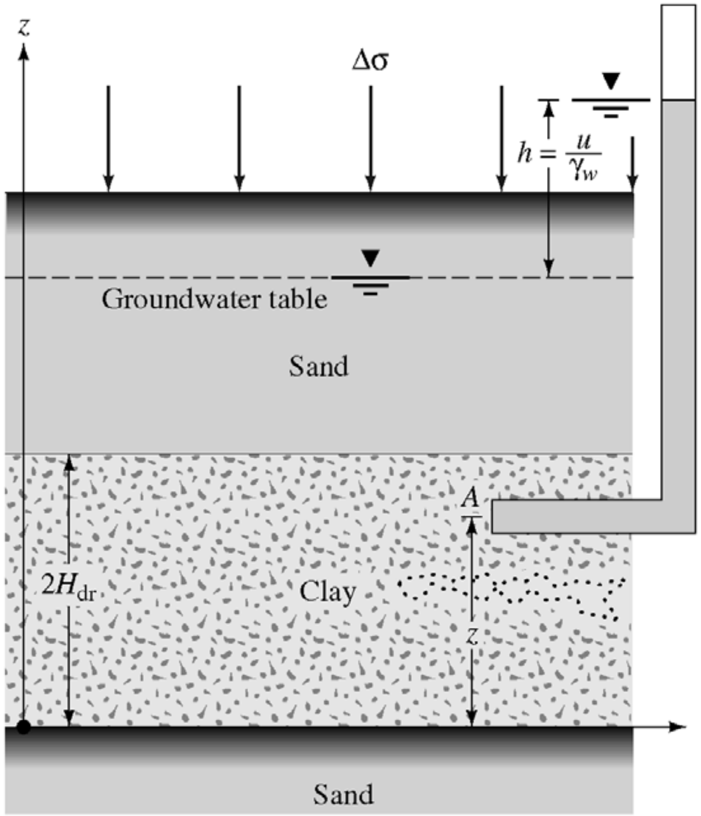
$$S_s = C'_\alpha H \log\left(\frac{t_2}{t_1}\right)$$

$$C'_\alpha = \frac{C_\alpha}{1 + e_p}$$

Where:

H = Height of Soil Layer
e_p = Void Ratio @ End of Primary Consolidation
t = Time

TIME RATE OF CONSOLIDATION



Clay Layer Undergoing Consolidation

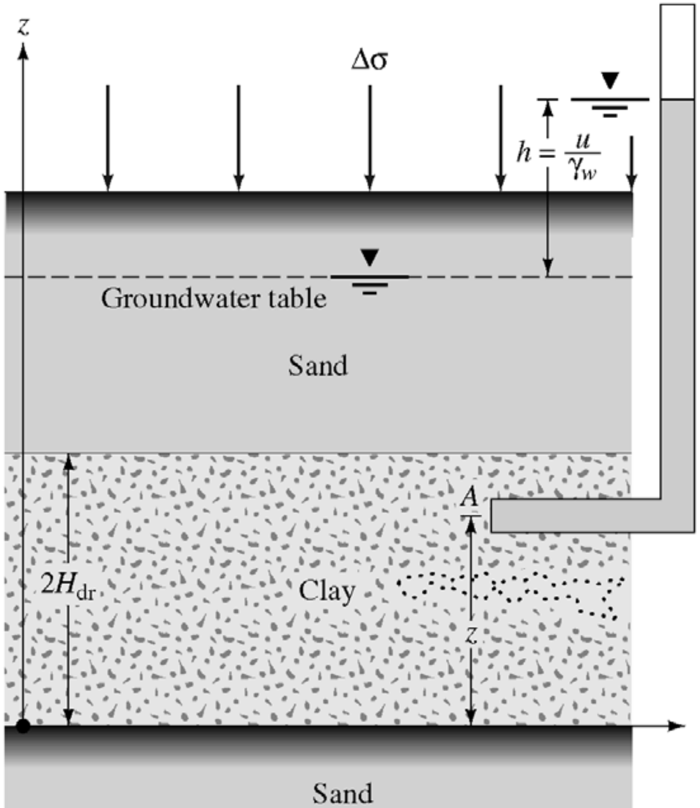
Figure 7.17a. Das FGE (2005).

Theory of 1D Consolidation (Terzaghi, 1925)

Assumptions:

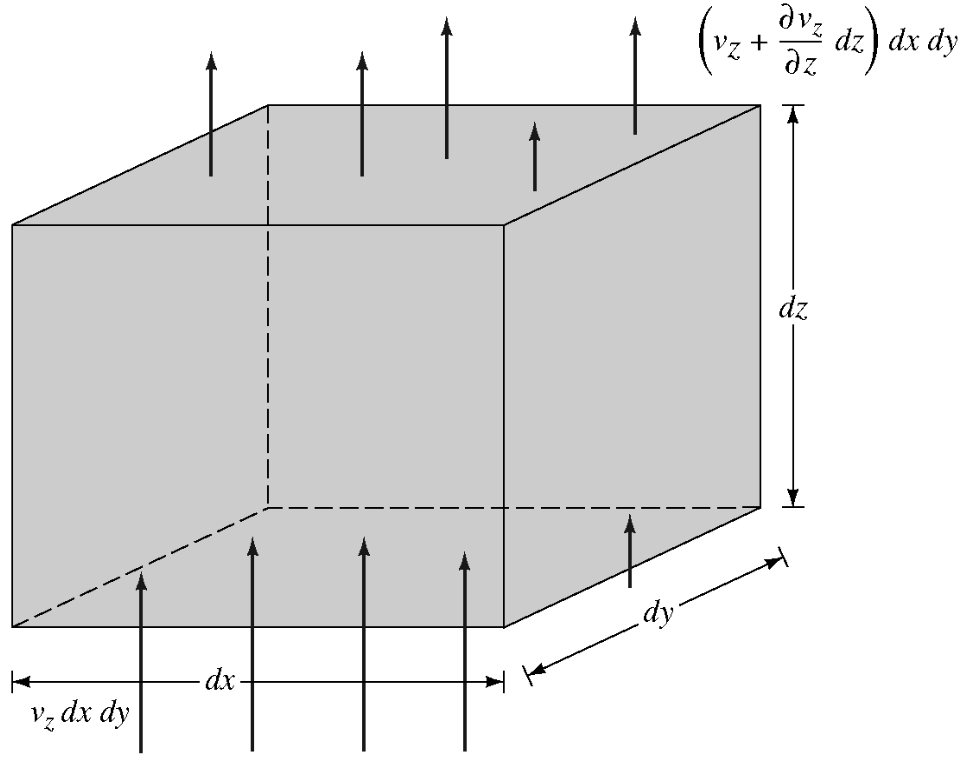
1. The clay-water system is homogenous.
2. Saturation is complete (S = 100%).
3. Compressibility of water is negligible.
4. Compressibility of soil grains is negligible (but soil particles rearrange).
5. Flow of water is in one direction only.
6. Darcy's Law is Valid.

Time Rate of Consolidation



Clay Layer Undergoing Consolidation

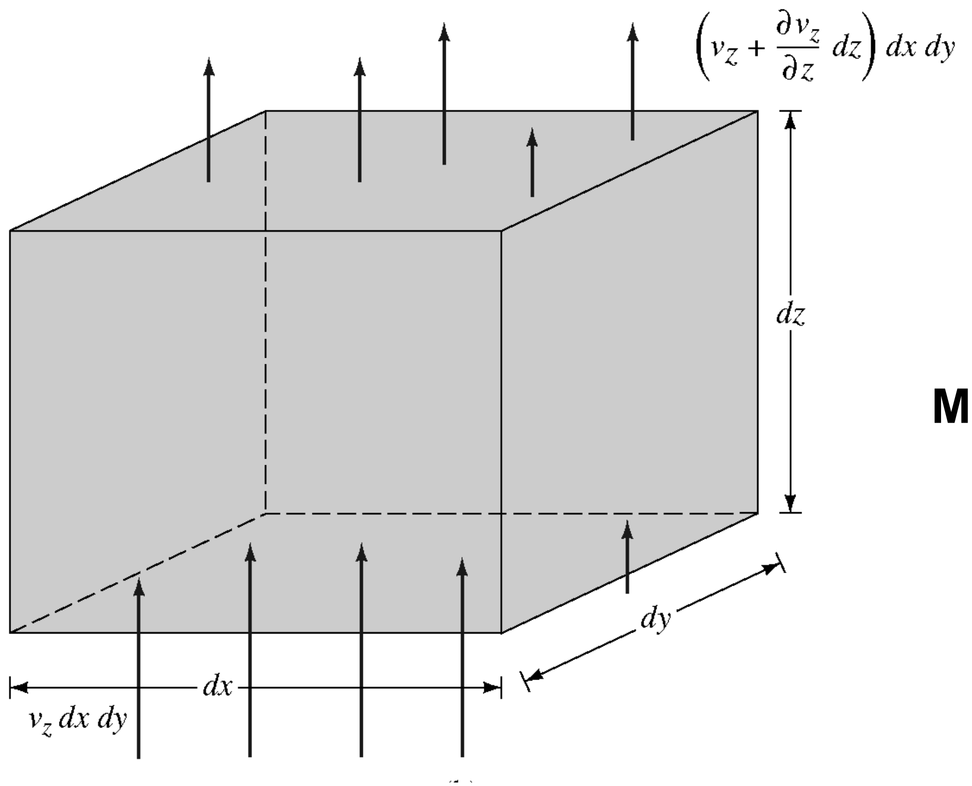
Figure 7.17a. Das FGE (2005).



Flow of Water @ Point A

Figure 7.17b. Das FGE (2005).

Time Rate of Consolidation



Flow of Water @ Point A
Figure 7.17b. Das FGE (2005).

(Rate of Water Outflow) –

(Rate of Water Inflow) =

(Rate of Volume Changes)

Mathematical Equation:

$$\left(v_z + \frac{\partial v_z}{\partial z} dz\right) dxdy - v_z dxdy = \frac{\partial V}{dt}$$

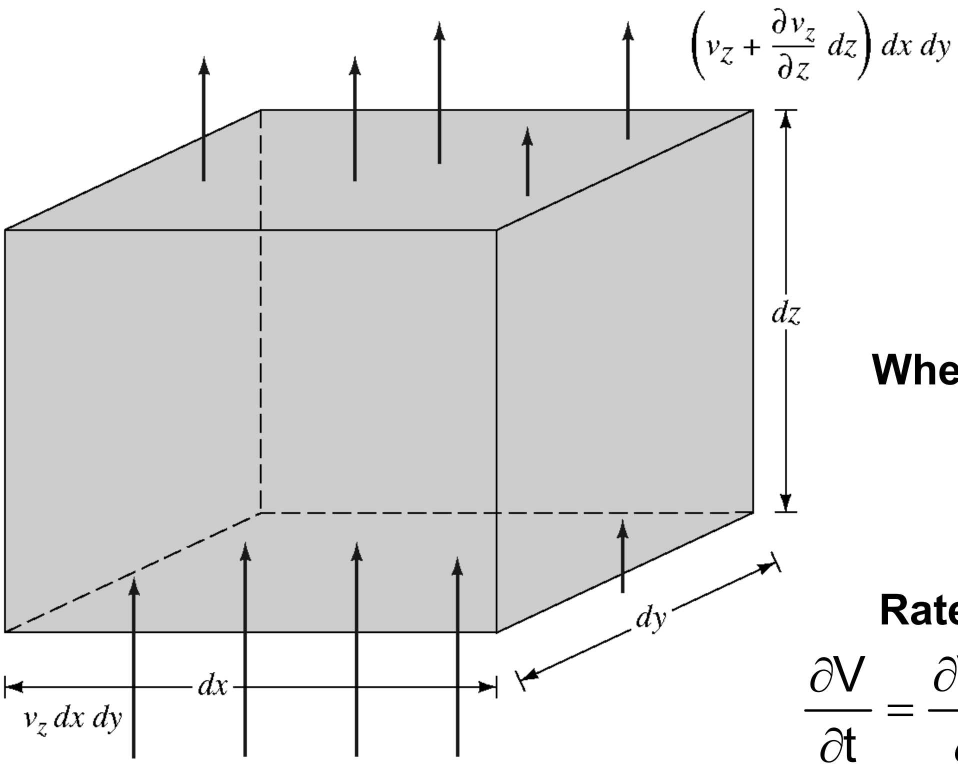
or

$$\frac{\partial v_z}{\partial z} dxdydz = \frac{\partial V}{dt}$$

Where:

V = Volume of Soil Element
v_z = Velocity of flow in z direction

TIME RATE OF CONSOLIDATION



Flow of Water @ Point A

Figure 7.17b. Das FGE (2005).

$$\frac{\partial v_z}{\partial z} dxdydz = \frac{\partial V}{\partial t}$$

Using Darcy's Law (*v* = *ki*)

$$v_z = ki = -k\frac{\partial h}{\partial z} = -\frac{k}{\gamma_w}\frac{\partial u}{\partial z}$$

Where *u* = excess pore pressure. From algebra:

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{dxdydz}\frac{\partial V}{\partial t}$$

Rate of change in *V* = Rate of Change in V_v

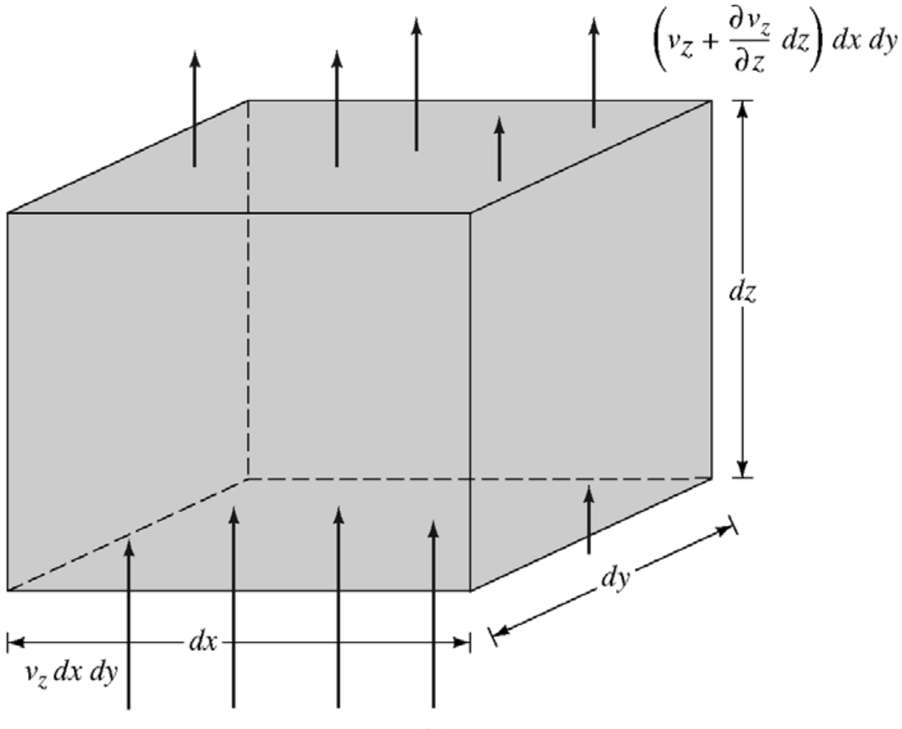
$$\frac{\partial V}{\partial t} = \frac{\partial V_v}{\partial t} = \frac{\partial (V_s + eV_s)}{\partial t} = \frac{\partial V_s}{\partial t} + V_s\frac{\partial e}{\partial t} + e\frac{\partial V_s}{\partial t}$$

Where:

V_s = Volume of Solids

V_v = Volume of Voids

TIME RATE OF CONSOLIDATION



Flow of Water @ Point A

Figure 7.17b. Das FGE (2005).

