Settlements of Shallow Foundations

- ***“The statistical accumulation of movements in the direction of interest is the settlement”.***
- **Foundation settlements must be estimated with great care**
  - for buildings, bridges, towers, power plants, and similar high-cost structures.
- **Greater margin of error in the settlements can usually be tolerated**
  - For structures such as fills, earth dams, levees, braced sheeting, and retaining walls
- **Two major problems with soil settlement analyses**
  - *Obtaining reliable values of the "elastic" parameters.*
  - *Obtaining a reliable stress profile from the applied load*

Settlement

- **Elastic or Immediate settlements**
  - Take place as the load applied or within 7 days
  - used for all fine-grained soils including silts and clays with a degree of saturation $S \leq 90\ \%$ and for all coarse-grained soils with a large coefficient of permeability
- **Consolidation Settlement**
  - Time dependent, take months to years to develop
  - Leaning tower of Pisa-700 years
  - for all saturated, or nearly saturated, fine grained
  - soils where the consolidation theory applies
  - Need to evaluate both settlement and time required

$$S_t = S_e + S_c + S_s$$

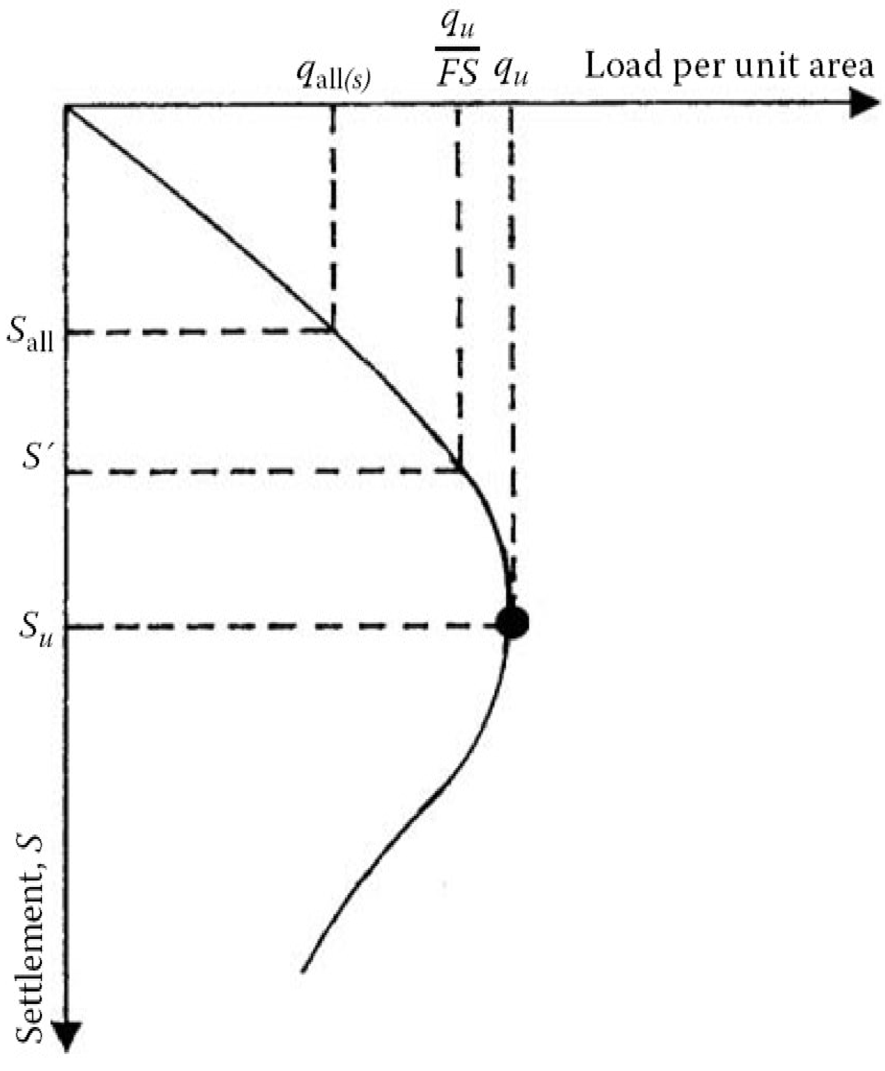
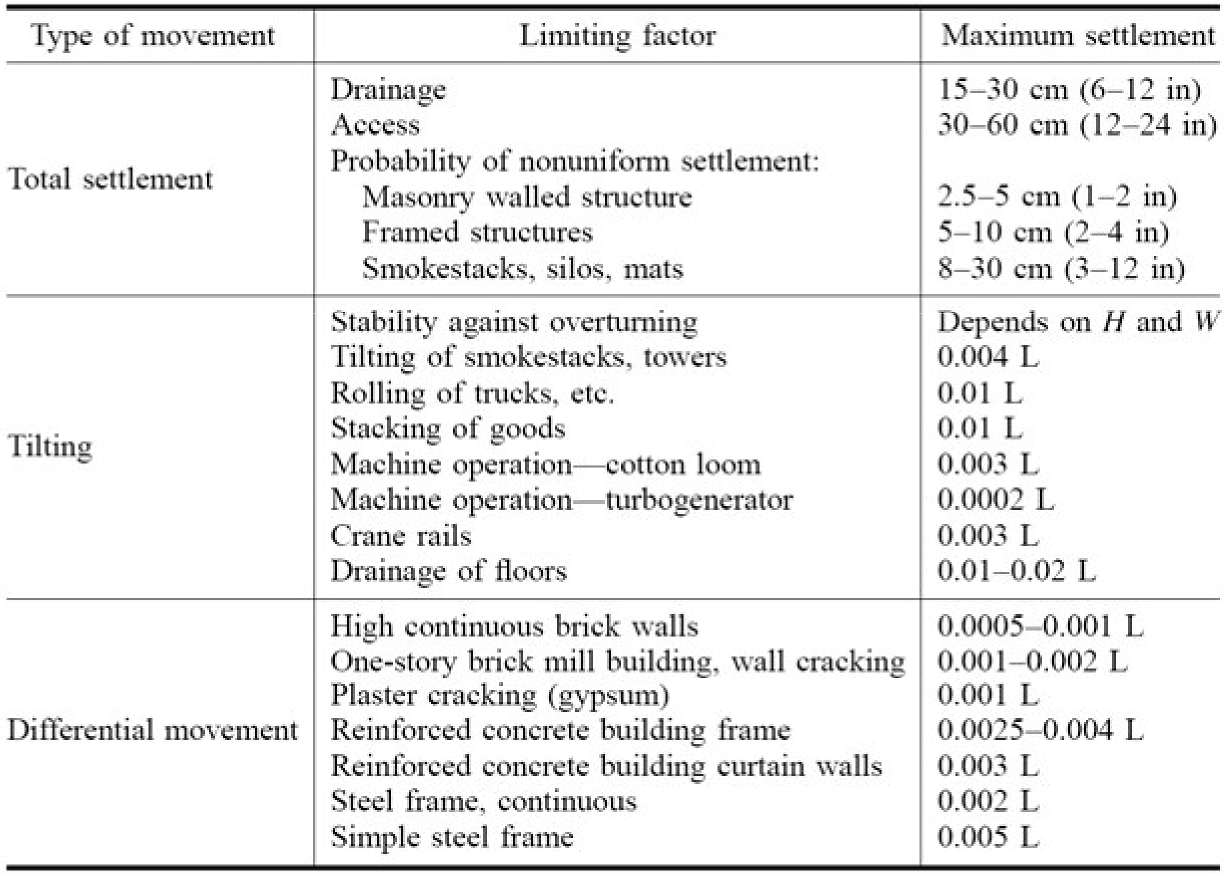


TABLE 6.13 Allowable Settlement

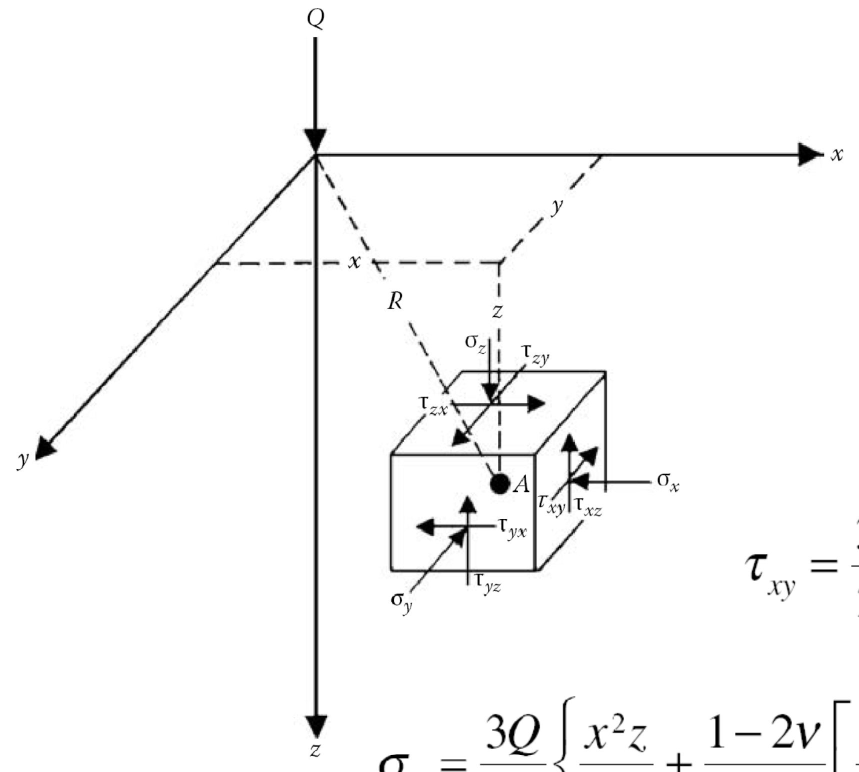


Type of movement	Limiting factor	Maximum settlement
Total settlement	Drainage	15–30 cm (6–12 in)
	Access	30–60 cm (12–24 in)
	Probability of nonuniform settlement:	
	Masonry walled structure	2.5–5 cm (1–2 in)
	Framed structures	5–10 cm (2–4 in)
	Smokestacks, silos, mats	8–30 cm (3–12 in)
Tilting	Stability against overturning	Depends on *H* and *W*
	Tilting of smokestacks, towers	0.004 L
	Rolling of trucks, etc.	0.01 L
	Stacking of goods	0.01 L
	Machine operation—cotton loom	0.003 L
	Machine operation—turbogenerator	0.0002 L
	Crane rails	0.003 L
	Drainage of floors	0.01–0.02 L
Differential movement	High continuous brick walls	0.0005–0.001 L
	One-story brick mill building, wall cracking	0.001–0.002 L
	Plaster cracking (gypsum)	0.001 L
	Reinforced concrete building frame	0.0025–0.004 L
	Reinforced concrete building curtain walls	0.003 L
	Steel frame, continuous	0.002 L
	Simple steel frame	0.005 L

L = distance between adjacent columns that settle different amounts, or between any two points that settle differently. Higher values are for regular settlements and more tolerant structures. Lower values are for irregular settlement and critical structures. *H* = height and *W* = width of structure.

