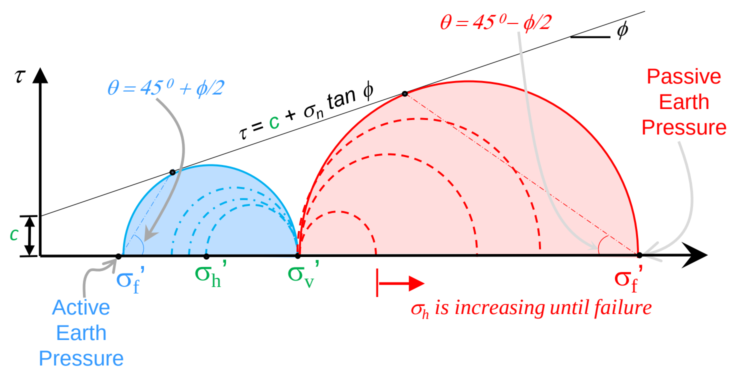
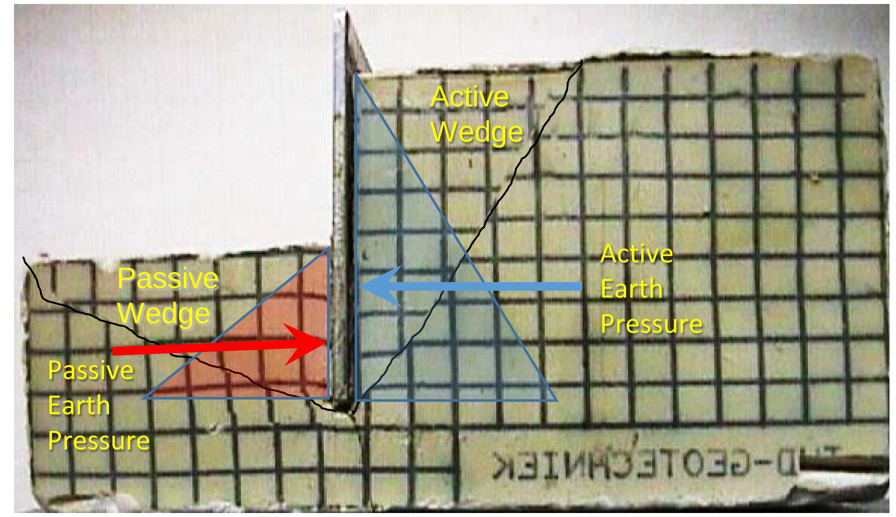
# Rankine's Earth Pressure Method for (c- ϕ) Soil

## Rankine's Active and Passive Earth Pressure in (c- f) Soil




## Rankine's Active Earth Pressure in ( f) Soil with inclined backfill

$$K_a = \cos\alpha \cdot \frac{\cos\alpha - \sqrt{(\cos^2\alpha - \cos^2\phi)}}{\cos\alpha + \sqrt{(\cos^2\alpha - \cos^2\phi)}}$$

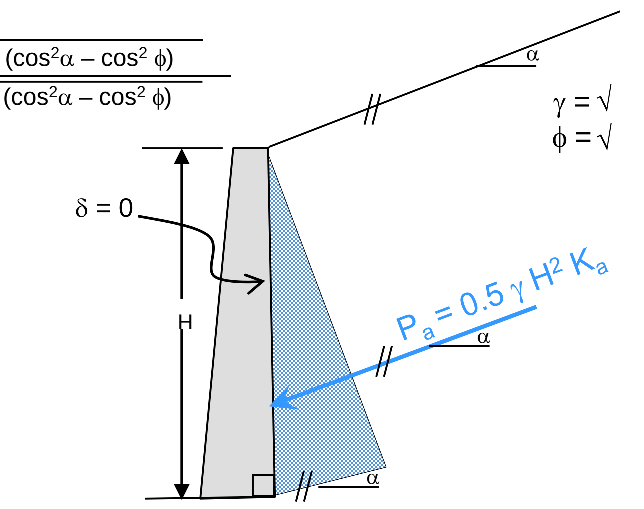



Fig.1

### Active Earth Pressure

$$\sigma_f' = \sigma_V' \tan^2\left(45^o - \frac{\phi}{2}\right) + 2\,c\,\tan\left(45^o - \frac{\phi}{2}\right)$$