From Previous Slide

$$\frac{\partial V}{\partial t} = \frac{\partial V_v}{\partial t} = \frac{\partial (V_s + eV_s)}{\partial t} = \frac{\partial V_s}{\partial t} + V_s \frac{\partial e}{\partial t} + e\frac{\partial V_s}{\partial t}$$

Assuming soil solids are incompressible

$$\frac{\partial V_s}{\partial t} = 0$$

and

$$V_s = \frac{V}{1+e_o} = \frac{dxdydz}{1+e_o}$$

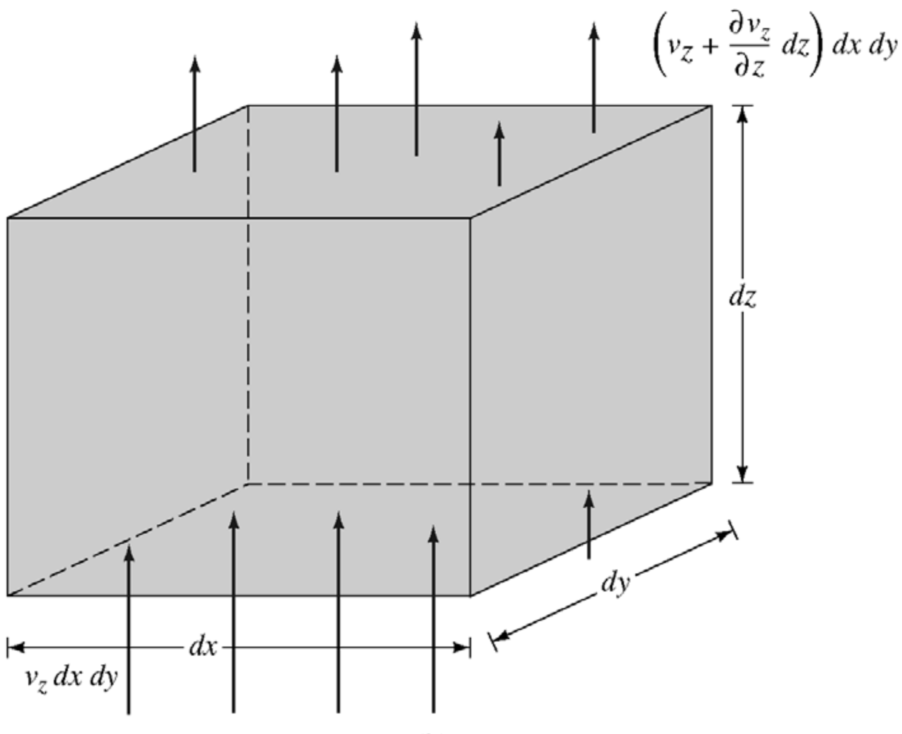
e_o = Initial Void Ratio. Substituting:

$$\frac{\partial V}{\partial t} = \frac{dxdydz}{1+e_o}\frac{\partial e}{\partial t}$$

Combining equations:

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e_o}\frac{\partial e}{\partial t}$$

TIME RATE OF CONSOLIDATION



Flow of Water @ Point A

Figure 7.17b. Das FGE (2005).

From Previous Slide

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e_o}\frac{\partial e}{\partial t}$$

The change in void ratio is caused by the increase in effective stress. Assuming linear relationship between the two:

$$\partial e = a_v \partial(\Delta\sigma') = -a_v \partial u$$

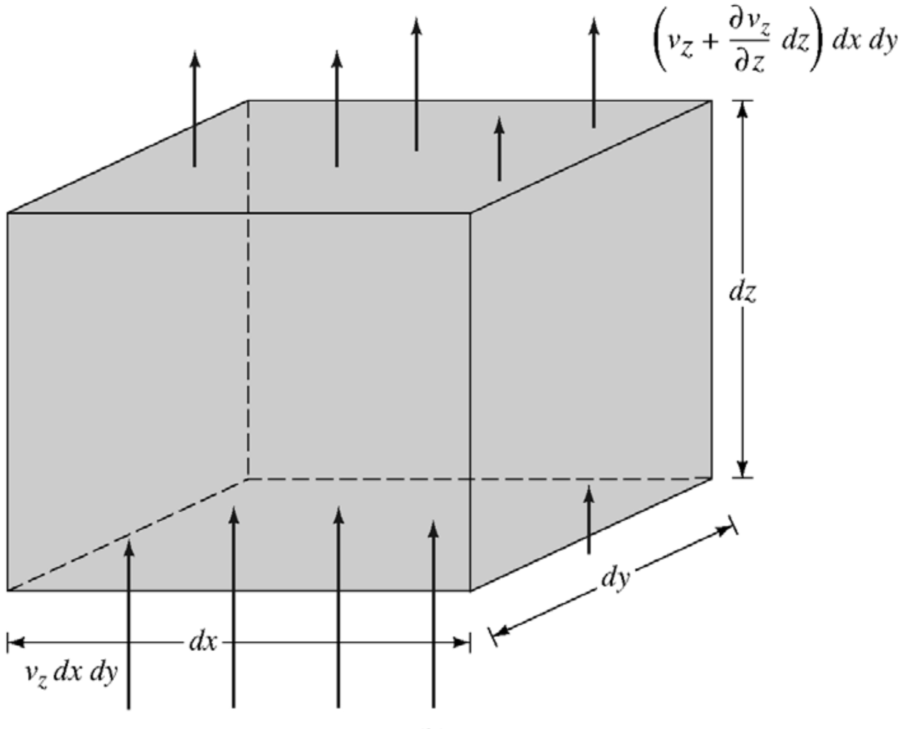
a_v = Coefficient of Compressibility. Can be considered constant over narrow pressure increases. Combining equations:

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = -\frac{a_v}{1+e_o}\frac{\partial u}{\partial t} = -m_v\frac{\partial u}{\partial t}$$

m_v = Coefficient of Volume Compressibility.

$$m_v = \frac{a_v}{1+e_o}$$

TIME RATE OF CONSOLIDATION



Flow of Water @ Point A

Figure 7.17b. Das FGE (2005).

From Previous Slide

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = -\frac{a_v}{1+e_o}\frac{\partial u}{\partial t} = -m_v\frac{\partial u}{\partial t}$$

a_v = Coefficient of Compressibility.
m_v = Coefficient of Volume Compressibility.

$$m_v = \frac{a_v}{1+e_o}$$

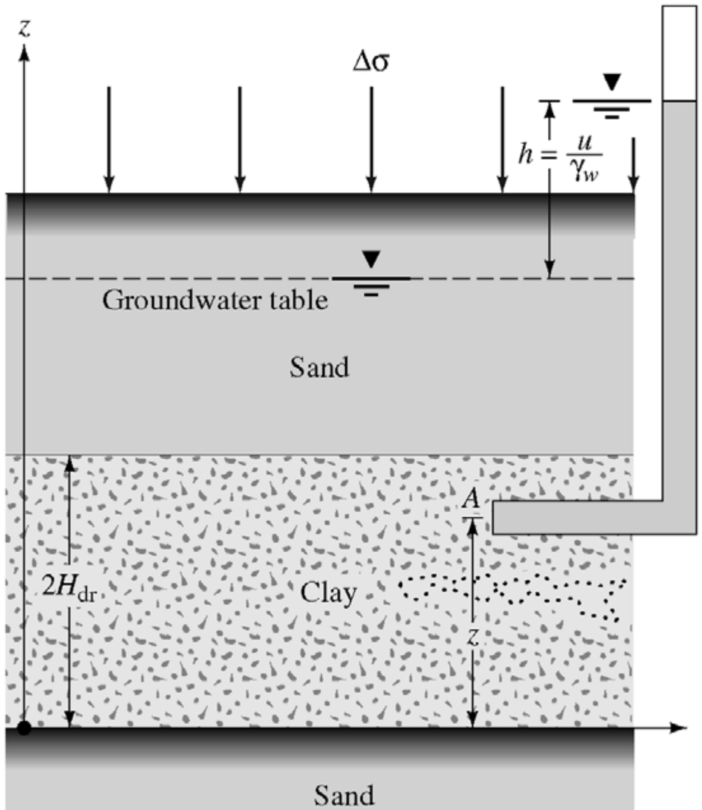
Rearranging Equations:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

Where c_v = Coefficient of Consolidation.

$$c_v = \frac{k}{(\gamma_w m_v)}$$

TIME RATE OF CONSOLIDATION



Clay Layer Undergoing Consolidation

Figure 7.17a. Das FGE (2005).

Basic Differential Equation of 1D Consolidation Theory

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

Can be solved with the following boundary conditions:

$$z = 0, u = 0$$

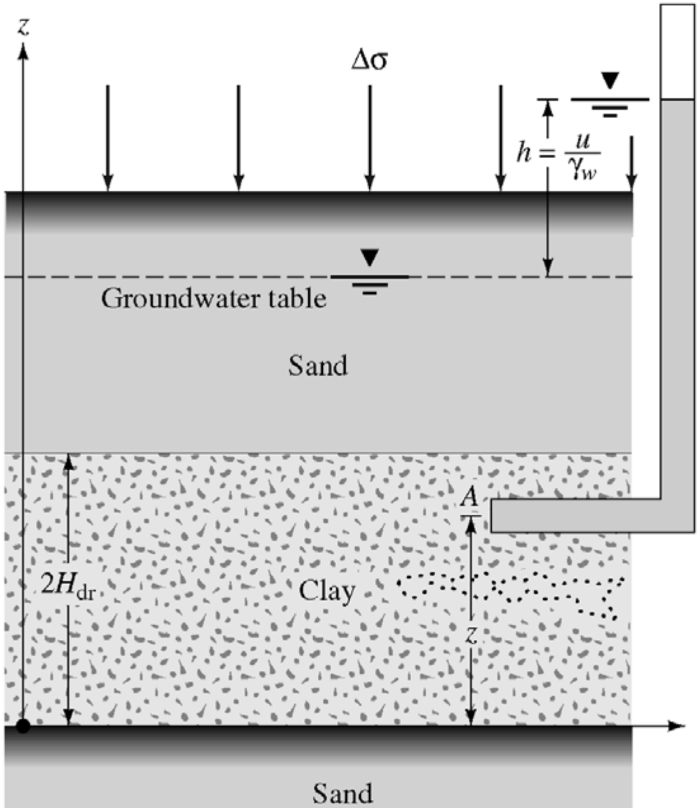
$$z = 2H_{dr}, u = 0$$

$$t = 0, u = u_0$$

The solution yields

$$u = \sum_{m=0}^{m=\infty} \left[\frac{2u_o}{M} \sin\left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$

TIME RATE OF CONSOLIDATION



Clay Layer Undergoing Consolidation
Figure 7.17a. Das FGE (2005).

From Previous Slide

$$u = \sum_{m=0}^{m=\infty} \left[\frac{2u_o}{M} \sin\left(\frac{Mz}{H_{dr}} \right) \right] e^{-M^2 T_v}$$

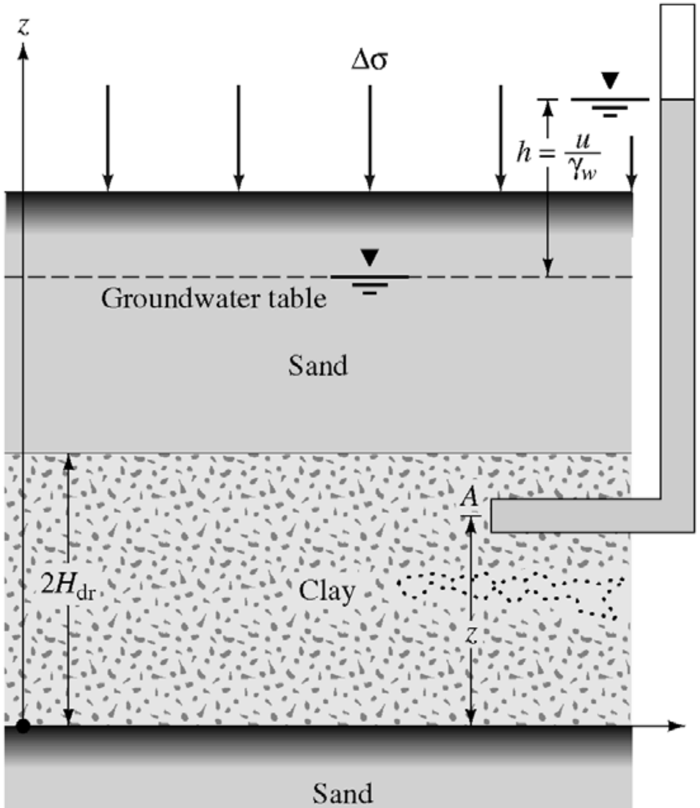
Where:

$$M = \frac{\pi}{2}(2m+1)$$

u_o = Initial excess pore water pressure

$$T_v = \frac{c_v t}{H^2_{dr}} =$$ **TIME FACTOR**

TIME RATE OF CONSOLIDATION



Clay Layer Undergoing Consolidation

Figure 7.17a. Das FGE (2005).