Source: G. F. Sowers, "Shallow Foundations," ch. 6 of "Foundation Engineerings," ed. G. A. Leonards, McGraw-Hill Publishing Co., New York.

Stress Increase in Soil Due to Applied Load—Boussinesq's Solution



$$\sigma_z = \frac{3Qz^3}{2\pi R^5}$$

where

σ = normal stress

τ = shear stress

$R = \sqrt{z^2 + r^2}$

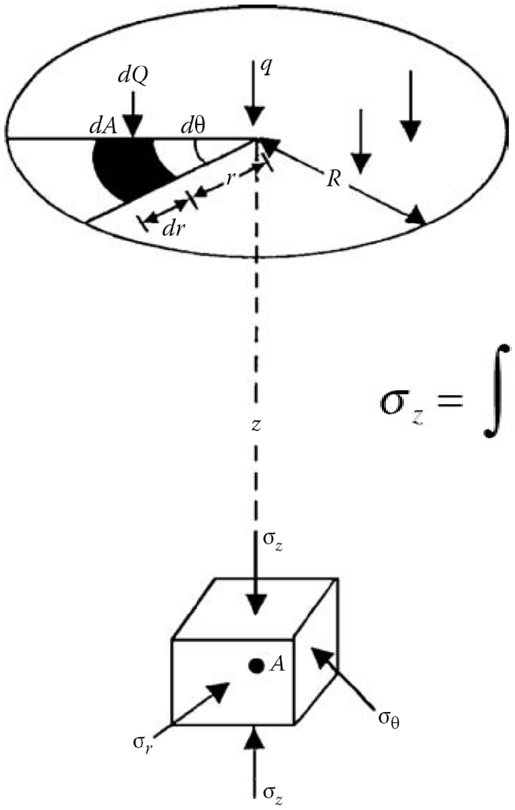
$r = \sqrt{x^2 + y^2}$

ν = Poisson's ratio

$$\tau_{xy} = \frac{3Q}{2\pi}\left[\frac{xyz}{R^5} - \frac{1-2\nu}{3}\,\frac{(2R+z)xy}{R^3(R+z)^2}\right]$$

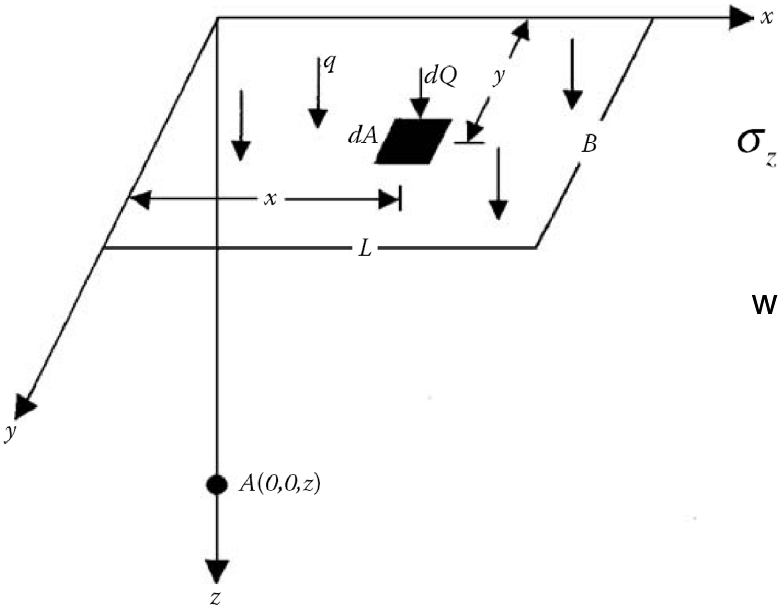
$$\sigma_x = \frac{3Q}{2\pi}\left\{\frac{x^2 z}{R^5} + \frac{1-2\nu}{3}\left[\frac{1}{R(R+z)} - \frac{(2R+z)x^2}{R^3(R+z)^2} - \frac{z}{R^3}\right]\right\}$$

Uniformly loaded circular area



$$\sigma_z = \int d\sigma_z = \int_{r=0}^{R}\int_{\theta=0}^{2\pi} \frac{3z^3 q r d\theta dr}{2\pi(r^2+z^2)^{5/2}} = q\left[1-\frac{z^3}{(R^2+z^2)^{3/2}}\right]$$

Uniformly loaded rectangular area



$$\sigma_z = \int d\sigma_z = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3 dxdy}{2\pi(r^2+z^2)^{5/2}} = qI$$

with

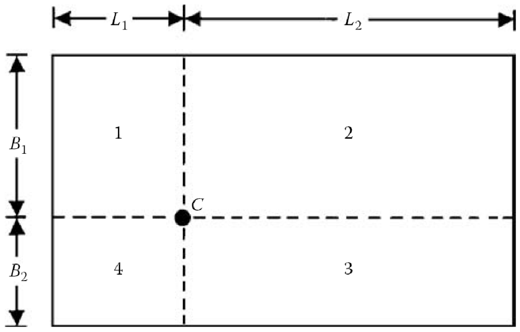
$$m = \frac{B}{z}$$

$$n = \frac{L}{z}$$

$$I = \frac{1}{4\pi}\left[\frac{2mn(m^2+n^2+1)^{0.5}}{m^2+n^2+m^2n^2+1} \times \frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\frac{2mn(m^2+n^2+1)^{0.5}}{m^2+n^2-m^2n^2+1}\right]$$

TABLE 5.2
Variation of *I* with *m* and *n*

	m																			
n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0
0.1	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279	0.0293	0.0301	0.0306	0.0309	0.0311	0.0314	0.0315	0.0316	0.0316	0.0316
0.2	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0474	0.0504	0.0528	0.0547	0.0573	0.0589	0.0599	0.0606	0.0610	0.0616	0.0618	0.0619	0.0620	0.0620
0.3	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794	0.0832	0.0856	0.0871	0.0880	0.0887	0.0895	0.0898	0.0901	0.0901	0.0902
0.4	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013	0.1063	0.1094	0.1114	0.1126	0.1134	0.1145	0.1150	0.1153	0.1154	0.1154
0.5	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1104	0.1158	0.1202	0.1263	0.1300	0.1324	0.1340	0.1350	0.1363	0.1368	0.1372	0.1374	0.1374
0.6	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1361	0.1431	0.1475	0.1503	0.1521	0.1533	0.1548	0.1555	0.1560	0.1561	0.1562
0.7	0.0242	0.0474	0.0686	0.0873	0.1034	0.1169	0.1277	0.1365	0.1436	0.1491	0.1570	0.1620	0.1652	0.1672	0.1686	0.1704	0.1711	0.1717	0.1719	0.1719
0.8	0.0258	0.0504	0.0731	0.0931	0.1104	0.1247	0.1365	0.1461	0.1537	0.1598	0.1684	0.1739	0.1774	0.1797	0.1812	0.1832	0.1841	0.1847	0.1849	0.1850
0.9	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1619	0.1684	0.1777	0.1836	0.1874	0.1899	0.1915	0.1938	0.1947	0.1954	0.1956	0.1957
1.0	0.0279	0.0547	0.0794	0.1013	0.1202	0.1361	0.1491	0.1598	0.1684	0.1752	0.1851	0.1914	0.1955	0.1981	0.1999	0.2024	0.2034	0.2042	0.2044	0.2045
1.2	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851	0.1958	0.2028	0.2073	0.2103	0.2124	0.2151	0.2163	0.2172	0.2175	0.2176
1.4	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914	0.2028	0.2102	0.2151	0.2184	0.2206	0.2236	0.2250	0.2260	0.2263	0.2264
1.6	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955	0.2073	0.2151	0.2203	0.2237	0.2261	0.2294	0.2309	0.2320	0.2323	0.2325
1.8	0.0309	0.0606	0.0880	0.1126	0.1340	0.1521	0.1672	0.1797	0.1899	0.1981	0.2103	0.2183	0.2237	0.2274	0.2299	0.2333	0.2350	0.2362	0.2366	0.2367
2.0	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999	0.2124	0.2206	0.2261	0.2299	0.2325	0.2361	0.2378	0.2391	0.2395	0.2397
2.5	0.0314	0.0616	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1938	0.2024	0.2151	0.2236	0.2294	0.2333	0.2361	0.2401	0.2420	0.2434	0.2439	0.2441
3.0	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034	0.2163	0.2250	0.2309	0.2350	0.2378	0.2420	0.2439	0.2455	0.2461	0.2463
4.0	0.0316	0.0619	0.0901	0.1153	0.1372	0.1560	0.1717	0.1847	0.1954	0.2042	0.2172	0.2260	0.2320	0.2362	0.2391	0.2434	0.2455	0.2472	0.2479	0.2481
5.0	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1719	0.1849	0.1956	0.2044	0.2175	0.2263	0.2324	0.2366	0.2395	0.2439	0.2460	0.2479	0.2486	0.2489
6.0	0.0316	0.0620	0.0902	0.1154	0.1374	0.1562	0.1719	0.1850	0.1957	0.2045	0.2176	0.2264	0.2325	0.2367	0.2397	0.2441	0.2463	0.2482	0.2489	0.2492



$$\sigma_z = q(I_1 + I_2 + I_3 + I_4)$$

- **Chart for calculating the increase in vertical stress beneath the corner of a uniformly loaded rectangular area**