Or

$$\sigma_f' = \sigma_V' K_a - 2\,c\,\sqrt{K_a}$$

$$K_a = \tan^2\left(45^o - \frac{\phi}{2}\right) = \frac{1 - \sin\phi}{1 + \sin\phi}$$

*Coefficient of active earth pressure*

### Passive Earth Pressure

$$\sigma_f' = \sigma_V' \tan^2\left(45^o + \frac{\phi}{2}\right) + 2\,c\,\tan\left(45^o + \frac{\phi}{2}\right)$$

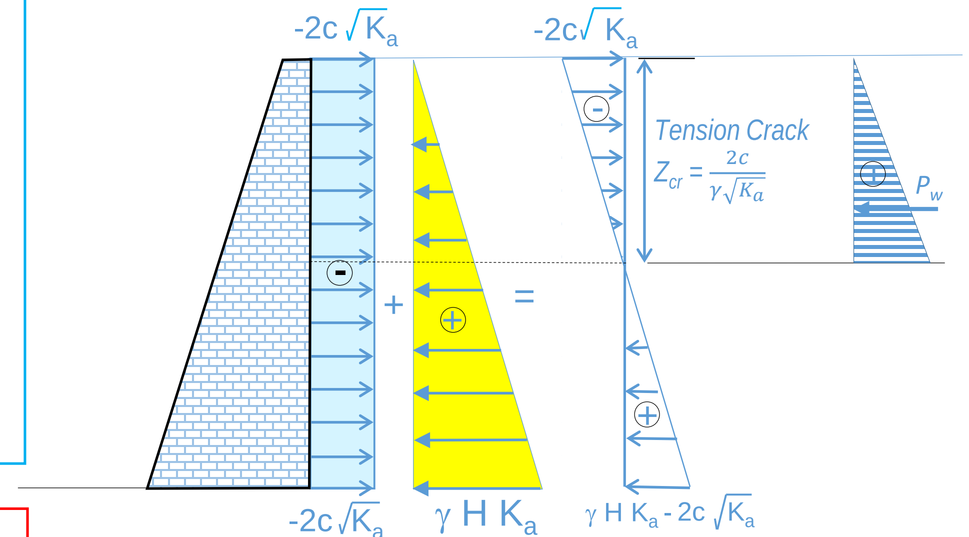
Or

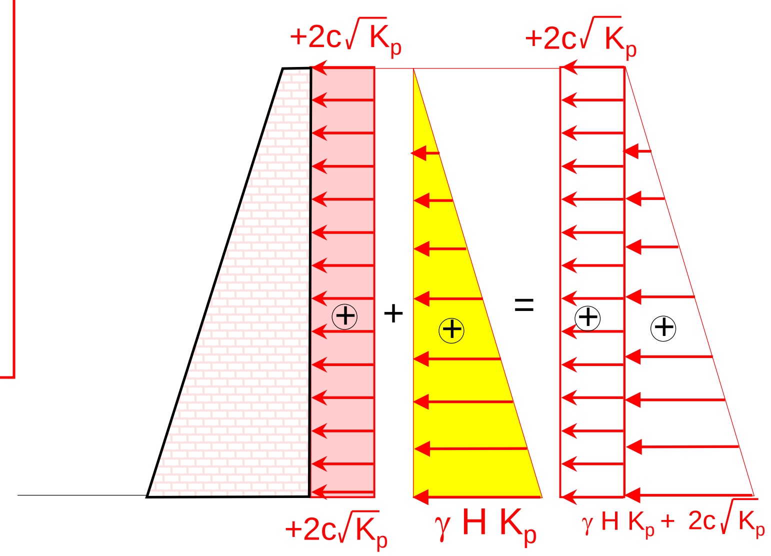
$$\sigma_f' = \sigma_V' K_p + 2\,c\,\sqrt{K_p}$$

$$K_p = \tan^2\left(45^o + \frac{\phi}{2}\right) = \frac{1 + \sin\phi}{1 - \sin\phi}$$