Because consolidation progress by dissipation of excess pore pressure, the degree of consolidation (U_z) at a distance z at any time t is:

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o}$$

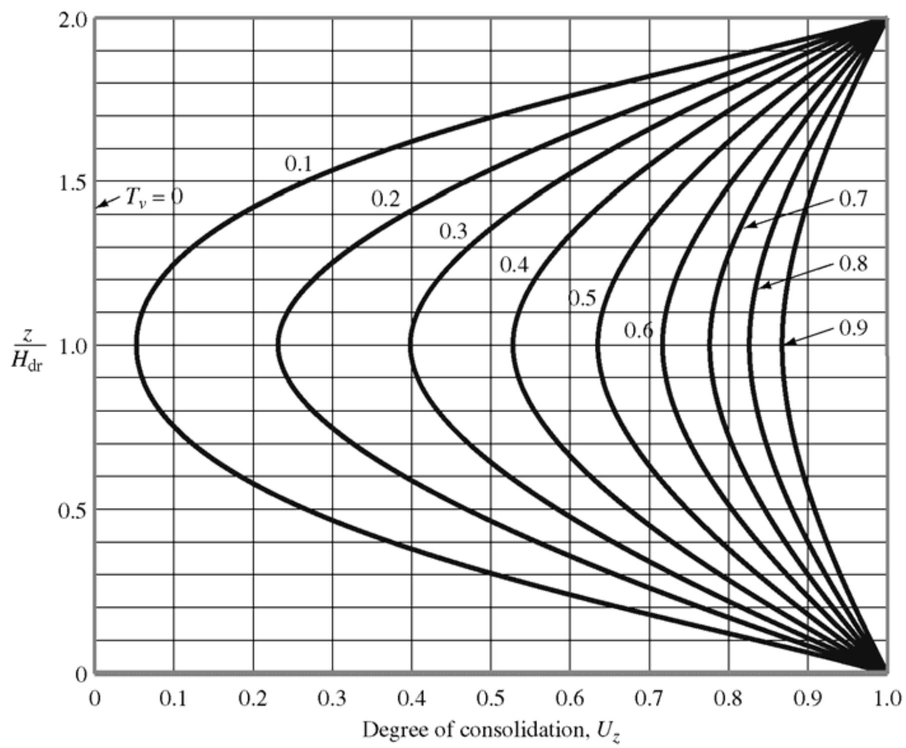


Figure 7.18. Das FGE (2006).

TIME RATE OF CONSOLIDATION

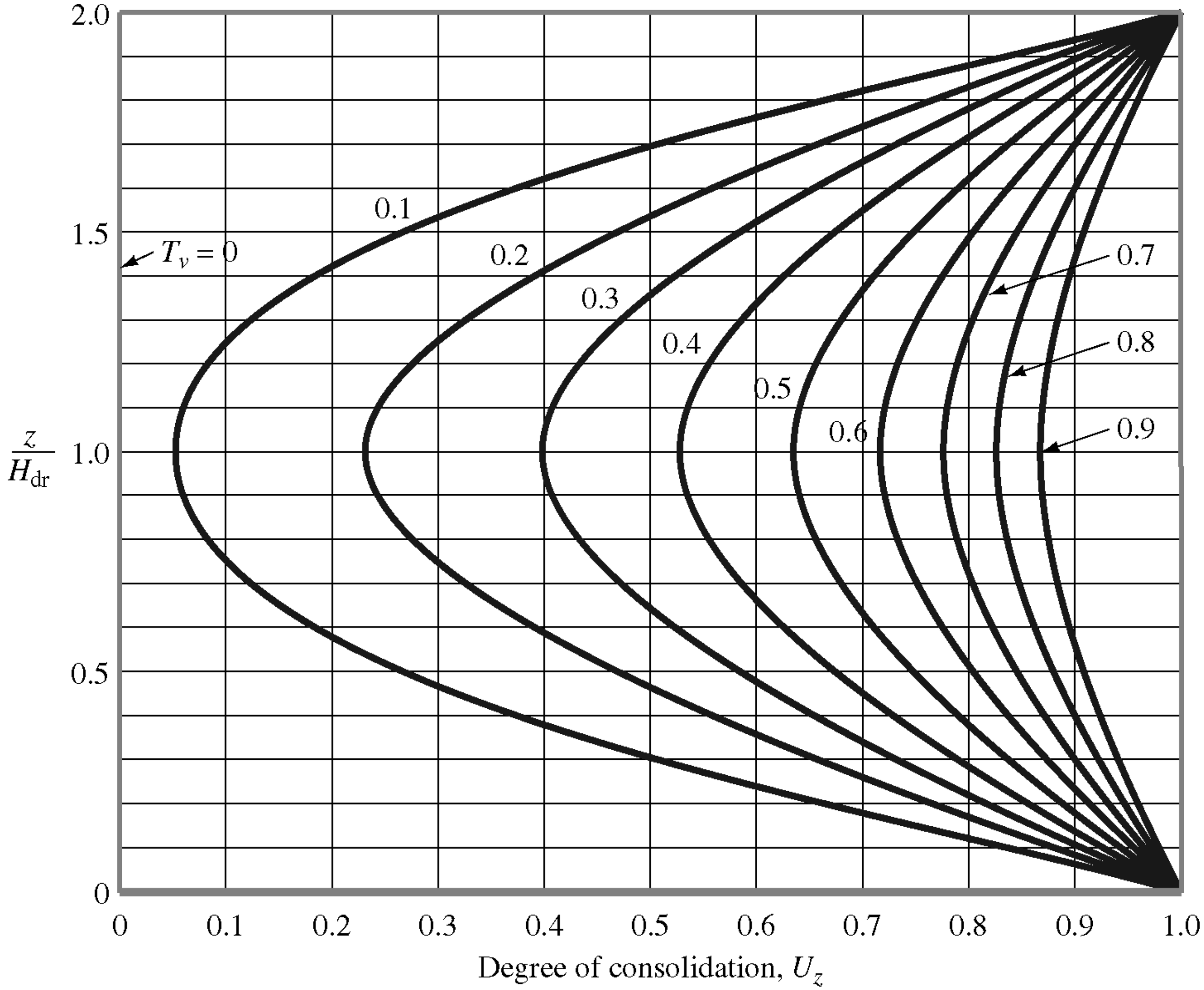
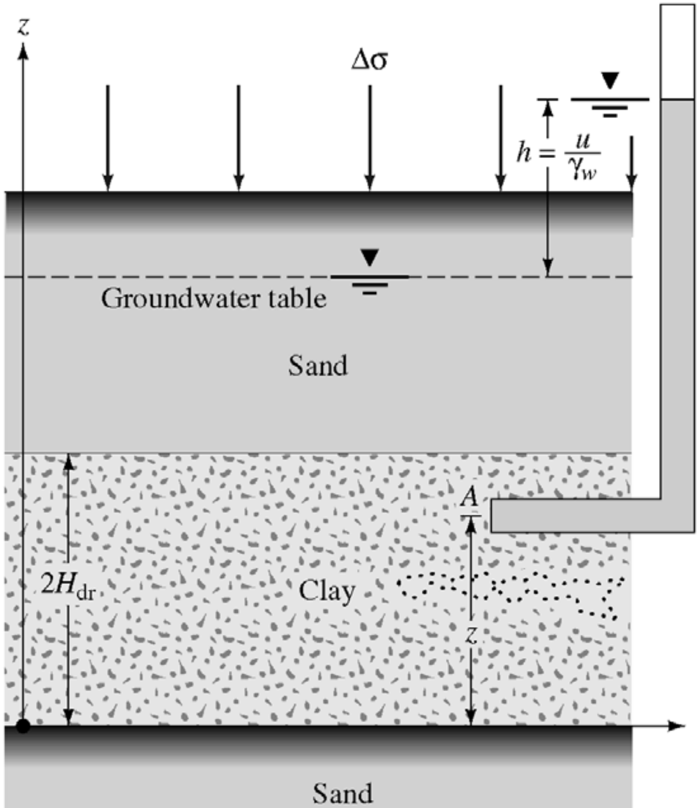


Figure 7.18. Das FGE (2006).

TIME RATE OF CONSOLIDATION



Clay Layer Undergoing Consolidation
Figure 7.17a. Das FGE (2005).

Average degree of consolidation (U) for the entire depth of the clay layer at any time t is:

$$U = \frac{S_t}{S_p} = 1 - \frac{\left(\frac{1}{2H_{dr}}\right)\int_0^{2H_{dr}} u_z dz}{u_o}$$

Where:

U = Average degree of Consolidation

S_t = Settlement of layer at time t

S_p = Settlement of Layer from Primary Consolidation

Substituting *U* for *u*

$$U = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} e^{-M^2 T_v}$$

***U* can be approximated by the following relationships:**

For $U = 0\%$ to 60%, $T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$

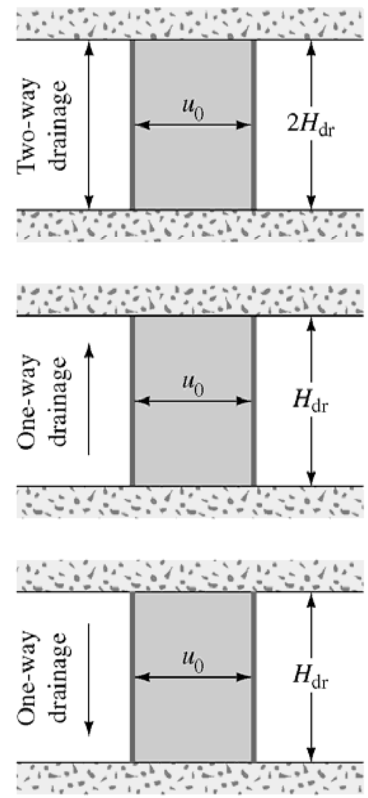
For $U > 60\%$, $T_v = 1.781 - 0.933\log(100 - U\%)$

TIME RATE OF CONSOLIDATION

Variation of T_v with U
Table 7.1 Das PGE (2006).

U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	34	0.0907	68	0.377
1	0.00008	35	0.0962	69	0.390
2	0.0003	36	0.102	70	0.403
3	0.00071	37	0.107	71	0.417
4	0.00126	38	0.113	72	0.431
5	0.00196	39	0.119	73	0.446
6	0.00283	40	0.126	74	0.461
7	0.00385	41	0.132	75	0.477
8	0.00502	42	0.138	76	0.493
9	0.00636	43	0.145	77	0.511
10	0.00785	44	0.152	78	0.529
11	0.0095	45	0.159	79	0.547
12	0.0113	46	0.166	80	0.567
13	0.0133	47	0.173	81	0.588
14	0.0154	48	0.181	82	0.610
15	0.0177	49	0.188	83	0.633
16	0.0201	50	0.197	84	0.658
17	0.0227	51	0.204	85	0.684
18	0.0254	52	0.212	86	0.712
19	0.0283	53	0.221	87	0.742
20	0.0314	54	0.230	88	0.774
21	0.0346	55	0.239	89	0.809
22	0.0380	56	0.248	90	0.848
23	0.0415	57	0.257	91	0.891
24	0.0452	58	0.267	92	0.938
25	0.0491	59	0.276	93	0.993
26	0.0531	60	0.286	94	1.055
27	0.0572	61	0.297	95	1.129
28	0.0615	62	0.307	96	1.219
29	0.0660	63	0.318	97	1.336
30	0.0707	64	0.329	98	1.500
31	0.0754	65	0.304	99	1.781
32	0.0803	66	0.352	100	∞
33	0.0855	67	0.364		

* u_0 constant with depth.