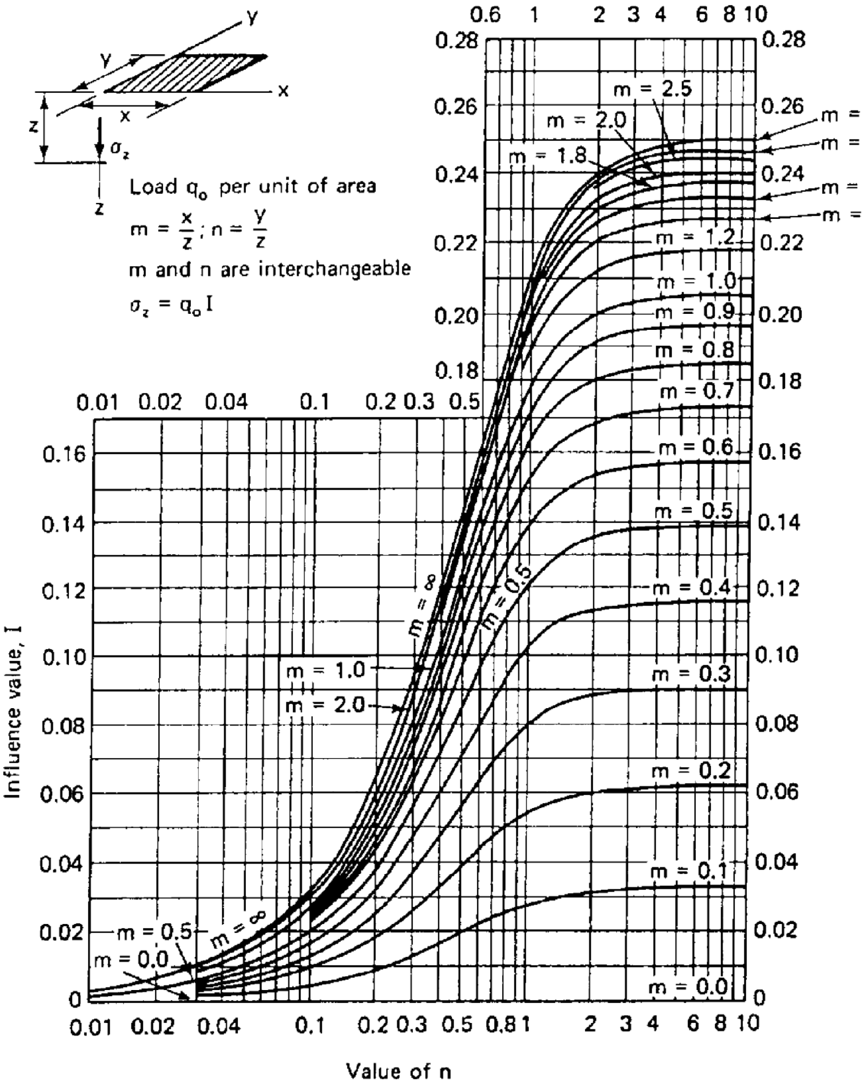
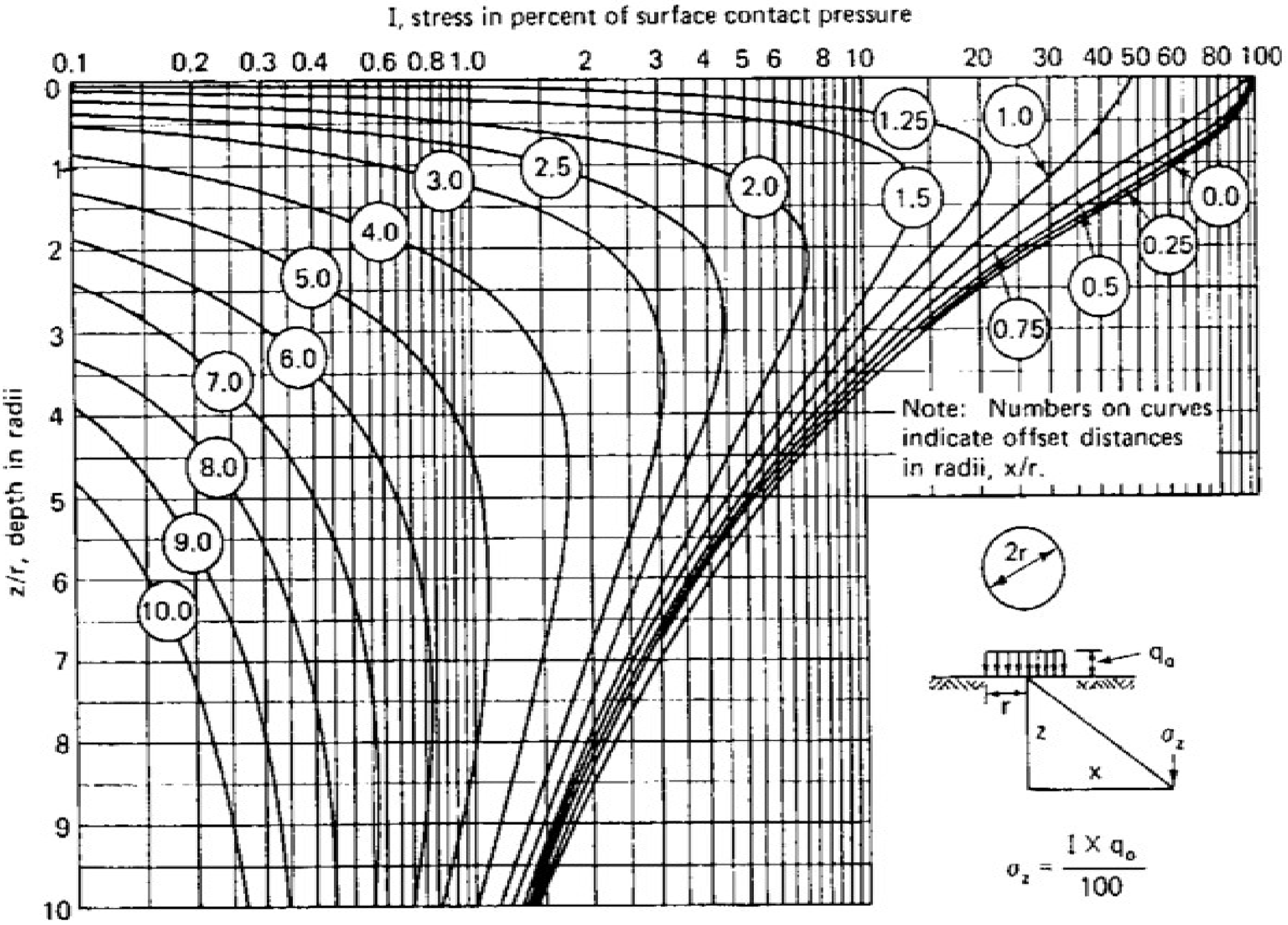


Chart for calculating the increase in vertical stress beneath a uniformly loaded circular area



Stress Increase Due to Applied Load—Westergaard's Solution

- **Point load *Q* in an elastic solid medium in which layers alternate with thin rigid reinforcements.**
  - an idealization of a clay layer with thin seams of sand.

$$\sigma_z = \frac{Q\eta}{2\pi z^2}\left[\frac{1}{\eta^2 + \left(\frac{r}{z}\right)^2}\right]^{-3/2}$$

where

$$\eta = \sqrt{\frac{1-2\nu}{2-2\nu}}$$

ν = Poisson's ratio

$r = \sqrt{x^2 + y^2}$

- **Uniformly Loaded Flexible Circular Area**

$$\sigma_z = q\left\{1 - \frac{\eta}{\left[\eta^2 + \left(\frac{R}{z}\right)^2\right]^{1/2}}\right\}$$

- **Uniformly Loaded Flexible Rectangular Area**

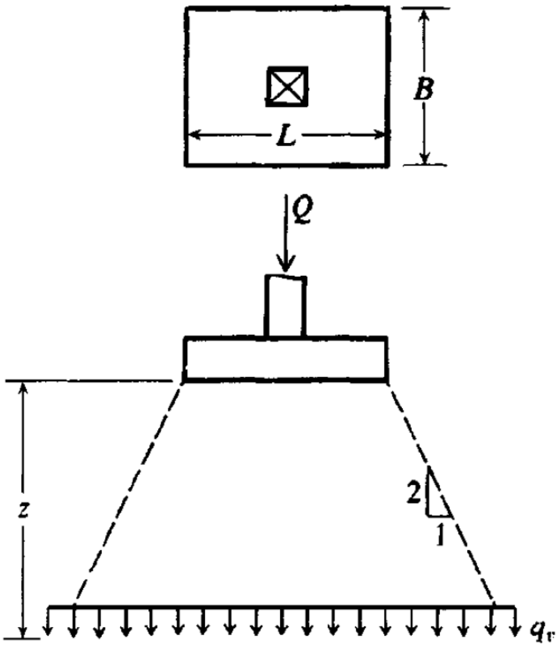
$$\sigma_z = \frac{q}{2\pi}\left[\cot^{-1}\sqrt{\eta^2\left(\frac{1}{m^2}+\frac{1}{n^2}\right)+\eta^4\left(\frac{1}{m^2n^2}\right)}\right]$$

where

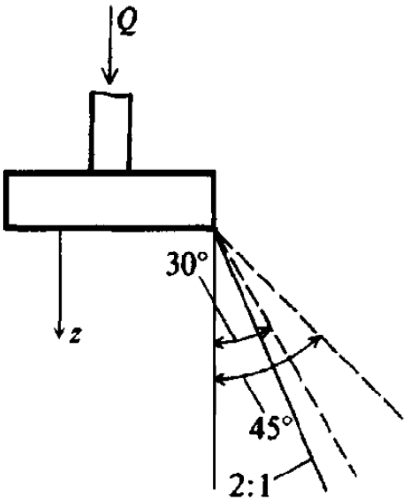
$$m = \frac{B}{z}$$

$$n = \frac{L}{z}$$

Approximate methods

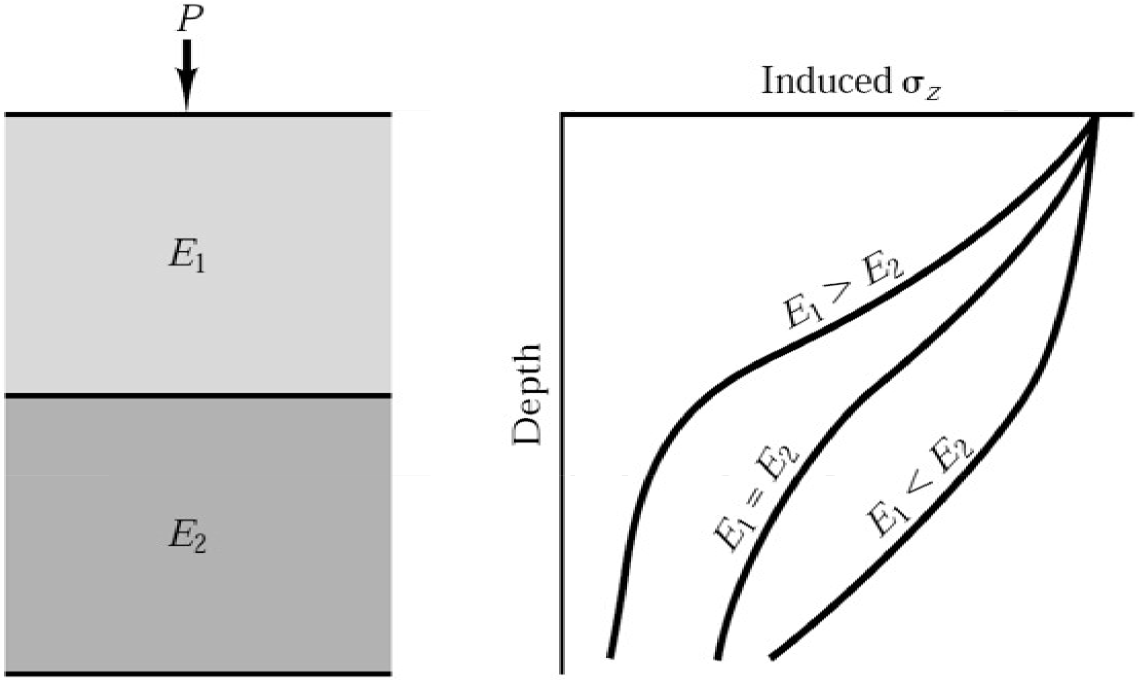


$$\Delta q = q_v = \frac{Q}{(B+z)(L+z)}$$

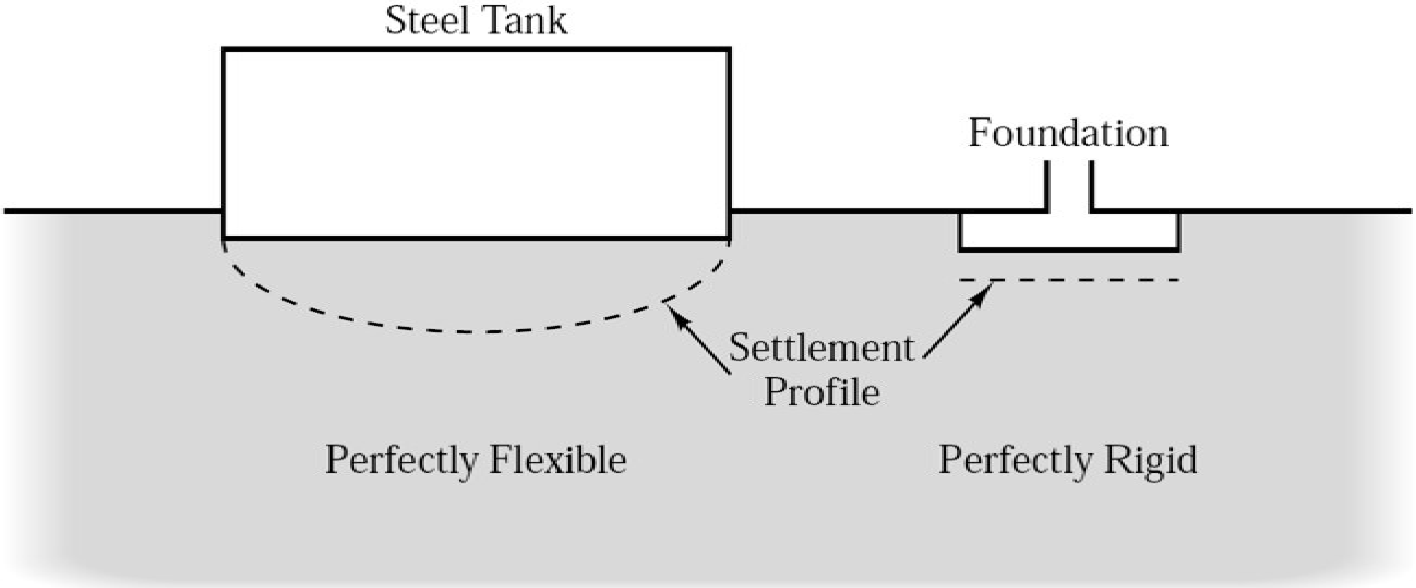


2 : 1 method compares reasonably well with more theoretical methods from ***z = B to*** **about *z = 4B***
but should not be used in the depth zone from *z* = 0 to *B.*

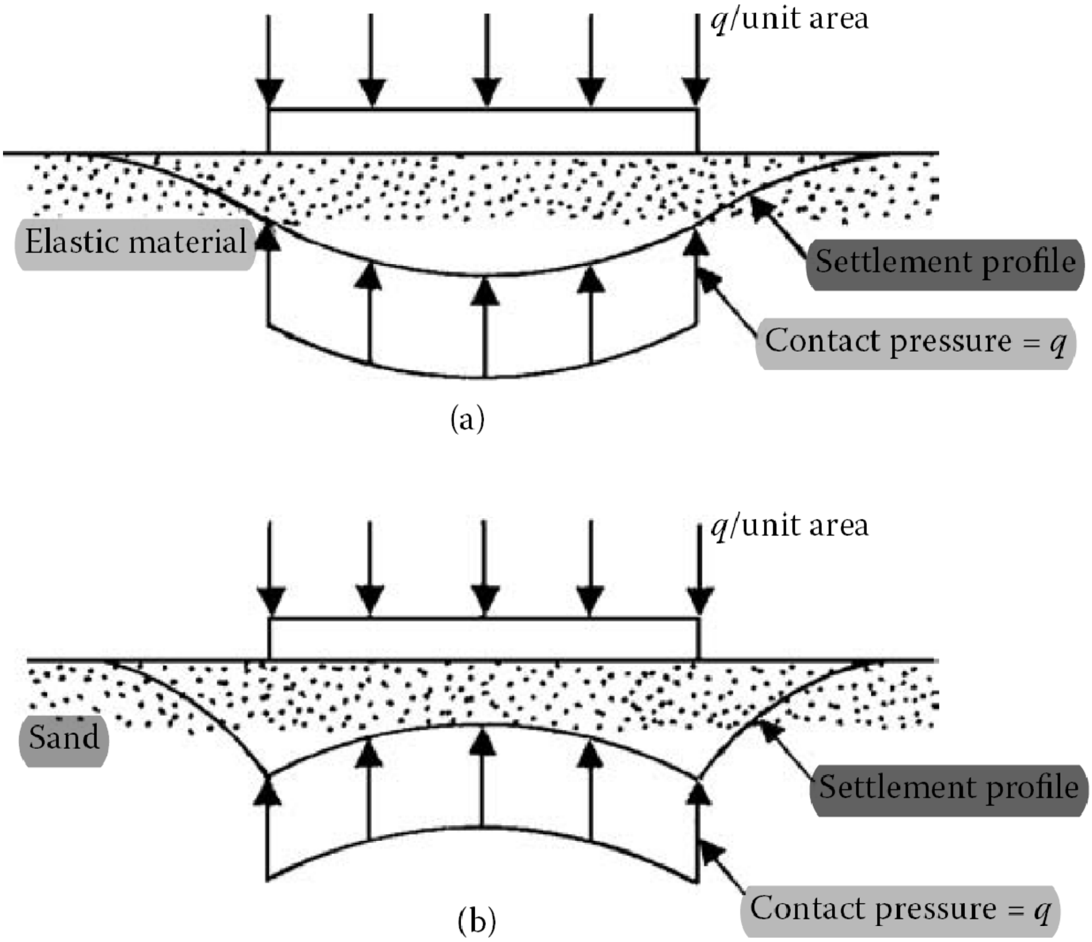
Distribution of induced stress *in layered strata*



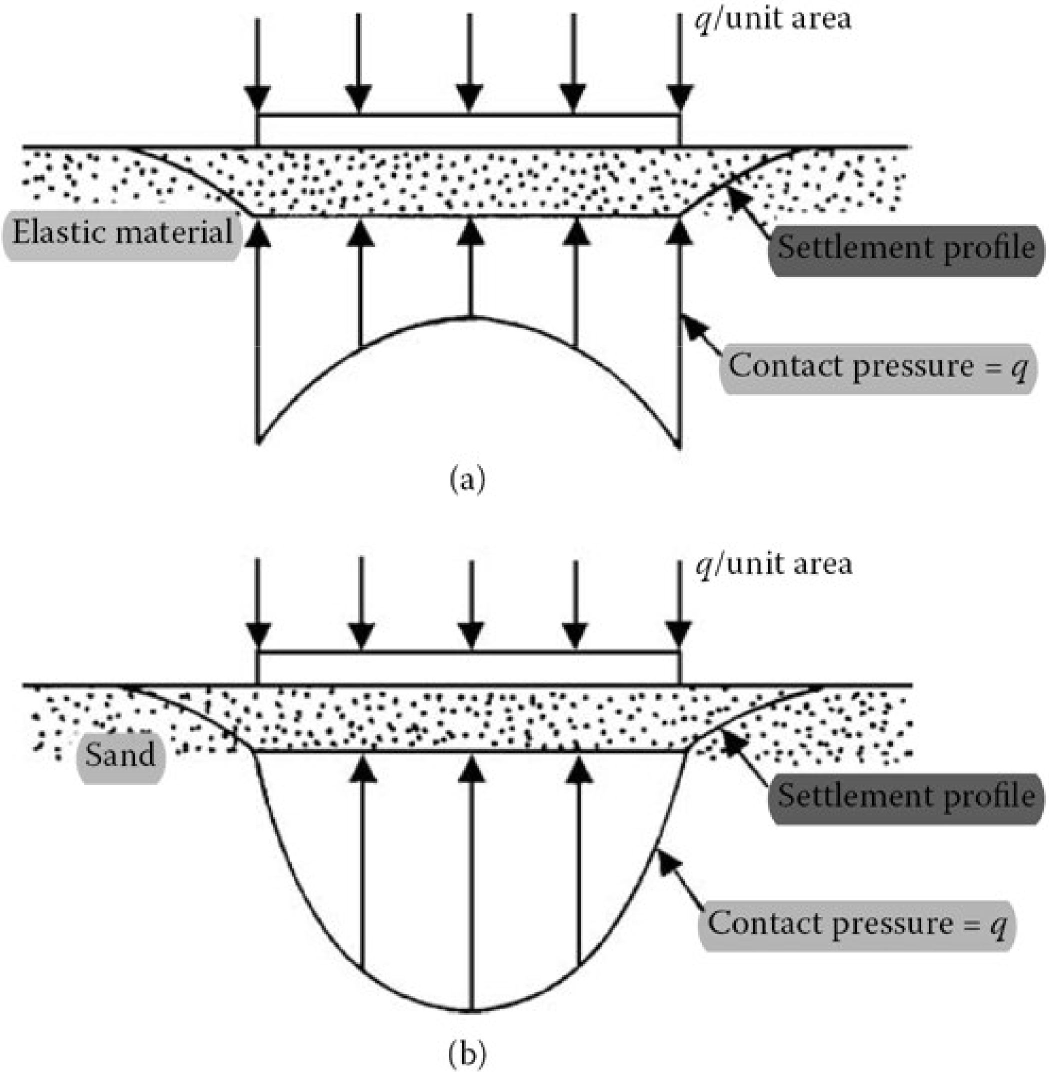
Influence of Foundation rigidity on settlement