Different types of drainage with u_0 constant

TIME RATE OF CONSOLIDATION

Difference between Average Degree of Consolidation and Midplane Degree of Consolidation

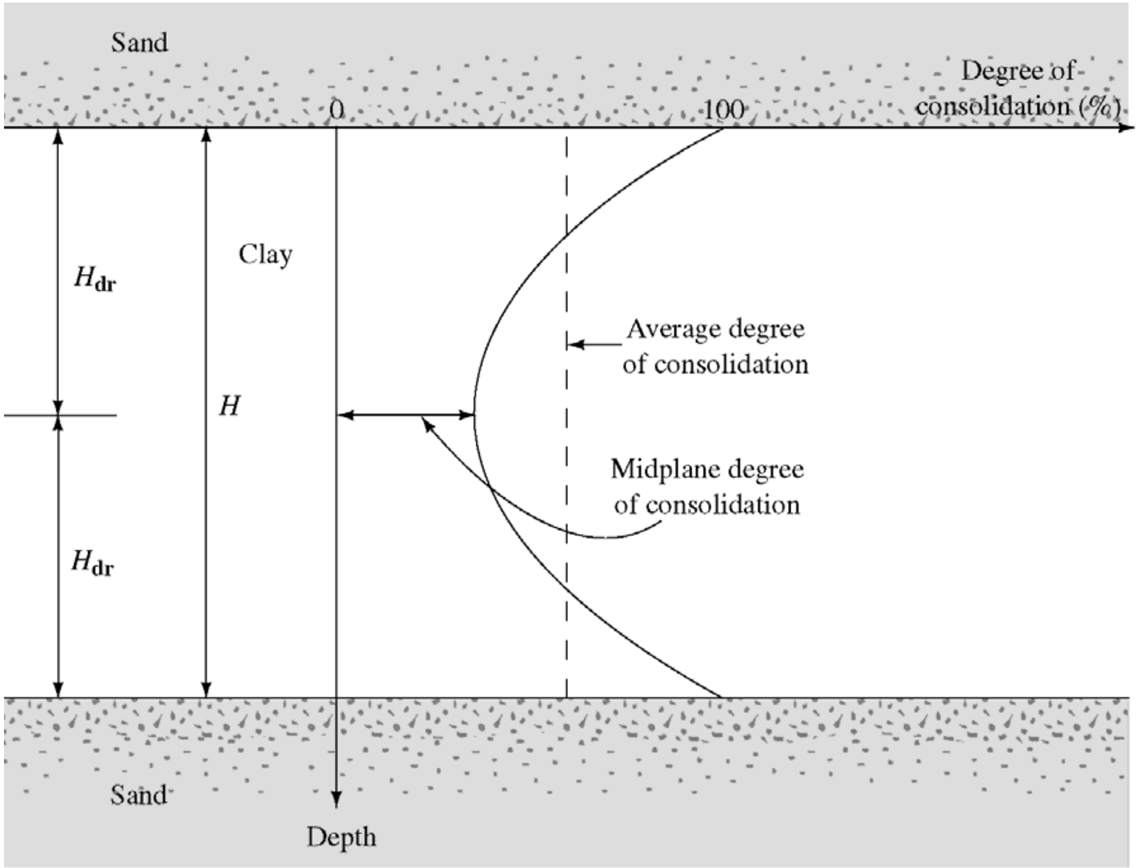
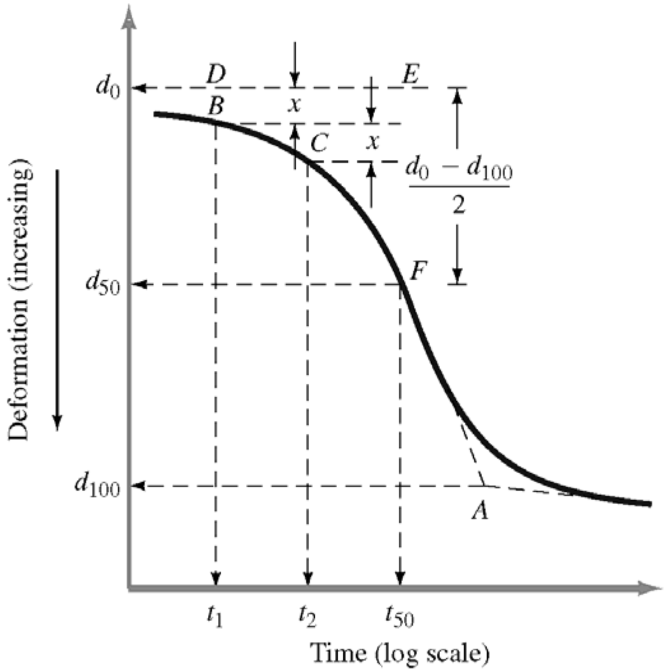


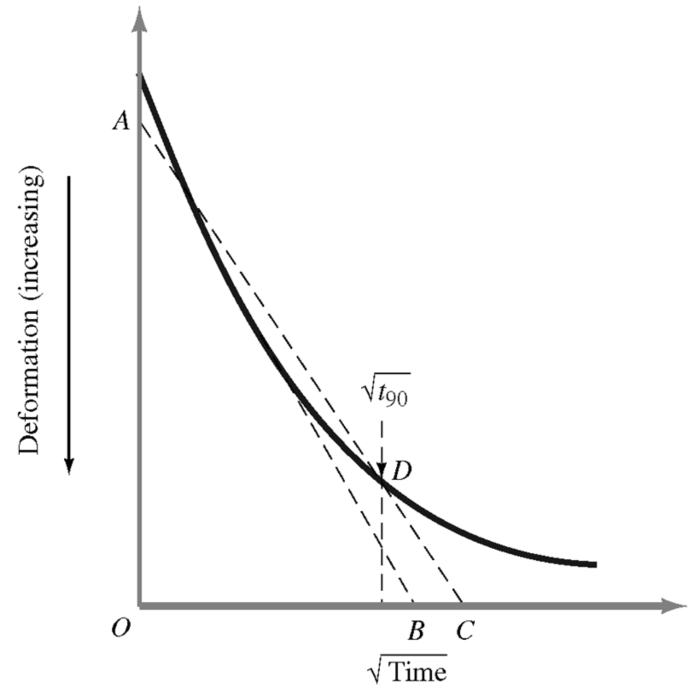
Figure 7.28. Das FGE (2006).

COEFFICIENT OF CONSOLIDATION (c_v)

- **Generally decreases as Liquid Limit (LL) increases.**
- **Determined from 1D Consolidation Test Lab per Load Increment.**



Logarithm of Time Method
(Casagrande and Fadum, 1940)
Figure 7.19 Das FGE (2006).



Square Root of Time Method
(Taylor, 1942)
Figure 7.20 Das FGE (2006).

COEFFICIENT OF CONSOLIDATION (c_v)

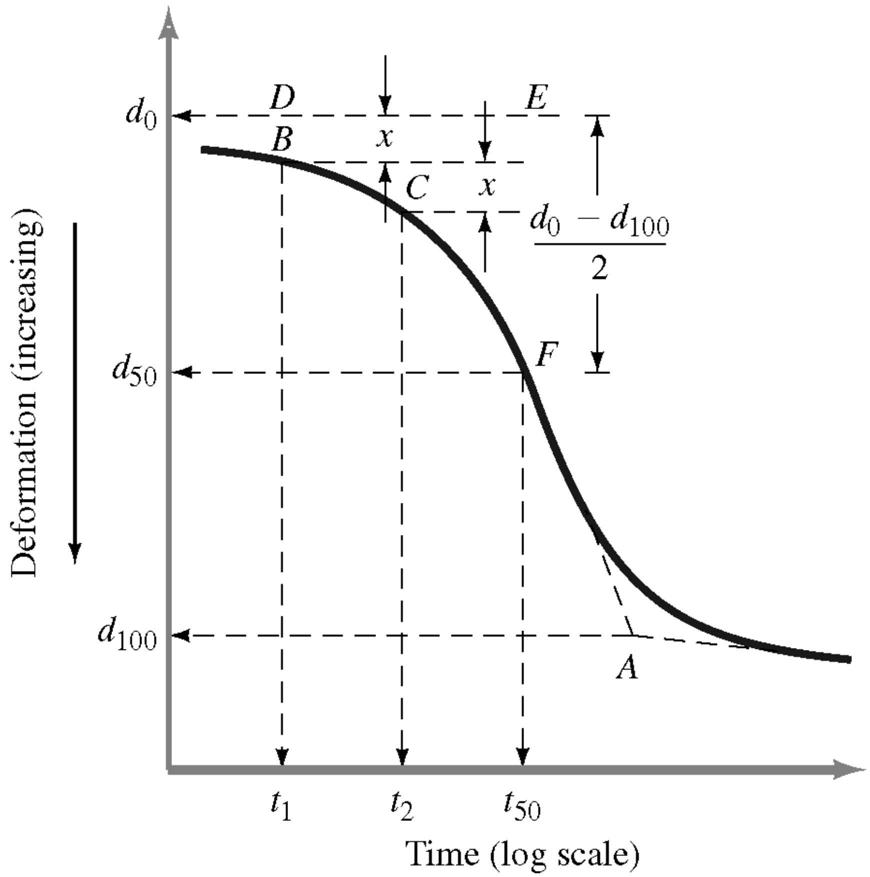


Figure 7.19. Das FGE (2006).

Logarithm of Time Method

1. Extend the straight line portion of primary and secondary consolidations to interest at Point A. Point A represents d_{100} (Deformation at 100% primary consolidation).

2. The initial curved portion of the deformation plot versus log t is approximated to be a parabola on a natural scale. Select times t_1 and t_2 on the curved portion such that $t_2 = 4t_1$. Let the difference of the specimen deformation between $(t_2 - t_1)$ be equal to x.

3. Draw a line horizontal to *DE* such that the vertical distance *BD* is equal to x. The deformation corresponding to the line *DE* is d_0 (Deformation at 0% primary consolidation).

COEFFICIENT OF CONSOLIDATION (c_v)

Logarithm of Time Method

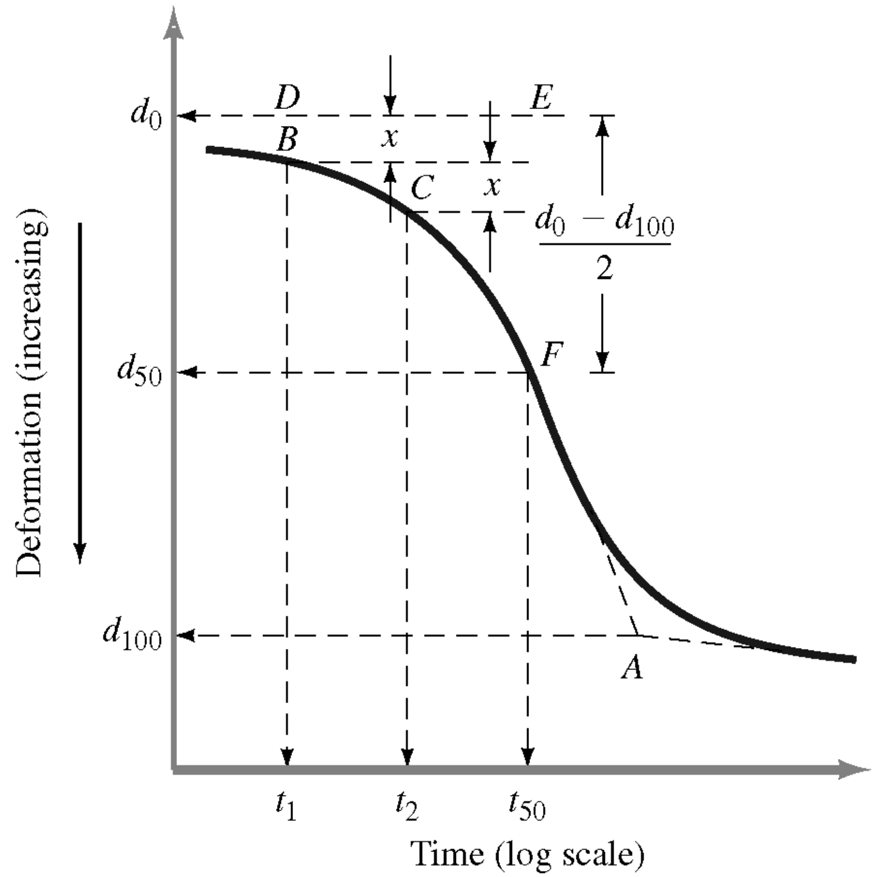


Figure 7.19. Das FGE (2006).

4. The ordinate of Point *F* on the consolidation curve represents the deformation at 50% primary consolidation (d_{50}).

5. For 50% average degree of consolidation (U = 50%), T_v = 0.197 (see Table 7.1, Das FGE 2006).

$$T_{50} = 0.197 = \frac{c_v t_{50}}{H_{dr}^2}$$

or

$$c_v = \frac{0.197 H_{dr}^2}{t_{50}}$$

Where:

H_{dr} = Average longest drain path during consolidation.

COEFFICIENT OF CONSOLIDATION (c_v)

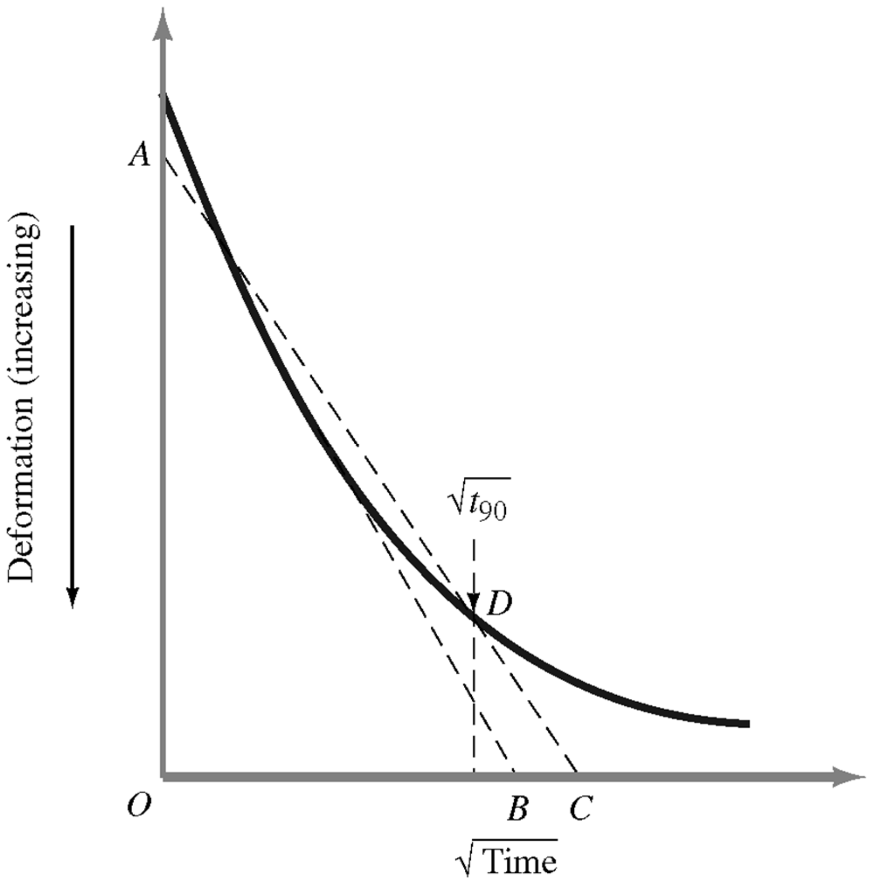


Figure 7.20. Das FGE (2006).

Square Root of Time Method

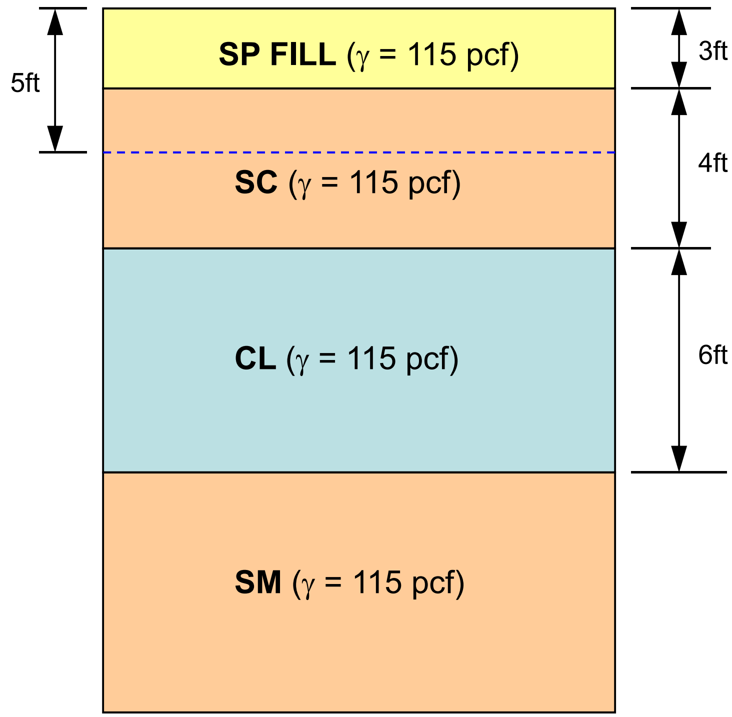
1. Draw a line *AB* trough the early portion of the curve.
2. Draw a line *AC* such that *OC* = 1.15*OB*. The time value for Point *D* (i.e. the intersection of line *AC* and the data) is the square root of time for t_{90} (i.e. the time to 90% primary consolidation).
3. For 90% consolidation, T_v = 0.848 (see Table 7.1, Das FGE 2006).

$$T_{90} = 0.848 = \frac{c_v t_{90}}{H_{dr}^2}$$

or

$$c_v = \frac{0.848 H_{dr}^2}{t_{90}}$$

COEFFICIENT OF CONSOLIDATION (c_v) Example



GIVEN: Soil Profile (NTS).
2 way drainage.

REQUIRED: Determine the following:

a. The change in pore pressure in the CL layer immediately after the application of the 3 ft of SP Fill.

b. The degree of consolidation in the middle of the clay layer when the excess pore pressure (u_e) is 170 psf.

c. How high would the water in a piezometer located in the middle of the layer rise above the GWT when u_e = 170 psf?

d. If c_v = 0.000004 ft²/sec, how long would it take to get to 25% average degree of consolidation? To U = 50%? To U = 99%?

PRECOMPRESSION – GENERAL CONSIDERATIONS

PRECOMPRESSION: Loading an area prior to placement of the planned structural loading to limit post-construction settlement. Also known as **Surcharging.**

Settlement caused by structural loading (S_p):

$$S_p = \frac{C_c H}{1+e_0}\log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

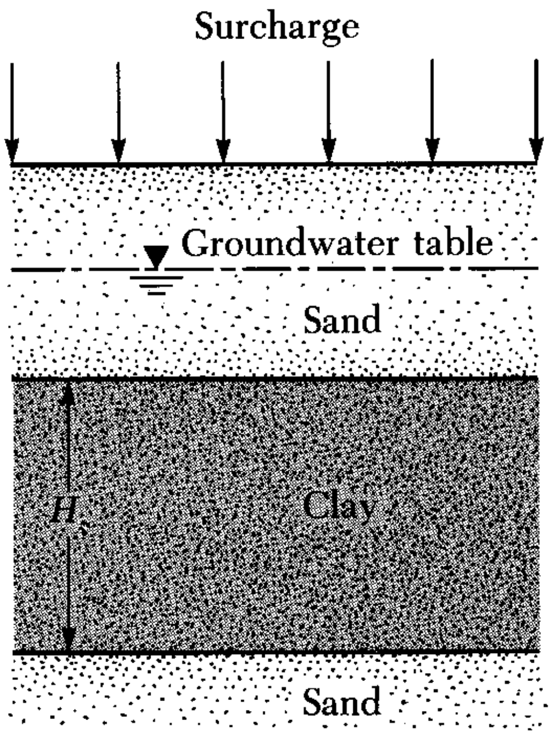
Settlement caused by structural loading and surcharging (S'_p or S_{p+f}):

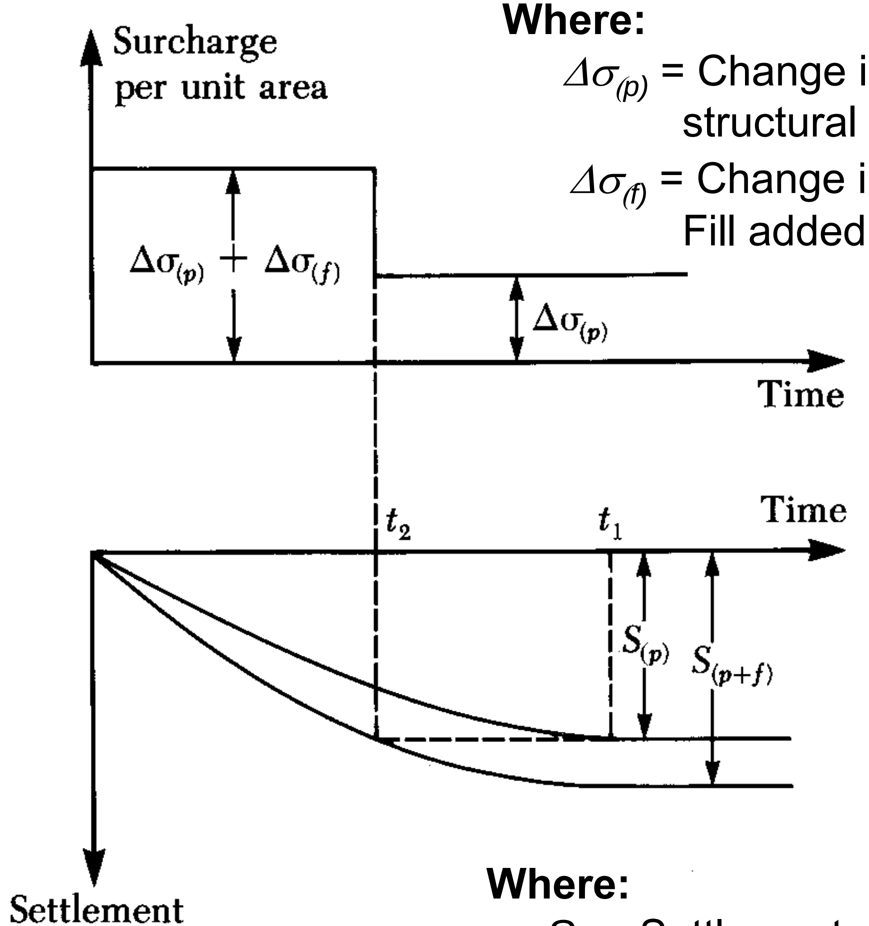
$$S'_p = S_{p+f} = \frac{C_c H}{1+e_0}\log\left(\frac{\sigma'_o + \left[\Delta\sigma' + \Delta\sigma_f\right]}{\sigma'_o}\right)$$

Where:

$\Delta\sigma_f$ = Change in vertical stress due to Fill added.

PRECOMPRESSION – GENERAL CONSIDERATIONS





Where:

$\Delta\sigma_{(p)}$ = Change in vertical stress due to structural load.

$\Delta\sigma_{(f)}$ = Change in vertical stress due to Fill added.

Where:

S_p = Settlement due to structural load.

S_{p+f} = Settlement due to structural load and Fill.

Figure 7.26. Das FGE (2006).

PRECOMPRESSION – PLANNING

Mathematical Equations

$$U = \frac{S_p}{S'_p}$$

Definition of average Degree of Consolidation U

$$U = \frac{\log\left[\dfrac{\sigma'_o + \Delta\sigma'_{(p)}}{\sigma'_o}\right]}{\log\left[\dfrac{\sigma'_o + \Delta\sigma'_{(p)} + \Delta\sigma'_{(f)}}{\sigma'_o}\right]}$$

Substitution

$$U = \frac{\log\left[1 + \dfrac{\Delta\sigma'_{(p)}}{\sigma'_o}\right]}{\log\left\{1 + \dfrac{\Delta\sigma'_{(p)}}{\sigma'_o}\left[1 + \dfrac{\Delta\sigma'_{(f)}}{\Delta\sigma'_{(p)}}\right]\right\}}$$

Re-arranging (Eqn 7.56 Das FGE 2006)

Place in graphical form for design use (Figure 7.27 Das FGE 2006)

PRECOMPRESSION – PLANNING

Where:

$\Delta\sigma_{(f)}$ = Change in vertical stress due to Fill added.

$\Delta\sigma_{(p)}$ = Change in vertical stress due to Structural Loading.

σ'_o = Initial vertical effective stress.

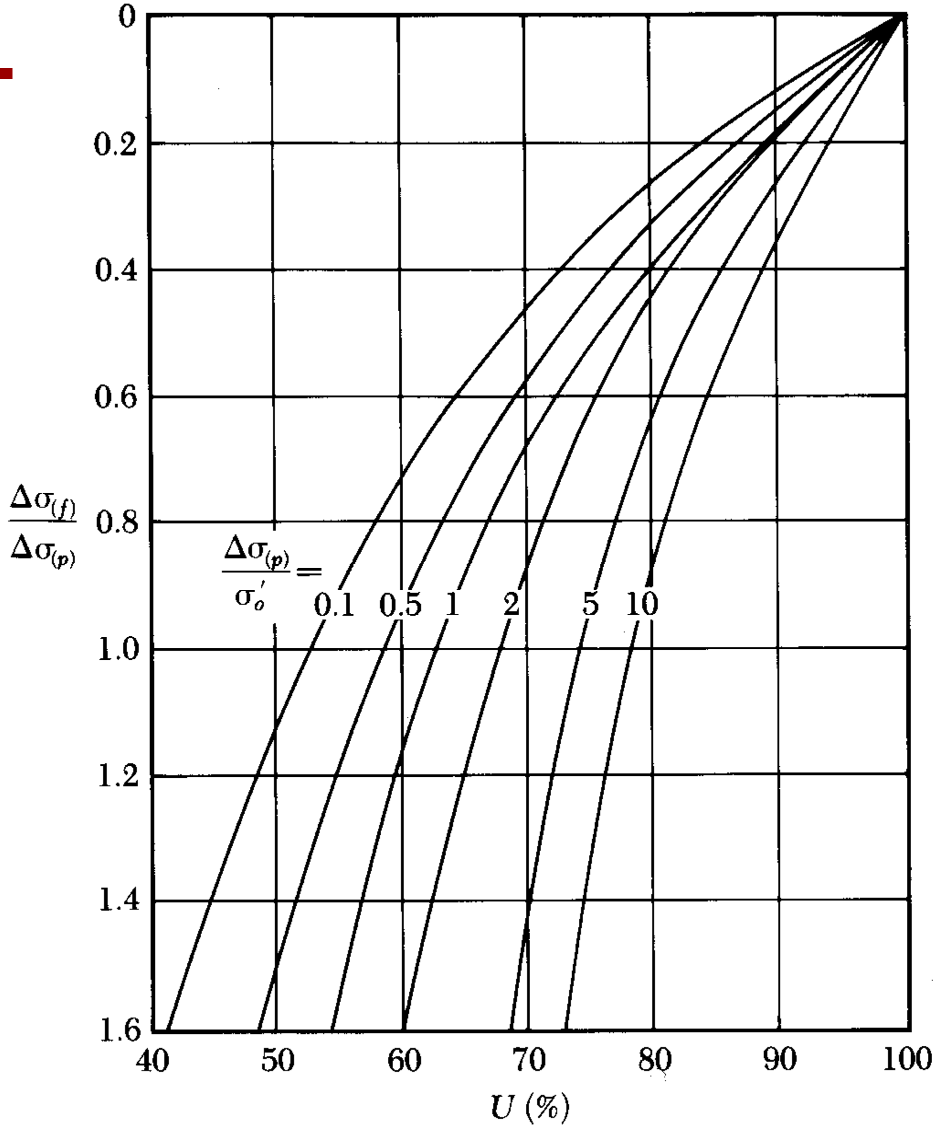


Figure 7.27. Das FGE (2006).