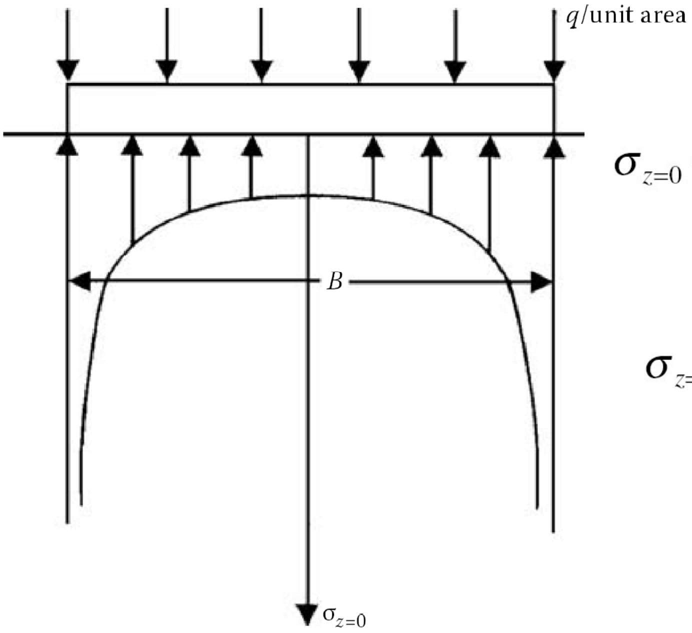
Flexible Foundation



Rigid Foundation



- **Contact pressure beneath a infinitely rigid foundation on perfectly elastic material**



$$\sigma_{z=0} = \frac{2q}{\pi\sqrt{1-\left(\frac{2x}{B}\right)^2}} \quad \text{(continuous foundation)}$$

$$\sigma_{z=0} = \frac{q}{2\sqrt{1-\left(\frac{2x}{B}\right)^2}} \quad \text{(circular foundation)}$$

Elastic Parameters

Suggested Values for Poisson's Ratio

Soil Type	Poisson's Ratio ν
Coarse sand	0.15–0.20
Medium loose sand	0.20–0.25
Fine sand	0.25–0.30
Sandy silt and silt	0.30–0.35
Saturated clay (undrained)	0.50
Saturated clay—lightly overconsolidated (drained)	0.2–0.4

$$\nu = 0.1 + 0.3\phi_{\text{rel}}$$

$$\phi_{\text{rel}} = \text{relative friction angle} = \frac{\phi_{\text{tc}} - 25^\circ}{45^\circ - 25^\circ} \qquad (0 \le \phi_{\text{rel}} \le 1)$$

$$\varphi_{\text{tc}} = \text{friction angle from drained triaxial compression test}$$

Modulus of elasticity

$$E_s\ (\text{kN/m}^2) = 766 N_{60}$$

$$E_s = 2.5 q_c$$

For Square and circular foundations

$$E_s = 3.5 q_c$$

For strip foundations
L/B ≥ 10

$$\frac{E_s}{c_u} = 1000 \text{ to } 1500$$

General Range of Modulus of Elasticity of Sand

Type	**E_s (kN/m²)**
Coarse and medium coarse sand	
Loose	25,000–35,000
Medium dense	30,000–40,000
De	40,000–45,000
Fine	
Loose	20,000–25,000
Medium dense	25,000–35,000
Dense	35,000–40,000
Sandy silt	
Loose	8,000–12,000
Medium dense	10,000–12,000
Dense	12,000–15,000

$$\frac{E_s}{c_u} = 1000 \text{ to } 1500$$

Settlement under circular area

- **Strain**

$$\in_z = \frac{1}{E_s}\left[\sigma_z - \nu(\sigma_r + \sigma_\theta)\right]$$

- **Elastic settlement**

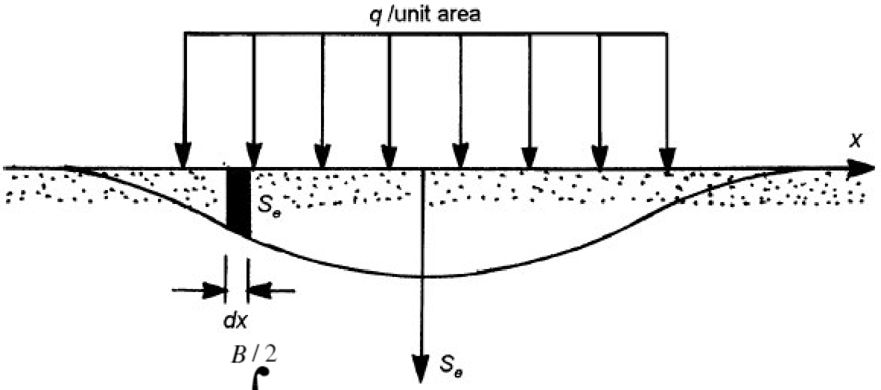
$$\int_z^\infty \in_z \, dz = \int_z^\infty \frac{1}{E_s}\left[\sigma_z - \nu(\sigma_r + \sigma_\theta)\right] dz$$

$$S_e = q\frac{1+\nu}{E_s}R \qquad \left[\frac{z}{R}I_1 + (1-\nu)I_2\right]$$

$$\text{where } I_1 \text{ and } I_2 = f\left(\frac{z}{R} \text{ and } \frac{r}{R}\right)$$

$$S_e(\text{center}) = \frac{2q(1-\nu^2)R}{E_s} = \frac{qB(1-\nu^2)}{E_s}$$

$$S_e(\text{edge}) = \frac{1.273qB(1-\nu^2)}{2E_s} = \frac{0.636qB(1-\nu^2)}{E_s}$$



$$S_e(\text{average}) = \frac{\int_{-B/2}^{B/2} S_e\,dx}{B}$$

$$S_e\ (\text{average}) \approx 0.85 S_e\ (\text{center})$$

$$S_e(\text{rigid}) \approx 0.93 S_e\ (\text{average flexible}) \approx \frac{0.79qB(1-\nu^2)}{E_s}$$

Settlement under rectangular area

$$S_e(\text{corner}) = \frac{qB}{2E_s}(1-\nu^2)\left[I_3 - \left(\frac{1-2\nu}{1-\nu}\right)I_4\right]$$

$$S_e(\text{corner}) = \frac{qB}{2E}(1-\nu^2)I_5$$

At the surface

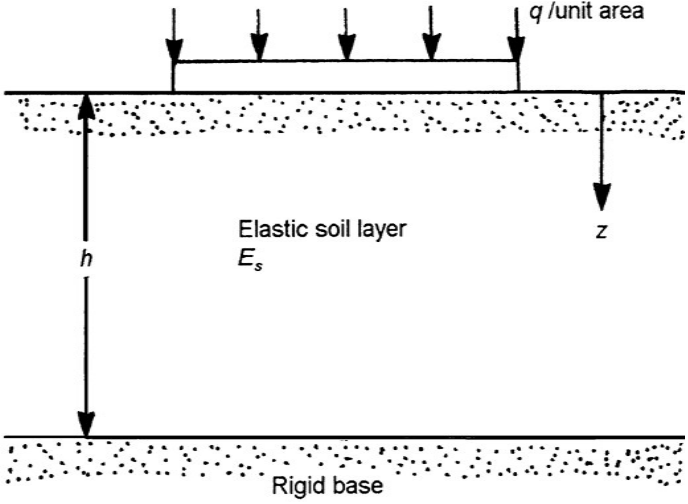
$$S_e(\text{center}) = \frac{qB}{E}(1-\nu^2)I_5$$

$$S_e(\text{average}) = \frac{0.85qB}{E_s}(1-\nu^2)I_5 = \frac{qB}{E_s}(1-\nu^2)I_6$$

$$S_e(\text{rigid}) = \frac{0.79qB}{E_s}(1-\nu^2)I_5 = \frac{qB}{E_s}(1-\nu^2)I_7$$

Effect of Rigid base at a limited Depth

$$S_e = \int_0^\infty \in_z \ dz - \int_h^\infty \in_z \ dz$$



Circular area:

$$S_e(\text{center, flexible}) = \frac{Rq}{E_s}(1-\nu^2)\alpha_1$$

$$S_e(\text{center, rigid}) = \frac{Rq}{E_s}(1-\nu^2)\alpha_2$$

α_1 and α_2 are functions of *h/R*

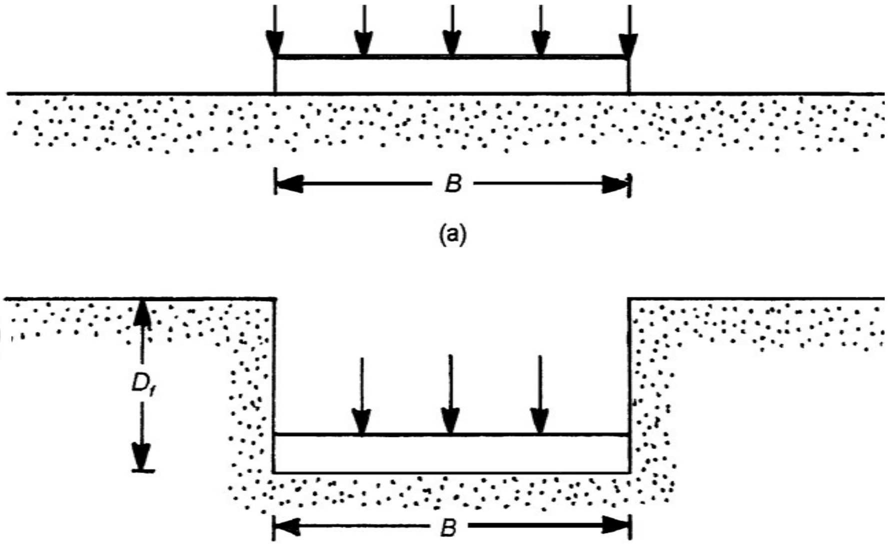
Rectangular Area ($B \times L$)

$$S_e(\text{center, flexible}) = \frac{Bq}{E_s}(1-\nu^2)\alpha_3$$

$$S_e(\text{center, rigid}) = \frac{Bq}{E_s}(1-\nu^2)\alpha_4$$

α_3 and α_4 are functions of *h/B, L/B*

Effect of Depth of Embedment



S_e (average, D_f)/S_e (average, D_f=0)