PRECOMPRESSION – PLANNING

STEPS:

1. Calculate primary consolidation settlement from planned loading (S_p).
2. Calculate primary consolidation settlement from planned loading plus surcharge (S_{p+f}).
3. Calculate average degree of consolidation U. Note $U = S_p/S_{p+f}$.
   Can also use Figure 7.27 or Eqn 7.56 (Das FGE 2006).
1. Find T_v from calculated U. To find time to when surcharge loading should be removed (i.e. t_2):

$$t_2 = \frac{T_v H_{dr}^2}{c_v}$$

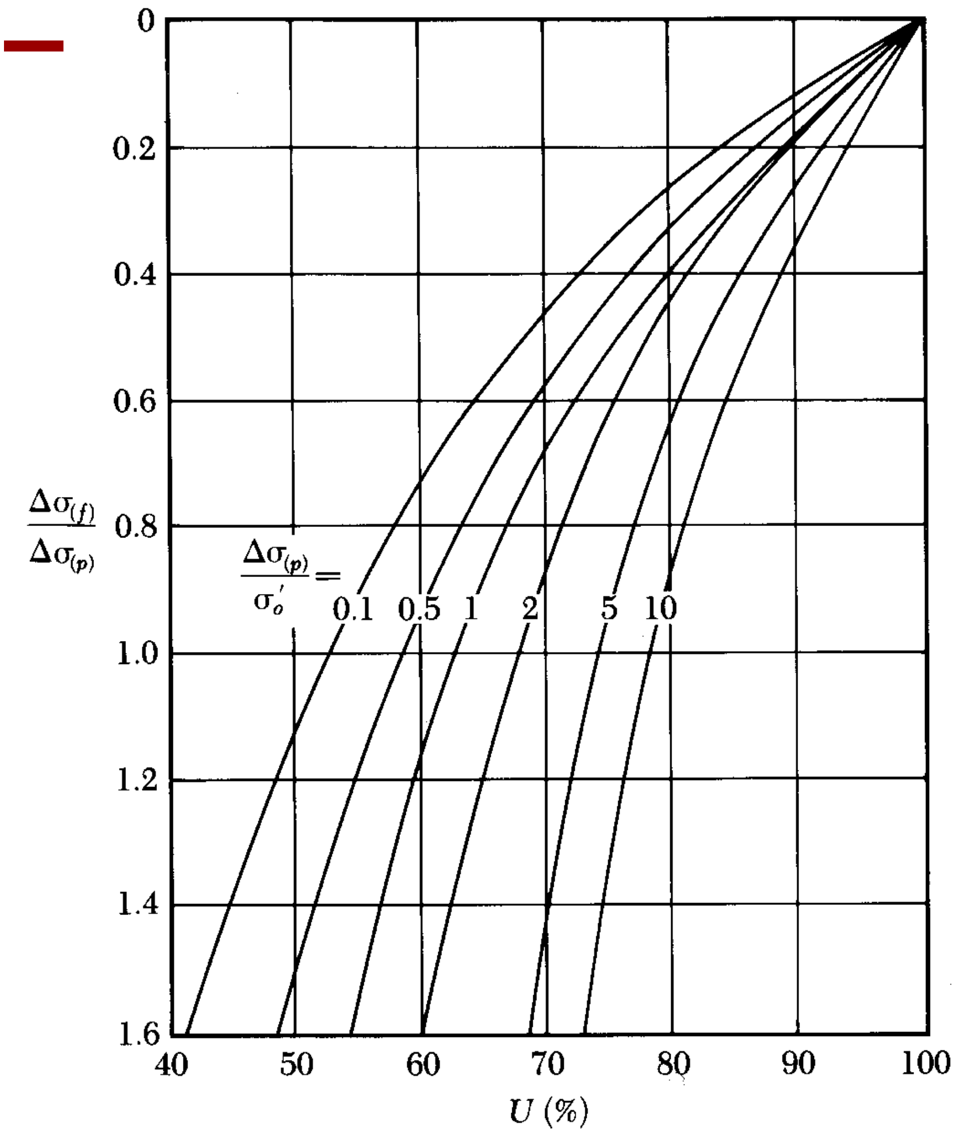


Figure 7.27. Das FGE (2006).

TIME RATE OF CONSOLIDATION

Difference between Average Degree of Consolidation and Midplane Degree of Consolidation

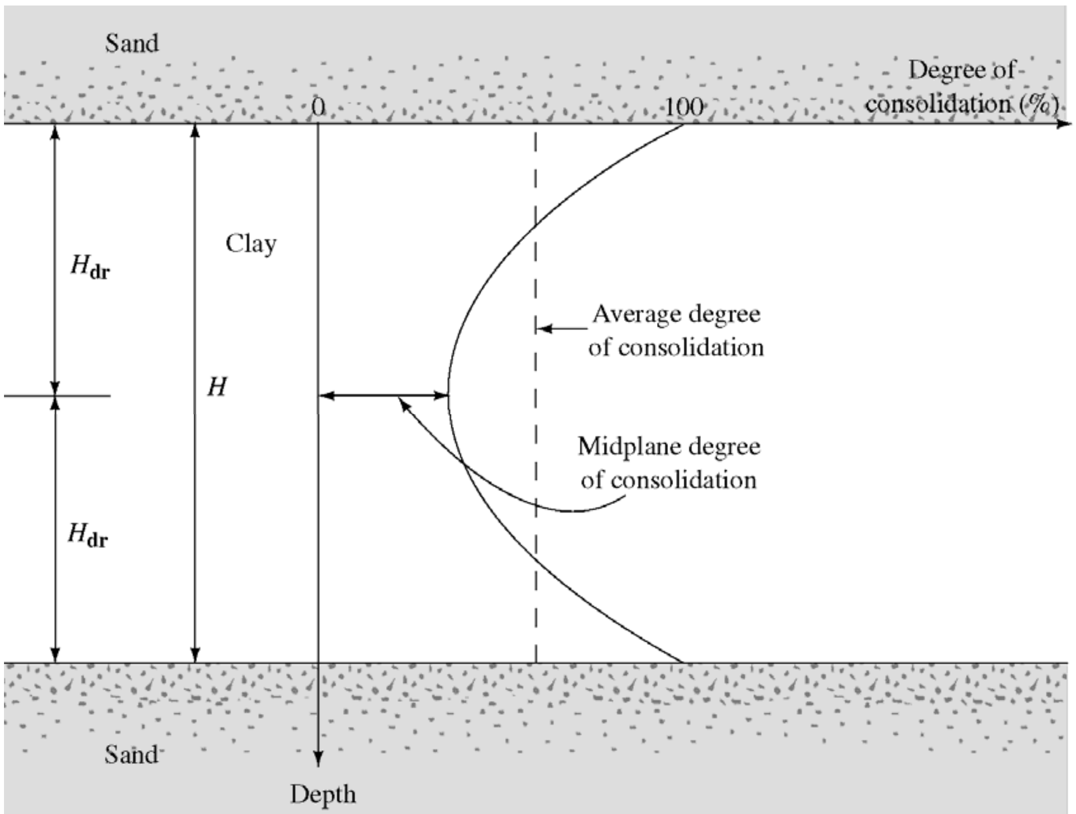


Figure 7.28. Das FGE (2006).

Removal of Surcharge may still cause net settlement
(swelling near drainage layers, settlement @ middle)

Conservative Approach:
Assume U is the midplane degree of consolidation.

TIME RATE OF CONSOLIDATION

Midplane Degree of Consolidation

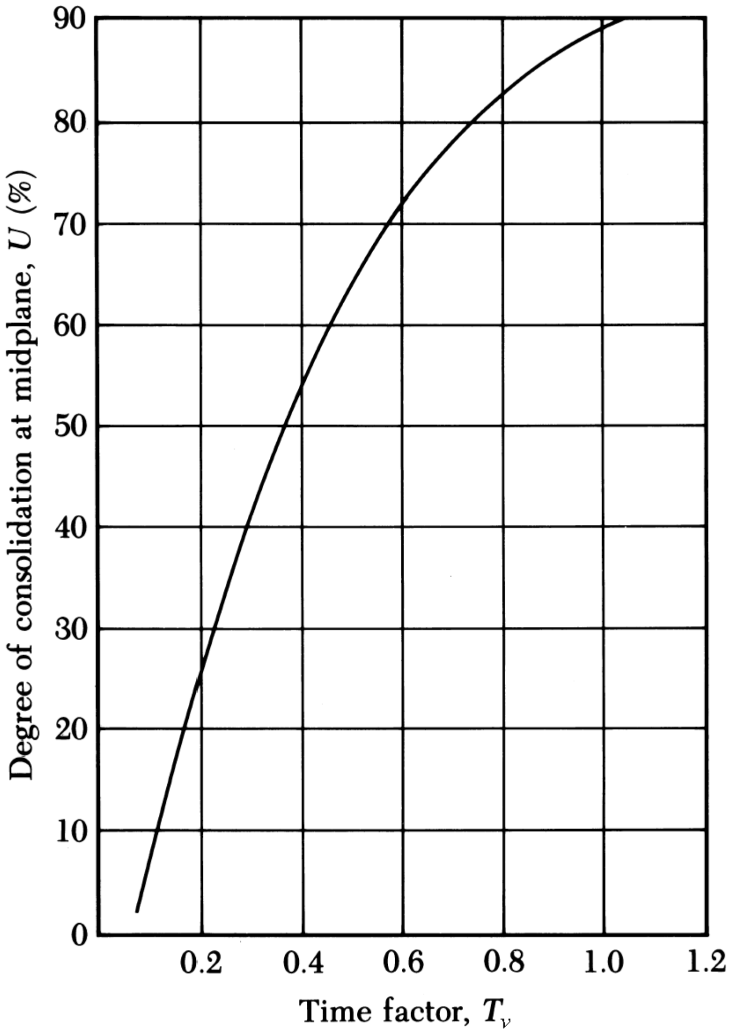
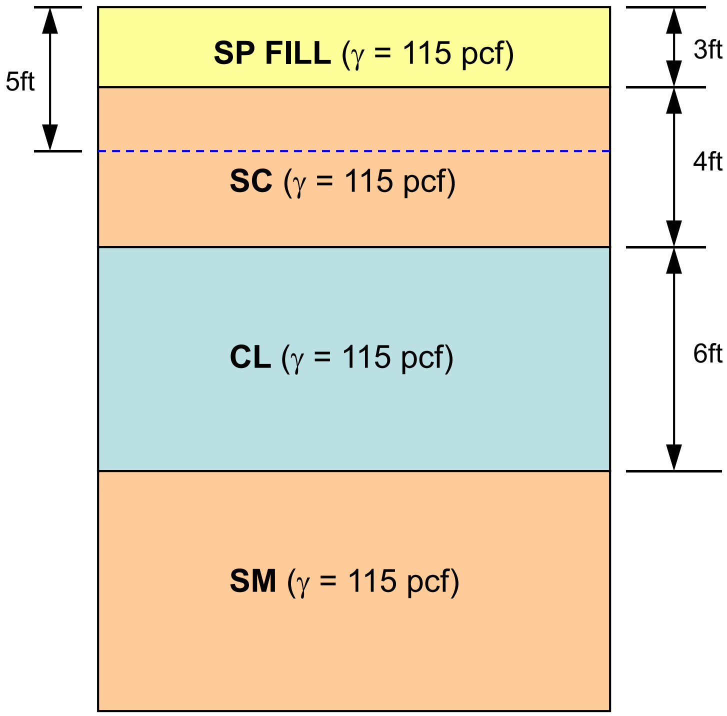


Figure 7.29. Das FGE (2006).

SURCHARGING EXAMPLE



GIVEN: Soil Profile (NTS).
2 way drainage.

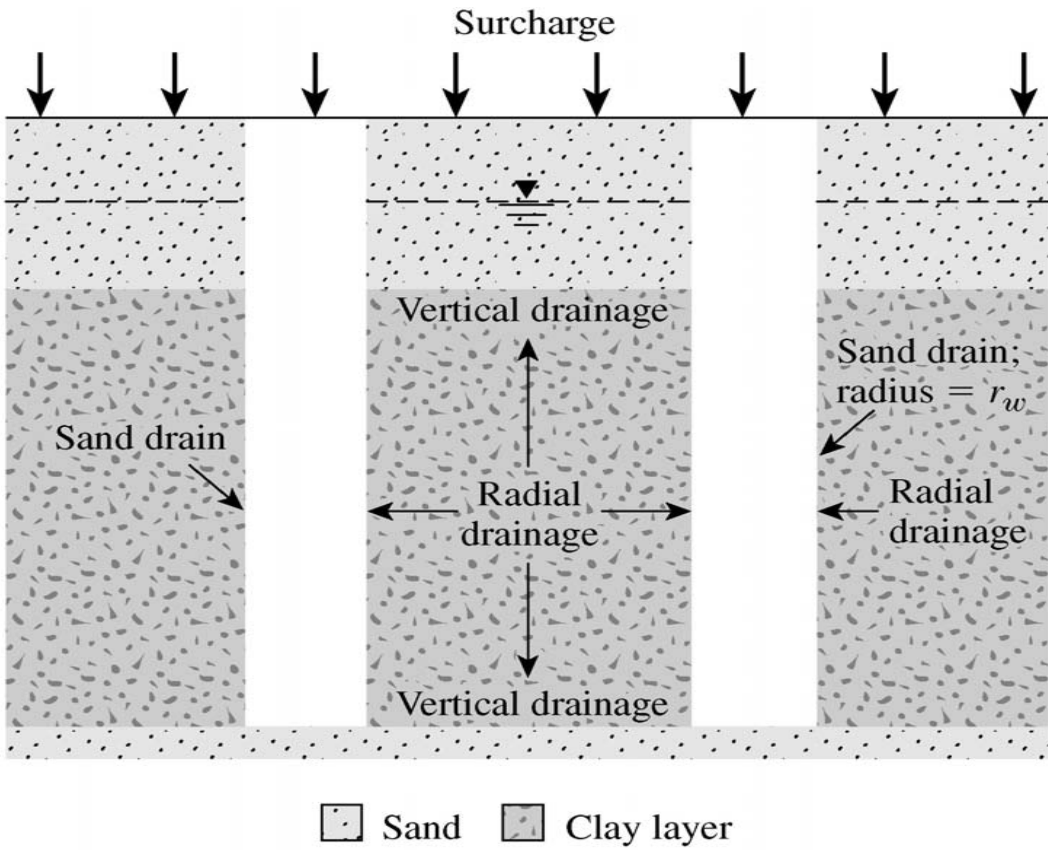
REQUIRED: Determine the following:

a. If c_v = 0.000004 ft²/sec, how long would it take to get to 99% average degree of consolidation?

b. If a surcharge of 4 ft of fill was placed in addition to the 3 ft of fill planned, when would you be able to remove the surcharge? Use the same value for c_v given in a.