$\frac{D_f}{\sqrt{LB}}$	L/B		
	1	5	10
0	1	1	1
0.1	0.981	0.975	0.969
0.2	0.959	0.947	0.931
0.3	0.928	0.906	0.893
0.4	0.894	0.875	0.856
0.5	0.863	0.844	0.831
0.6	0.825	0.819	0.813
0.8	0.763	0.775	0.781
1.0	0.732	0.737	0.738

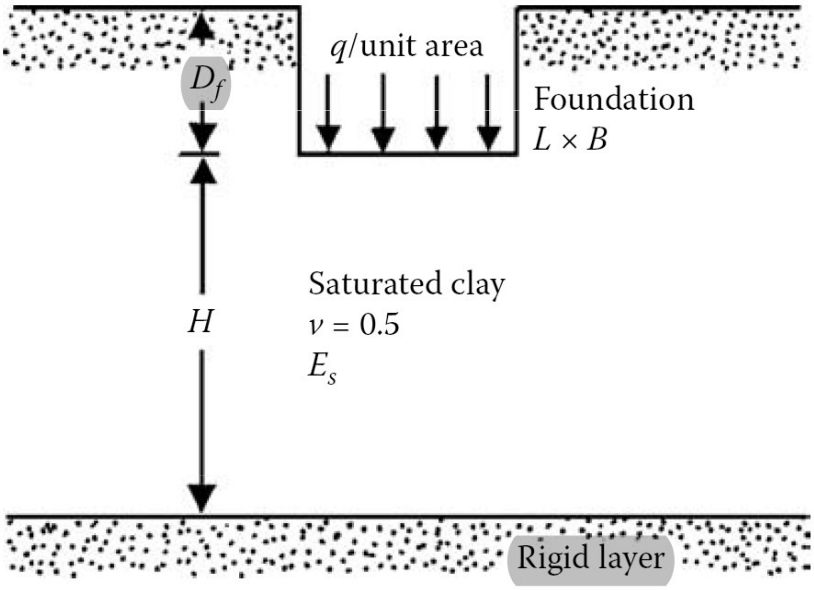
Settlement of Foundations on saturated clays ($\nu = 0.5$) *Janbu et al. (1956)*

$$S_e = \mu_1 \mu_2 \frac{qB}{E_s}$$

$$\mu_1 = f\left(\frac{D_f}{B}\right)$$

$$\mu_2 = \left(\frac{H}{B}, \frac{L}{B}\right)$$

L = foundation length
B = foundation width



Variation of μ_1 with D_f/B

D_f/B	μ_1
0	1.0
2	0.9
4	0.88
6	0.875
8	0.87
10	0.865
12	0.863
14	0.860
16	0.856
18	0.854
20	0.850

Variation of μ_2 with H/B and L/B

	L/B					
H/B	Circle	1	2	5	10	∞
1	0.36	0.36	0.36	0.36	0.36	0.36
2	0.47	0.53	0.63	0.64	0.64	0.64
4	0.58	0.63	0.82	0.94	0.94	0.94
6	0.61	0.67	0.88	1.08	1.14	1.16
8	0.62	0.68	0.90	1.13	1.22	1.26
10	0.63	0.70	0.92	1.18	1.30	1.42
20	0.64	0.71	0.93	1.26	1.47	1.74
30	0.66	0.73	0.95	1.29	1.54	1.84

Foundations on Sand-Correlation with SPT

- **Terzaghi and Peck's correlation** $\frac{S_e}{S_{e(1)}} = \frac{4}{\left[1+\left(\frac{B_1}{B}\right)\right]^2}$
  - Based on plate load test B_1 X B_1

$$S_e = C_W C_D \frac{3q}{N_{60}}\left(\frac{B}{B+0.3}\right)^2$$

q = bearing pressure in kN/m^2
B = width of foundation in m
C_W = groundwater table correction
C_D = correction for depth of embedment $= 1-\left(\frac{D_f}{4B}\right)$
D_f = depth of embedment

C_W = 1 if GWL ≥ 2B
= 2 if GWL ≤ B

Mayerhof's Correlataion

$$S_e\ (\text{mm}) = C_W C_D \frac{1.25q}{N} \quad (\text{for } B \le 1.22\ \text{m})$$

$$S_e\ (\text{mm}) = C_W C_D \frac{2q}{N_{60}}\left(\frac{B}{B+0.3}\right)^2 \quad (\text{for } B > 1.22\ \text{m})$$

S_e is in mm, B is in m, and q is in kN/m^2

Peck and Bazaraa (1969) method

$$\boxed{S_e = C_W C_D \frac{2q}{(N_1)_{60}} \left(\frac{B}{B+0.3} \right)^2}$$

S_e is in mm, q is in kN/m², and B is in m

$$C_W = \frac{\sigma_o \text{ at } 0.5B \text{ below the bottom of the foundation}}{\sigma'_o \text{ at } 0.5B \text{ below the bottom of the foundation}}$$

σ_o = total overburden pressure

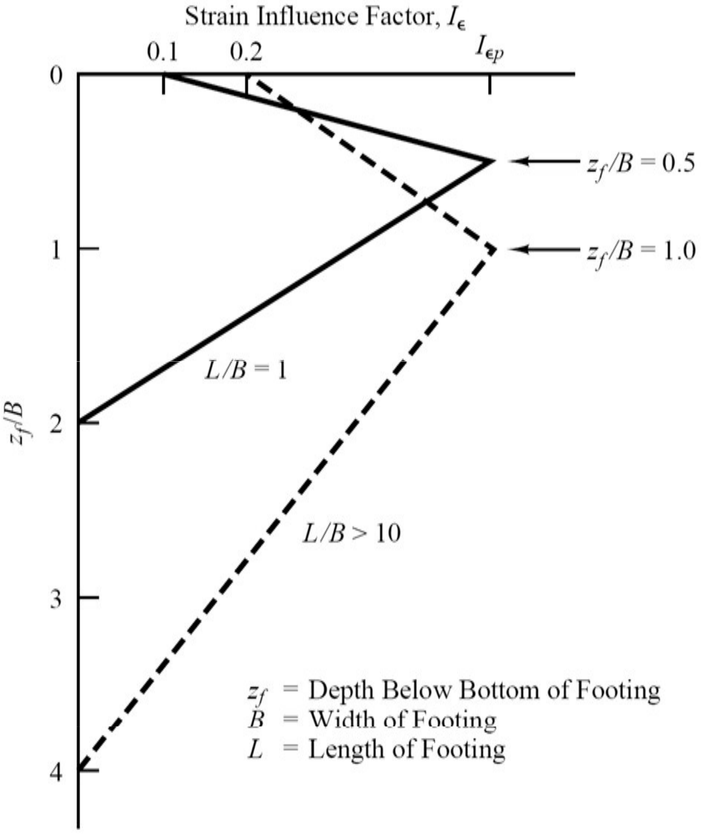
σ'_o = effective overburden pressure

$$C_D = 1.0 - 0.4 \left(\frac{\gamma D_f}{q} \right)^{0.5}$$

$$(N_1)_{60} = \frac{4N_{60}}{1 + 0.04\sigma'_o} \quad (\text{for } \sigma'_o \le 75 \text{ kN/m}^2)$$

$$(N_1)_{60} = \frac{4N_{60}}{3.25 + 0.01\sigma'_o} \quad (\text{for } \sigma'_o > 75 \text{ kN/m}^2)$$

Schmertman's Method



$$\delta = C_1 C_2 C_3 (q - \sigma'_{zD}) \sum \frac{I_\epsilon \Delta z}{E_s}$$

$$I_{\epsilon p} = 0.5 + 0.1 \sqrt{\frac{q - \sigma'_{zD}}{\sigma'_{zp}}}$$

$$\boxed{C_1 = 1 - 0.5 \left(\frac{\sigma'_{zD}}{q - \sigma'_{zD}} \right)}$$

$$\boxed{C_2 = 1 + 0.2 \log \left(\frac{t}{0.1} \right)}$$

$$\boxed{C_3 = 1.03 - 0.03\, L/B \ge 0.73}$$

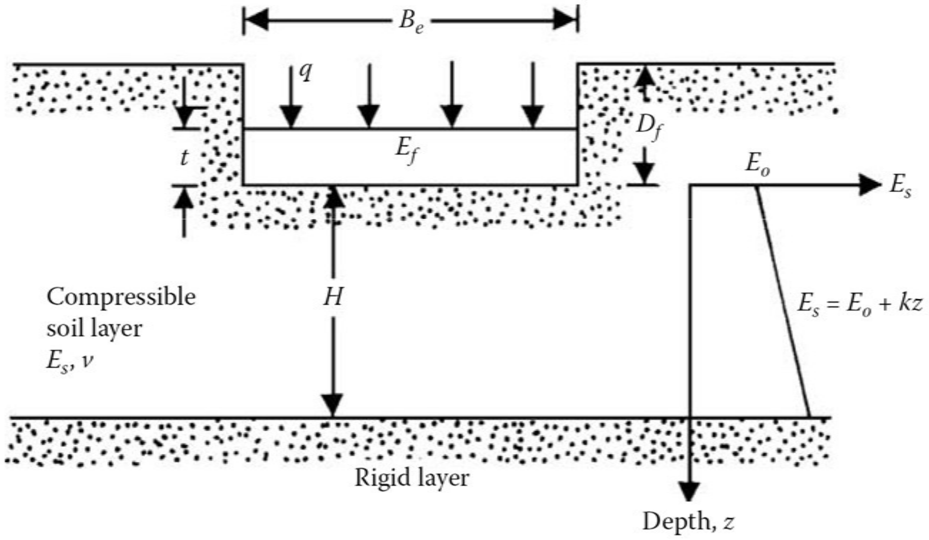
$I_{\epsilon p}$ = peak strain influence factor

q = bearing pressure

σ'_{zD} = vertical effective stress at a depth D below the ground surface

σ'_{zp} = initial vertical effective stress at depth of peak strain influence factor (for square and circular foundations ($L/B = 1$), compute σ'_{zp} at a depth of $D+B/2$ below the ground surface; for continuous footings ($L/B \ge 10$), compute it at a depth of $D+B$)

Mayne and Poulos method (1999)



- **Based on Theory of elasticity**
- **For granular soil**
- **Accounts the *Rigidity* of the foundation**
- **Depth of Embedment**
- **Increase in modulus with depth**
- **Location of rigid layer**