GROUND MODIFICATION FOR CONSOLIDATION

SAND DRAINS

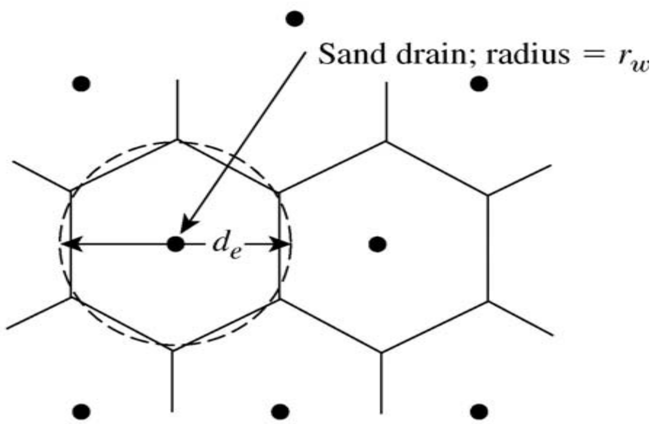


Section View

Figure 10.38. Das PGE (2006).

r_w = Sand Drain Radius

d_e = Effective Diameter



Plan View – Triangular Spacing

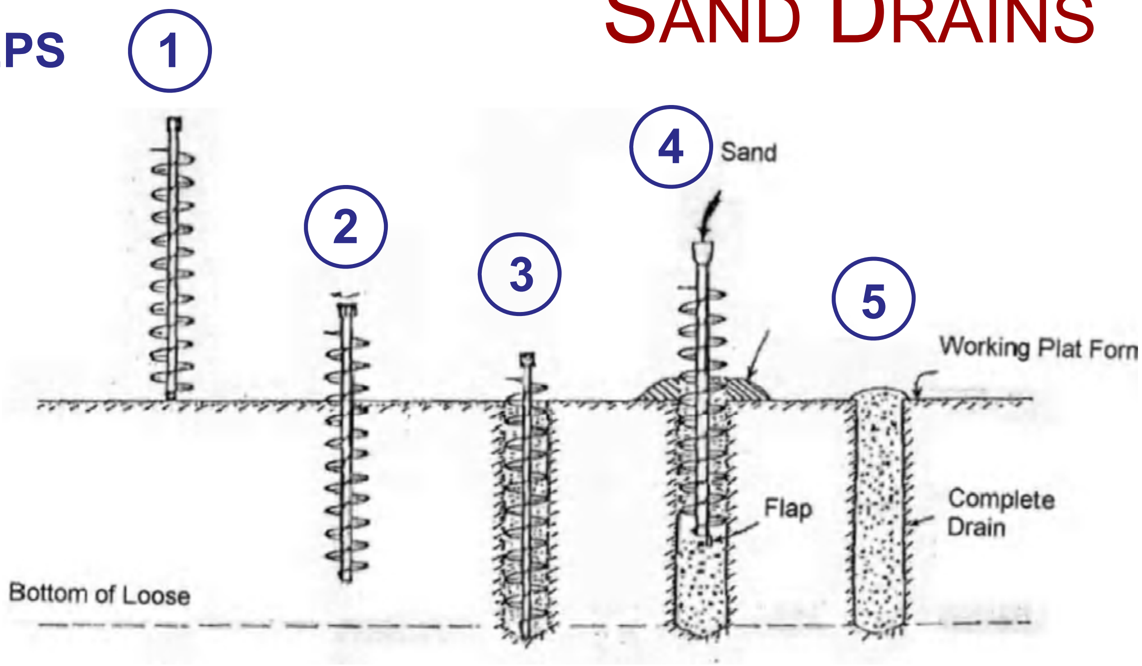
Figure 10.38. Das PGE (2006)

Reduction Drainage Path = Reduction in Drainage Time

GROUND MODIFICATION FOR CONSOLIDATION

SAND DRAINS

STEPS



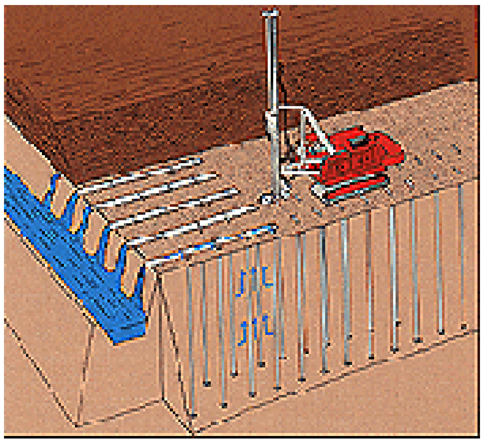
1. Place auger at drain location.
2. Screw auger to selected depth.
3. Rotate auger at selected depth to remove soil.
4. Inject sand while auger is extracted.
5. Complete sand drain to working platform level.

Sand Drain Installation: Auger Method
(Kirmani, 2004)

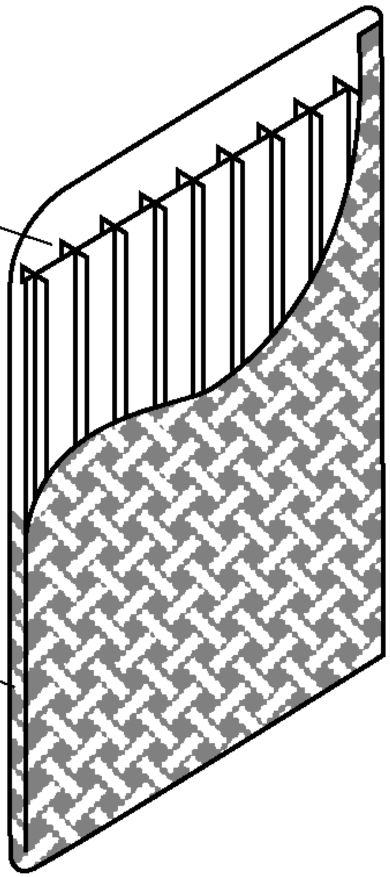
Figure 10.39. Das PGE (2006).

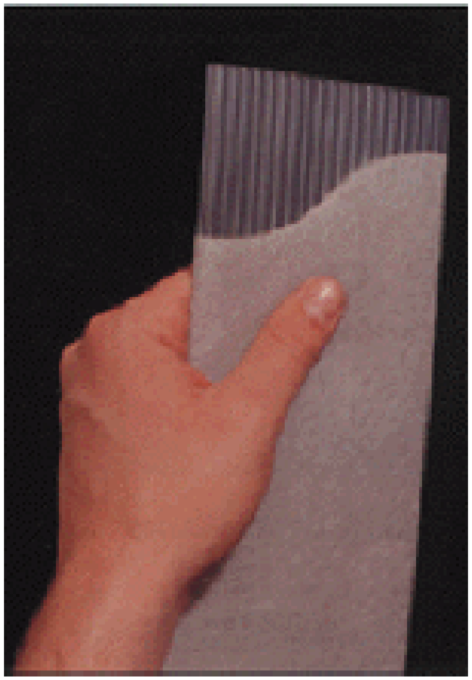
GROUND MODIFICATION FOR CONSOLIDATION

PREFABRICATED VERTICAL DRAINS (PVD'S) (A.K.A. WICK DRAINS)



Conceptual Concept
Courtesy of www.americanwick.com





Courtesy of www.americandrainagesystems.com

Figure 7.31. Das FGE (2006).

GROUND MODIFICATION FOR CONSOLIDATION

PREFABRICATED VERTICAL DRAINS (PVD'S) (A.K.A. WICK DRAINS)

Courtesy of www.americandrainagesystems.com

Courtesy of www.nilex.com

Courtesy of www.nilex.com

GROUND MODIFICATION FOR CONSOLIDATION

RADIAL CONSOLIDATION

U_r = Average Degree of Radial Consolidation

$$U_r = 1 - \exp\left(\frac{-8T_r}{m}\right)$$

Barron (1948)

$$m = \left(\frac{n^2}{n^2 - 1}\right)\ln(n) - \frac{3n^2 - 1}{4n^2}$$

$$n = \frac{d_e}{2r_w}$$

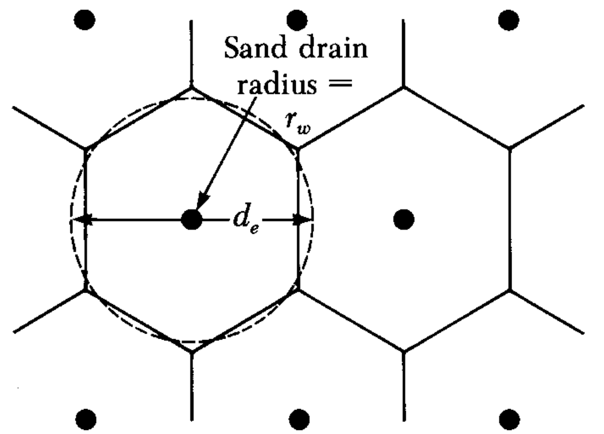
d_e = Effective Diameter
r_w = Sand Drain Radius

$$T_r = \frac{c_{vr} t}{d_e^2}$$

c_{vr} = Coefficient of Radial Consolidation
T_r = Time Factor for Radial Consolidation

$$c_{vr} = \frac{k_h}{\left[\frac{\Delta e}{\Delta\sigma'(1 + e_o)}\right]\gamma_w}$$

k_h = Coefficient of Horizontal Permeability
T_r = Time Factor for Radial Consolidation
e_o = Initial Void Ratio



Plan View – Sand Drain Triangular Spacing
Figure 7.30. Das FGE (2006).

TIME RATE OF RADIAL CONSOLIDATION

Variation of T_r with U - Table 7.3 Das PGE (2006).

Degree of consolidation, U_r (%)	Time factor, T_r, for values of n				
	5	10	15	20	25
0	0	0	0	0	0
1	0.0012	0.0020	0.0025	0.0028	0.0031
2	0.0024	0.0040	0.0050	0.0057	0.0063
3	0.0036	0.0060	0.0075	0.0086	0.0094
4	0.0048	0.0081	0.0101	0.0115	0.0126
5	0.0060	0.0101	0.0126	0.0145	0.0159
6	0.0072	0.0122	0.0153	0.0174	0.0191
7	0.0085	0.0143	0.0179	0.0205	0.0225
8	0.0098	0.0165	0.0206	0.0235	0.0258
9	0.0110	0.0186	0.0232	0.0266	0.0292
10	0.0123	0.0208	0.0260	0.0297	0.0326
11	0.0136	0.0230	0.0287	0.0328	0.0360
12	0.0150	0.0252	0.0315	0.0360	0.0395
13	0.0163	0.0275	0.0343	0.0392	0.0431
14	0.0177	0.0298	0.0372	0.0425	0.0467
15	0.0190	0.0321	0.0401	0.0458	0.0503
16	0.0204	0.0344	0.0430	0.0491	0.0539
17	0.0218	0.0368	0.0459	0.0525	0.0576
18	0.0232	0.0392	0.0489	0.0559	0.0614
19	0.0247	0.0416	0.0519	0.0594	0.0652
20	0.0261	0.0440	0.0550	0.0629	0.0690
21	0.0276	0.0465	0.0581	0.0664	0.0729
22	0.0291	0.0490	0.0612	0.0700	0.0769
23	0.0306	0.0516	0.0644	0.0736	0.0808
24	0.0321	0.0541	0.0676	0.0773	0.0849
25	0.0337	0.0568	0.0709	0.0811	0.0890
26	0.0353	0.0594	0.0742	0.0848	0.0931
27	0.0368	0.0621	0.0776	0.0887	0.0973
28	0.0385	0.0648	0.0810	0.0926	0.1016
29	0.0401	0.0676	0.0844	0.0965	0.1059
30	0.0418	0.0704	0.0879	0.1005	0.1103
31	0.0434	0.0732	0.0914	0.1045	0.1148
32	0.0452	0.0761	0.0950	0.1087	0.1193
33	0.0469	0.0790	0.0987	0.1128	0.1239
34	0.0486	0.0820	0.1024	0.1171	0.1285
35	0.0504	0.0850	0.1062	0.1214	0.1332
36	0.0522	0.0881	0.1100	0.1257	0.1380
37	0.0541	0.0912	0.1139	0.1302	0.1429
38	0.0560	0.0943	0.1178	0.1347	0.1479
39	0.0579	0.0975	0.1218	0.1393	0.1529
40	0.0598	0.1008	0.1259	0.1439	0.1580
41	0.0618	0.1041	0.1300	0.1487	0.1632
42	0.0638	0.1075	0.1342	0.1535	0.1685
43	0.0658	0.1109	0.1385	0.1584	0.1739
44	0.0679	0.1144	0.1429	0.1634	0.1793
45	0.0700	0.1180	0.1473	0.1684	0.1849
46	0.0721	0.1216	0.1518	0.1736	0.1906
47	0.0743	0.1253	0.1564	0.1789	0.1964
48	0.0766	0.1290	0.1611	0.1842	0.2023
49	0.0788	0.1329	0.1659	0.1897	0.2083
50	0.0811	0.1368	0.1708	0.1953	0.2144
51	0.0835	0.1407	0.1758	0.2020	0.2206
52	0.0859	0.1448	0.1809	0.2068	0.2270
53	0.0884	0.1490	0.1860	0.2127	0.2335
54	0.0909	0.1532	0.1913	0.2188	0.2402
55	0.0935	0.1575	0.1968	0.2250	0.2470
56	0.0961	0.1620	0.2023	0.2313	0.2539
57	0.0988	0.1665	0.2080	0.2378	0.2610
58	0.1016	0.1712	0.2138	0.2444	0.2683
59	0.1044	0.1759	0.2197	0.2512	0.2758
60	0.1073	0.1808	0.2258	0.2582	0.2834
61	0.1102	0.1858	0.2320	0.2653	0.2912
62	0.1133	0.1909	0.2384	0.2726	0.2993
63	0.1164	0.1962	0.2450	0.2801	0.3075
64	0.1196	0.2016	0.2517	0.2878	0.3160
65	0.1229	0.2071	0.2587	0.2958	0.3247
66	0.1263	0.2128	0.2658	0.3039	0.3337
67	0.1298	0.2187	0.2732	0.3124	0.3429
68	0.1334	0.2248	0.2808	0.3210	0.3524
69	0.1371	0.2311	0.2886	0.3300	0.3623
70	0.1409	0.2375	0.2967	0.3392	0.3724
71	0.1449	0.2442	0.3050	0.3488	0.3829
72	0.1490	0.2512	0.3134	0.3586	0.3937
73	0.1533	0.2583	0.3226	0.3689	0.4050
74	0.1577	0.2658	0.3319	0.3795	0.4167
75	0.1623	0.2735	0.3416	0.3906	0.4288
76	0.1671	0.2816	0.3517	0.4021	0.4414
77	0.1720	0.2900	0.3621	0.4141	0.4546
78	0.1773	0.2988	0.3731	0.4266	0.4683
79	0.1827	0.3079	0.3846	0.4397	0.4827
80	0.1884	0.3175	0.3966	0.4534	0.4978
81	0.1944	0.3277	0.4090	0.4679	0.5137
82	0.2007	0.3383	0.4225	0.4831	0.5304
83	0.2074	0.3496	0.4366	0.4922	0.5481
84	0.2146	0.3616	0.4516	0.5163	0.5668

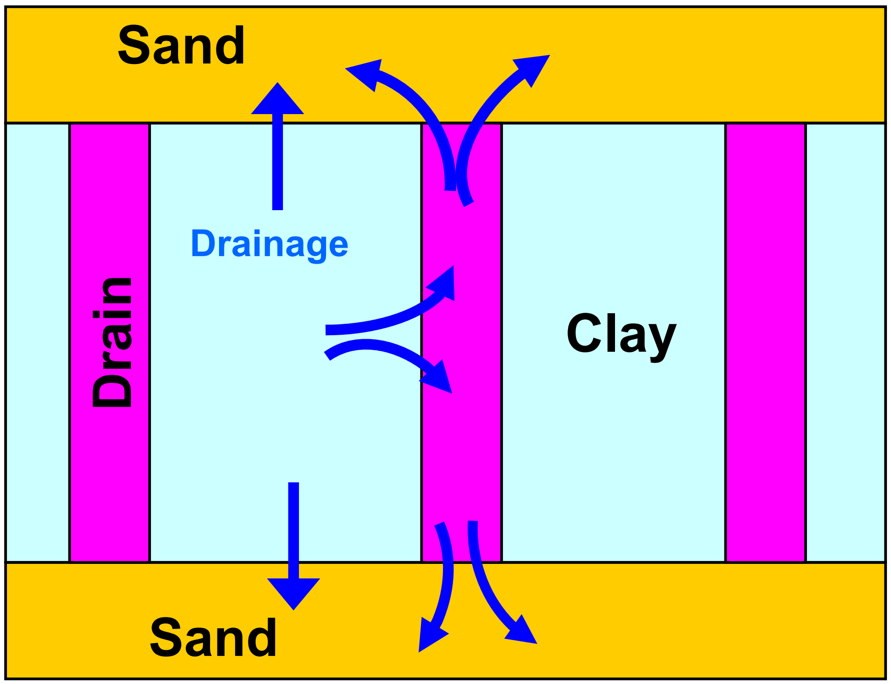
TIME RATE OF RADIAL CONSOLIDATION

Variation of T_r with U - Table 7.3 Das PGE (2006).

Degree of consolidation, U_r (%)	Time factor, T_r, for values of n				
	5	10	15	20	25
85	0.2221	0.3743	0.4675	0.5345	0.5868
86	0.2302	0.3879	0.4845	0.5539	0.6081
87	0.2388	0.4025	0.5027	0.5748	0.6311
88	0.2482	0.4183	0.5225	0.5974	0.6558
89	0.2584	0.4355	0.5439	0.6219	0.6827
90	0.2696	0.4543	0.5674	0.6487	0.7122
91	0.2819	0.4751	0.5933	0.6784	0.7448
92	0.2957	0.4983	0.6224	0.7116	0.7812
93	0.3113	0.5247	0.6553	0.7492	0.8225
94	0.3293	0.5551	0.6932	0.7927	0.8702
95	0.3507	0.5910	0.7382	0.8440	0.9266
96	0.3768	0.6351	0.7932	0.9069	0.9956
97	0.4105	0.6918	0.8640	0.9879	1.0846
98	0.4580	0.7718	0.9640	1.1022	1.2100
99	0.5391	0.9086	1.1347	1.2974	1.4244

GROUND MODIFICATION FOR CONSOLIDATION

AVERAGE DEGREE OF CONSOLIDATION DUE TO VERTICAL & RADIAL DRAINAGE



Vertical and Radial Drainage
Courtesy of www.nhi.fhwa.dot.gov

$$U_{v,r} = 1 - (1 - U_r)(1 - U_v)$$

Where:

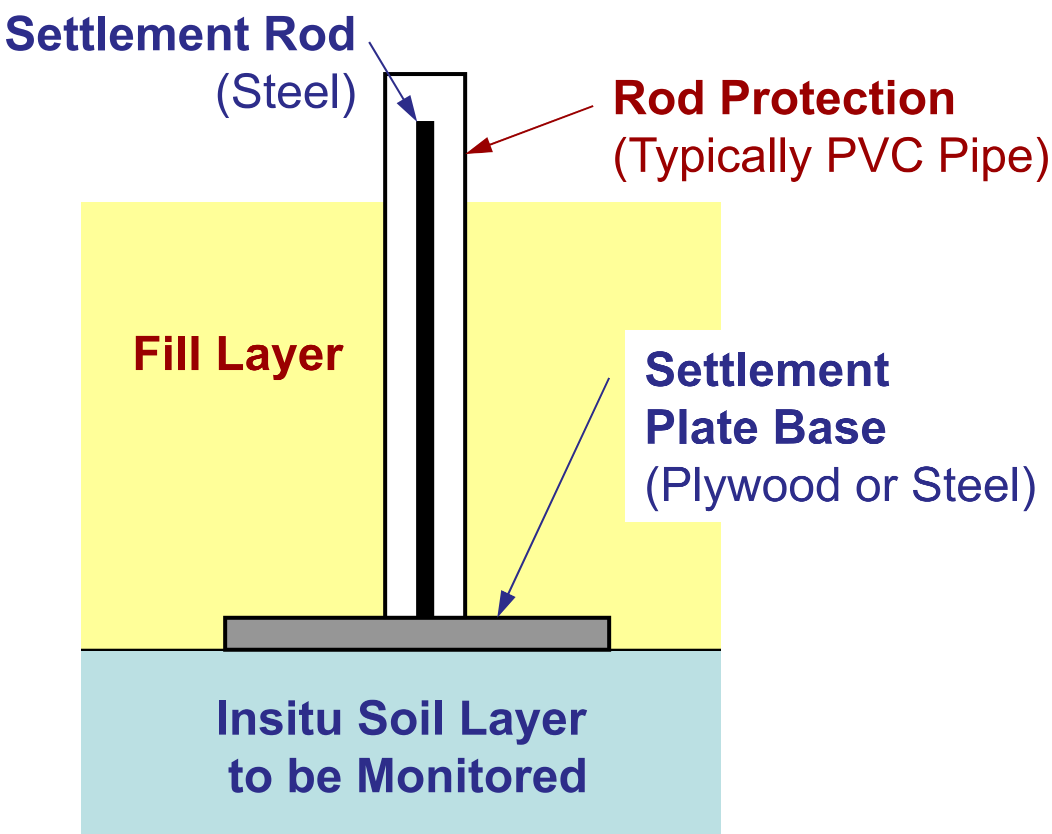
$U_{v,r}$ = Average Degree of Consolidation due to Vertical & Radial Drainage

U_v = Average Degree of Consolidation due to Vertical Drainage

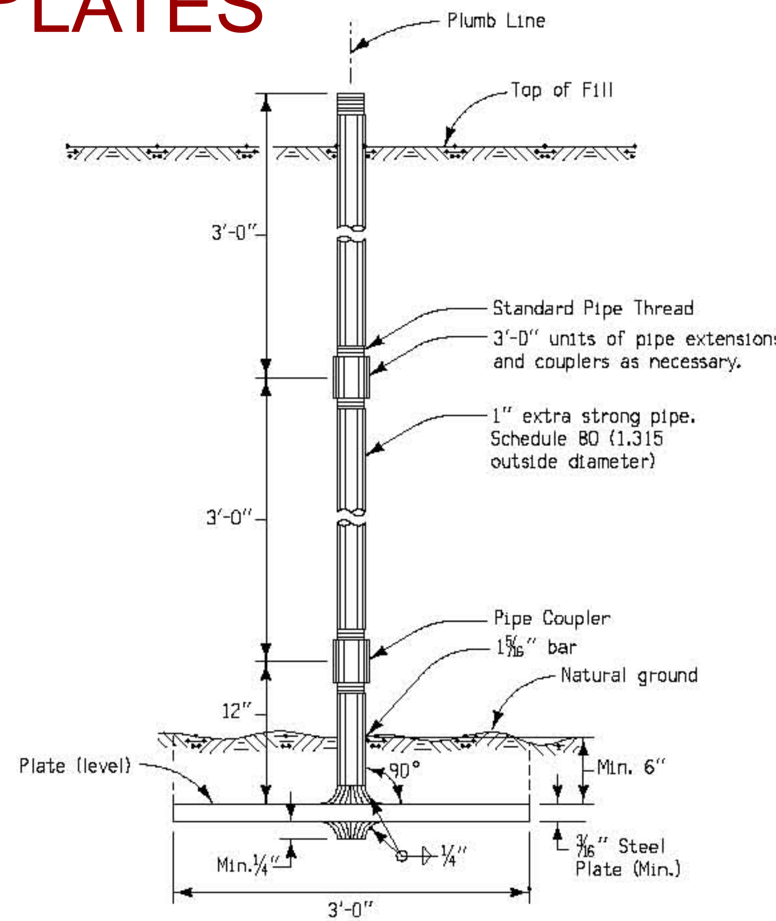
U_r = Average Degree of Consolidation due to Radial Drainage

Consolidation Monitoring

Settlement Plates



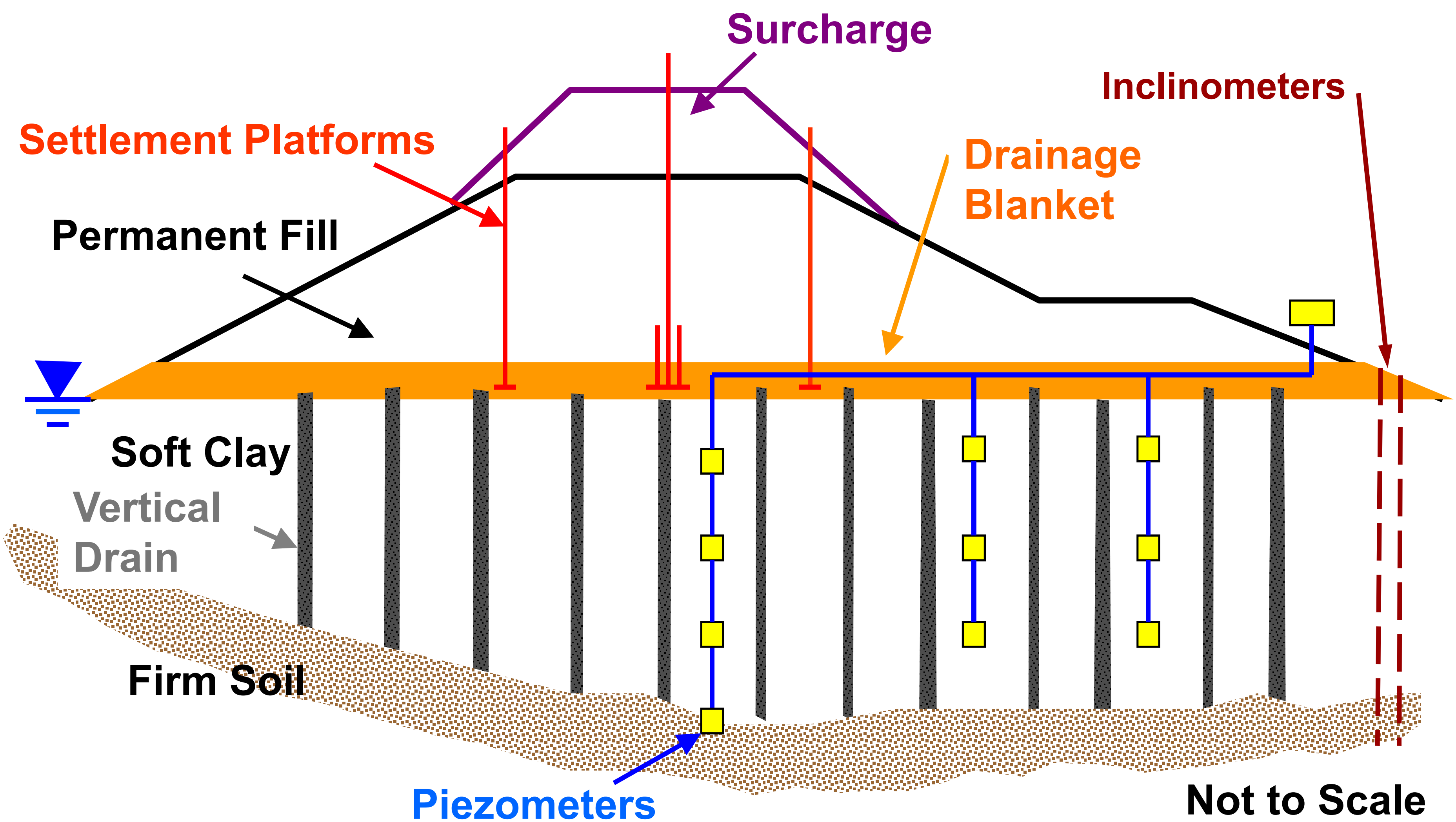
General Concept



Standard Plan Detail

(Courtesy of Iowa DOT)

SURCHARGING INSTRUMENTATION EXAMPLE



Courtesy of www.nhi.fhwa.dot.gov