- **Elastic settlement below the centre of Footing**

$$S_e = \frac{qB_e I_G I_R I_E}{E_o}(1-\nu^2)$$

  – Equivalent dia of Rect. Foundation $B_e = \sqrt{\frac{4BL}{\pi}}$

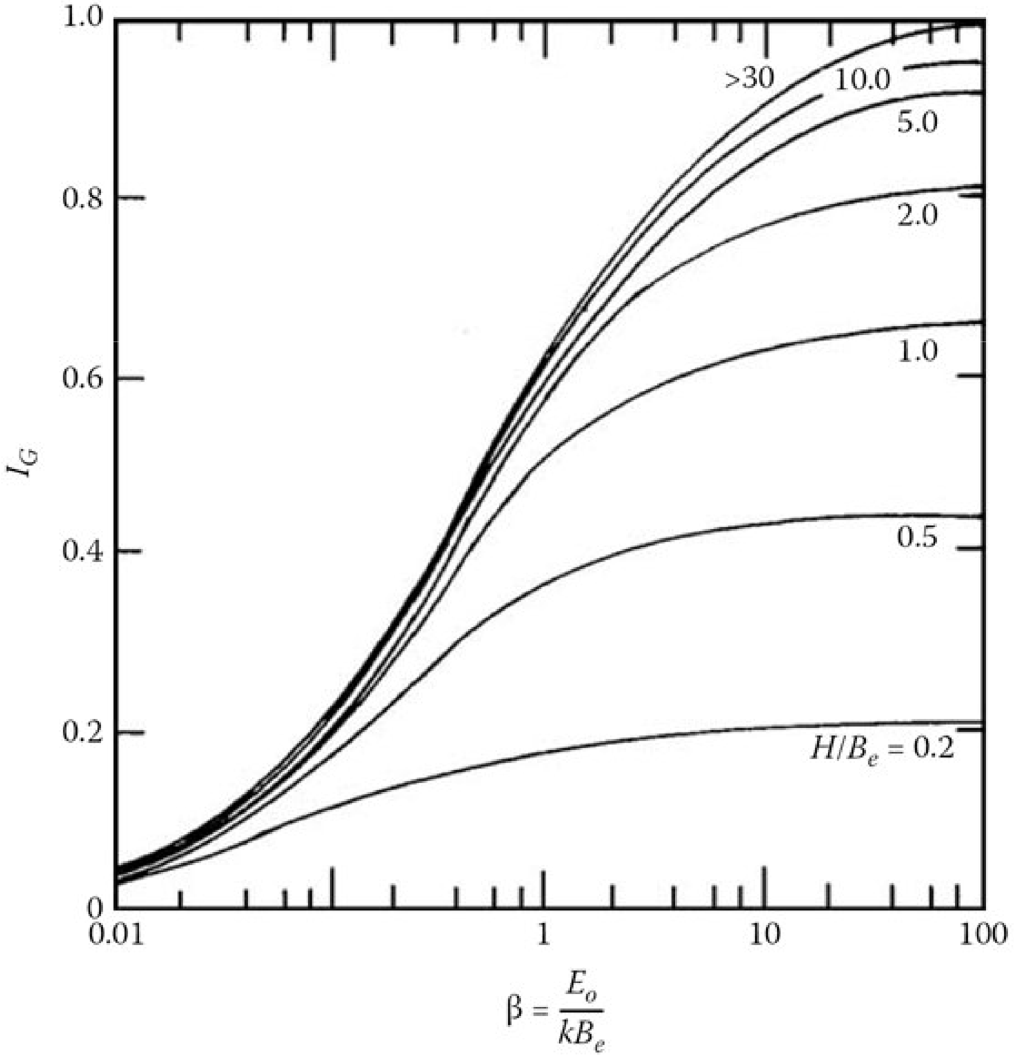
I_G = influence factor for the variation of E_s with depth $= f\left(\beta = \frac{E_o}{kB}, \frac{H}{B}\right)$

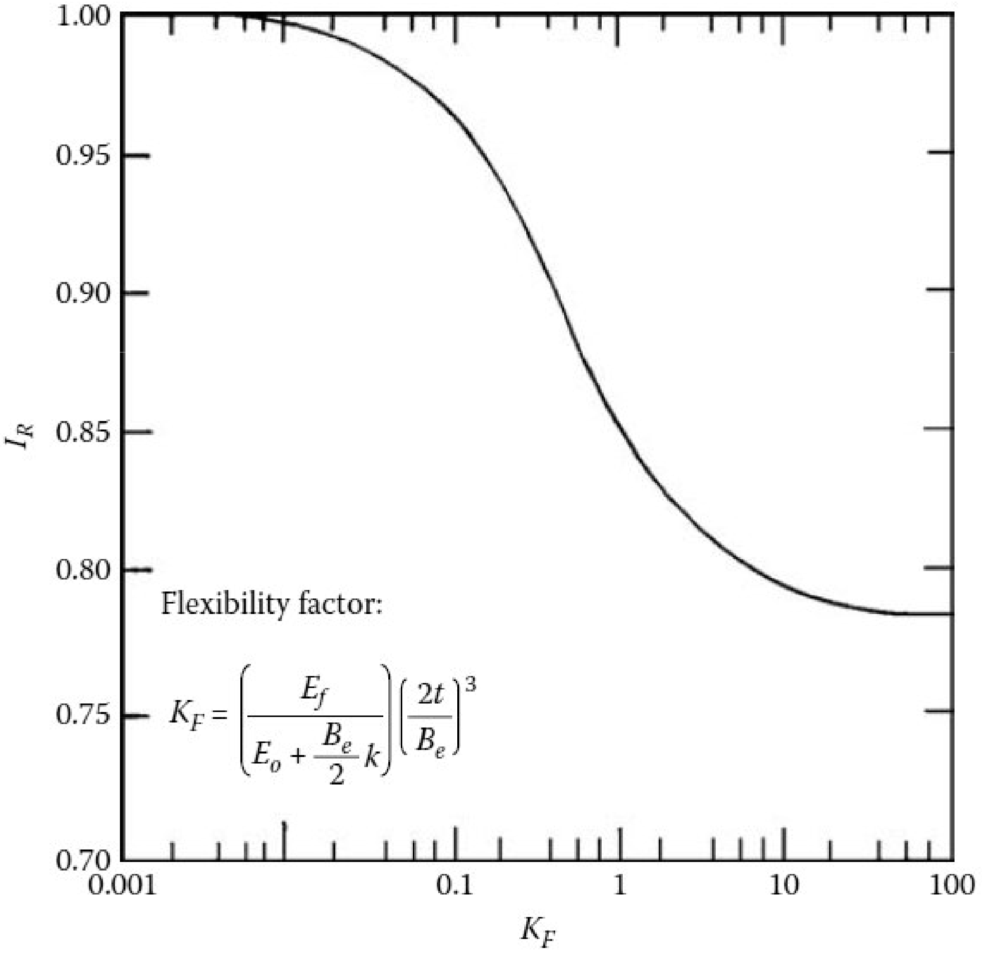
I_R = foundation rigidity correction factor

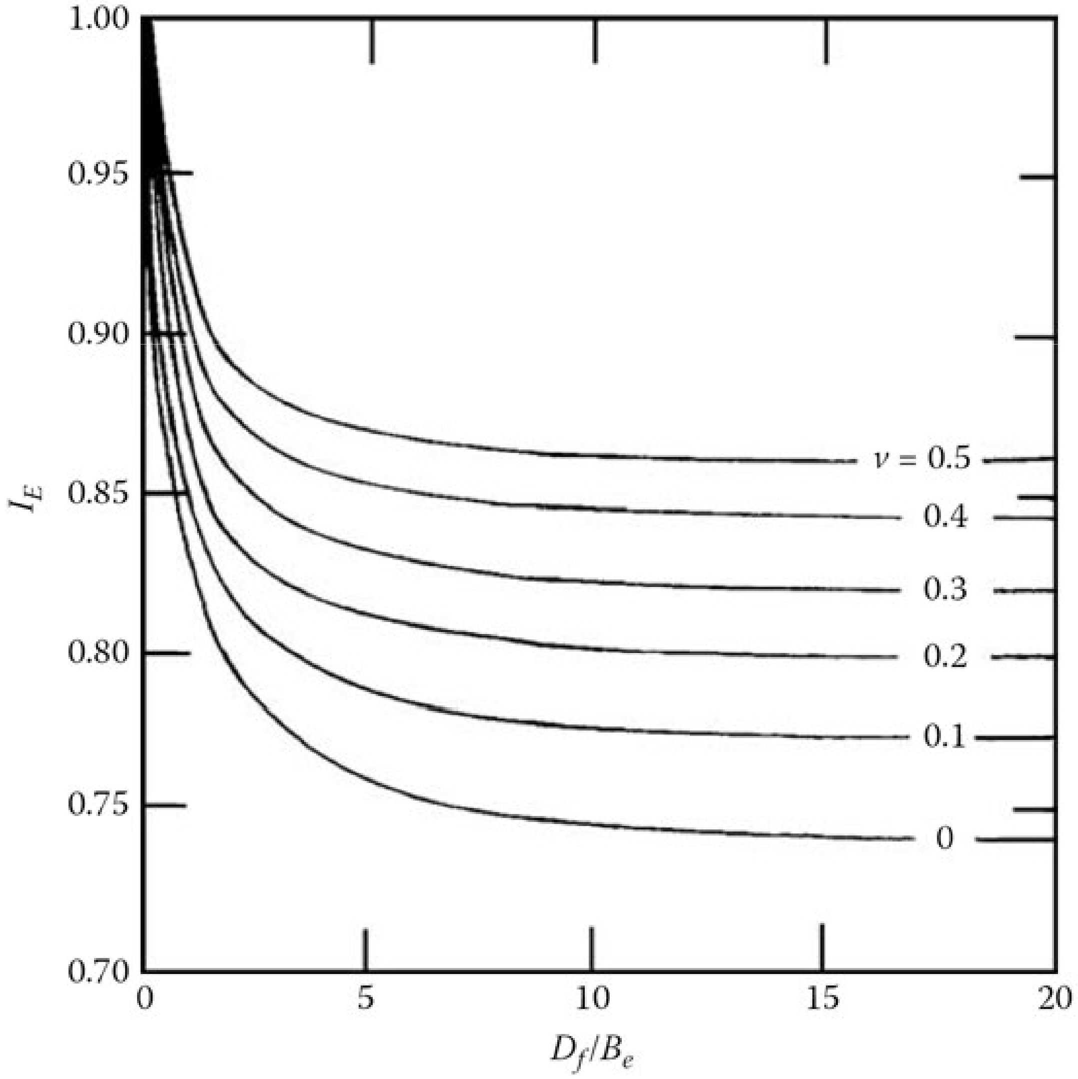
I_E = foundation embedment correction factor

$$I_R = \frac{\pi}{4} + \frac{1}{4.6 + 10\left(\frac{E_f}{E_o + \frac{B_e}{2}k}\right)\left(\frac{2t}{B_e}\right)^3}$$

$$I_E = 1 - \frac{1}{3.5\exp(1.22\nu - 0.4)\left(\frac{B_e}{D_f} + 1.6\right)}$$

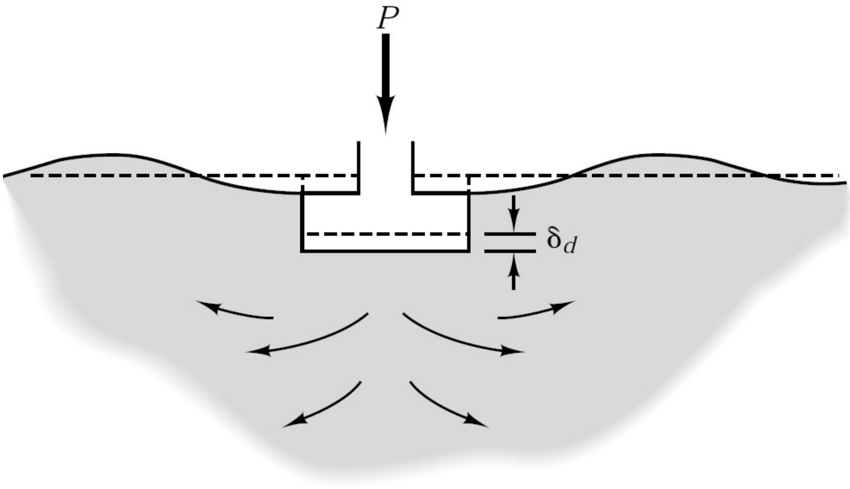

1.0
0.8
0.6
0.4
0.2
0
I_G
0.01
1
10
100
$\beta = \frac{E_o}{kB_e}$
>30
10.0
5.0
2.0
1.0
0.5
$H/B_e = 0.2$


1.00
0.95
0.90
0.85
0.80
0.75
0.70
I_R
0.001
0.1
1
10
100
K_F
Flexibility factor:
$K_F = \left(\frac{E_f}{E_o + \frac{B_e}{2}k}\right)\left(\frac{2t}{B_e}\right)^3$



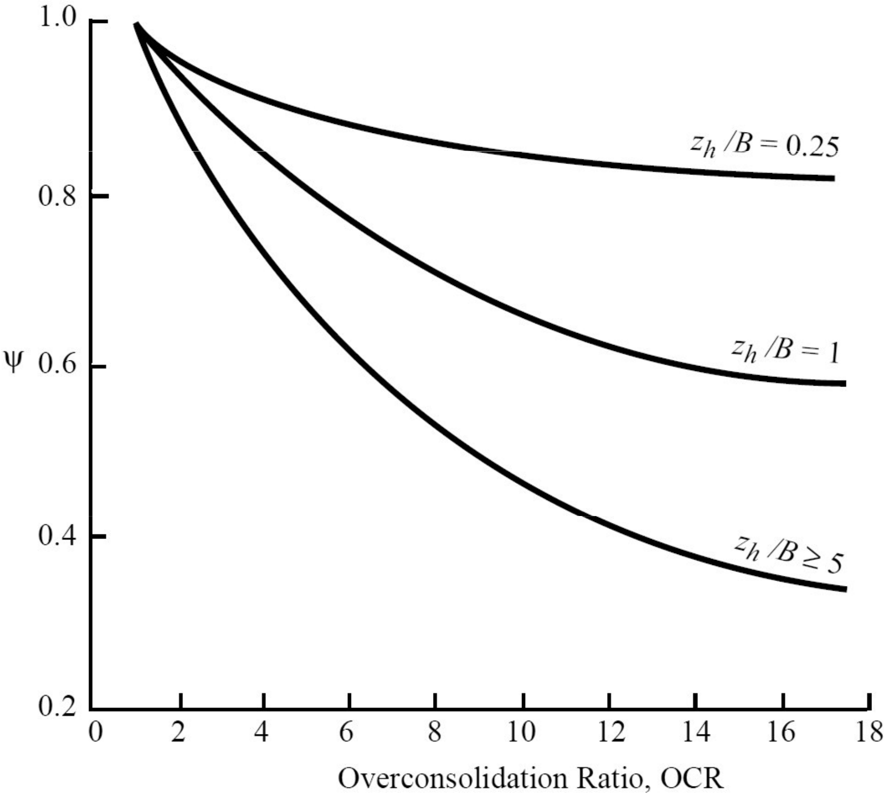
Skempton and Bjerrum method

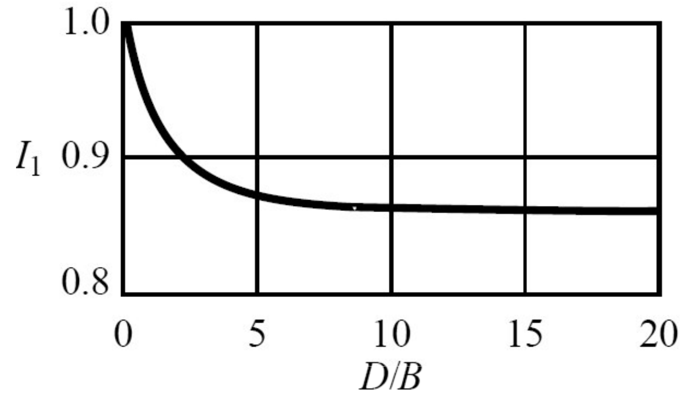
- **Accounts for 3D settlement**
- **Two Components**
  - Distortion (initial)
  - Consolidation

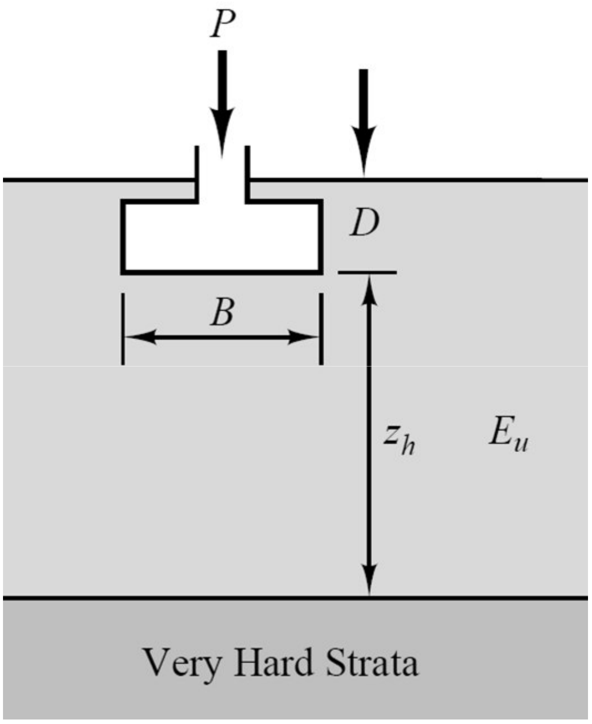


$$\delta = \delta_d + \psi\delta_c$$

$$\delta_d = \frac{(q - \sigma'_{zD})B}{E_u} I_1 I_2$$


1.0
0.8
ψ 0.6
0.4
0.2
0
2
4
6
8
10
12
14
16
18
Overconsolidation Ratio, OCR
$z_h/B = 0.25$
$z_h/B = 1$
$z_h/B \geq 5$


1.0
I_1 0.9
0.8
0
5
10
15
20
D/B


P
D
B
z_h
E_u
Very Hard Strata

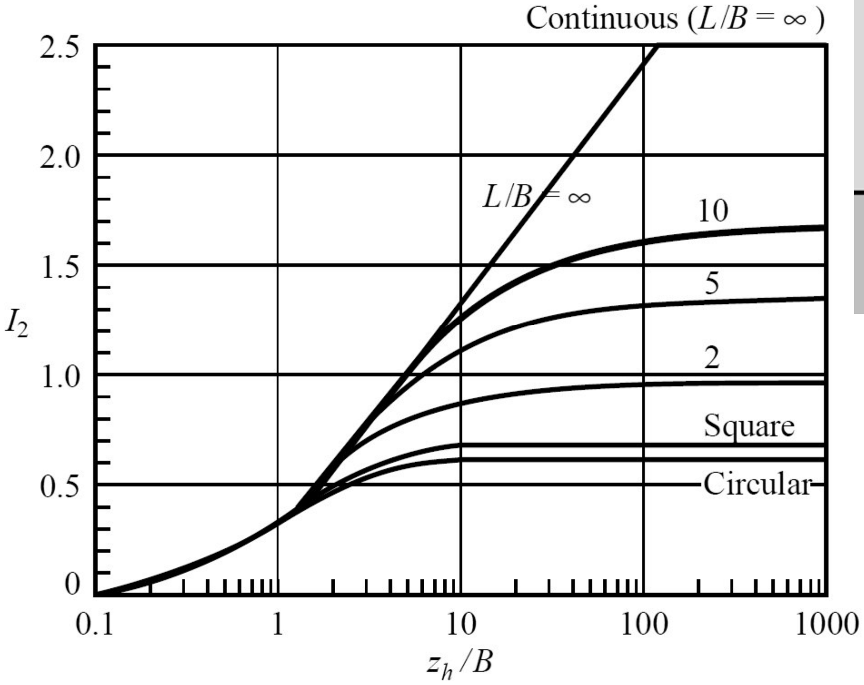

Continuous ($L/B = \infty$)
2.5
2.0
1.5
1.0
0.5
0
I_2
$L/B = \infty$
10
5
2
Square
Circular
0.1
1
10
100
1000
z_h/B

Table 13.2 Equations for computing E_s by making use of SPT and CPT values (in kPa)

Soil	SPT	CPT
Sand (normally consolidated)	$500 (N_{cor} + 15)$ $(35000 \text{ to } 50000) \log N_{cor}$ (U.S.S.R Practice)	$2 \text{ to } 4\, q_c$ $(1 + D_r^2)\, q_c$
Sand (saturated)	$250 (N_{cor} + 15)$	
Sand (overconsolidated)	–	$6 \text{ to } 30\, q_c$
Gravelly sand and gravel	$1200 (N_{cor} + 6)$	
Clayey sand	$320 (N_{cor} + 15)$	$3 \text{ to } 6\, q_c$
Silty sand	$300 (N_{cor} + 6)$	$1 \text{ to } 2\, q_c$
Soft clay	–	$3 \text{ to } 8\, q_c$

For normally consolidated sands,

$E_s = 4\, q_c$ for $q_c < 10$

$E_s = (2q_c + 20)$ for $10 < q_c < 50$

$E_s = 120$ for $q_c > 50$

For overconsolidated sands

$E_s = 5\, q_c$ for $q_c < 50$

$E_s = 250$ for $q_c > 50$

E_s and q_c are expressed in MPa.