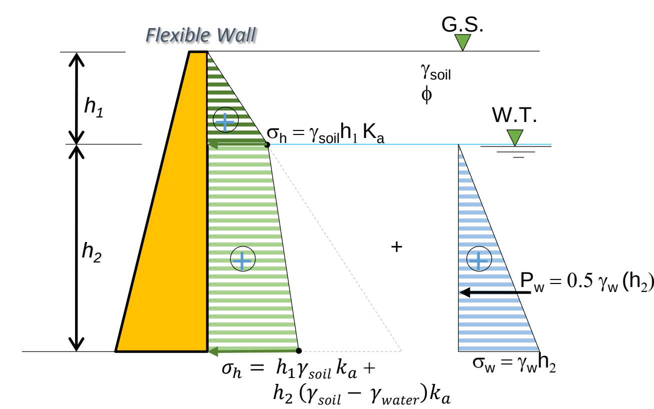
*Coefficient of passive earth pressure*

## Effect of Cohesion of the Rankine's Active and Passive Earth Pressure

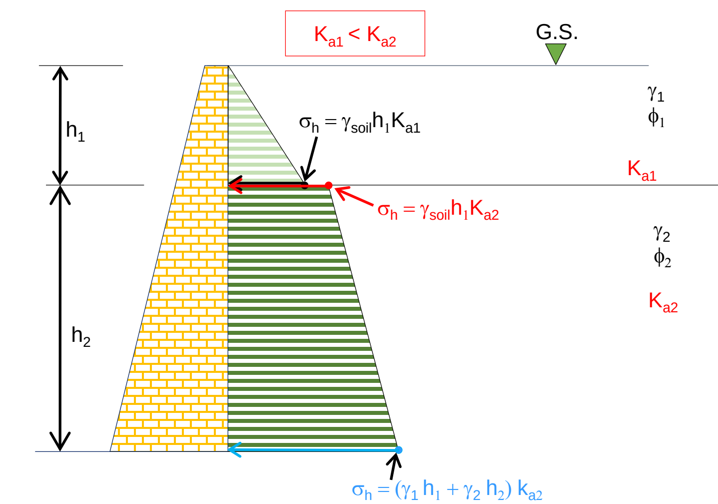


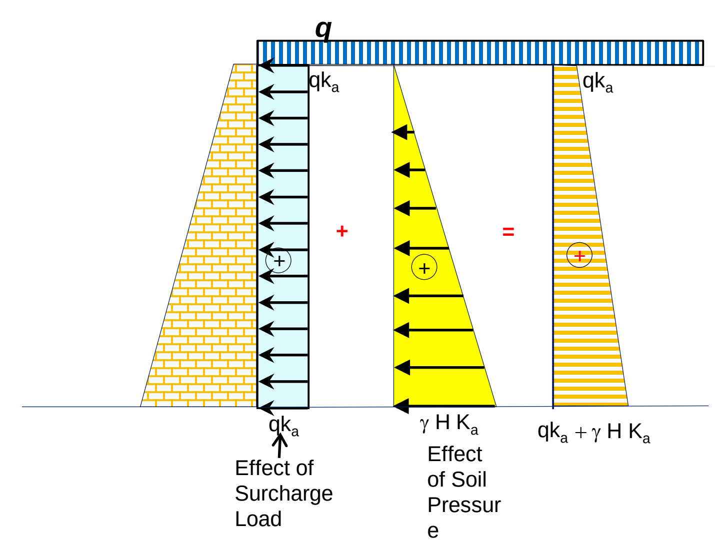

## Rankine's Active Earth Pressure in f - Soil & Water Table

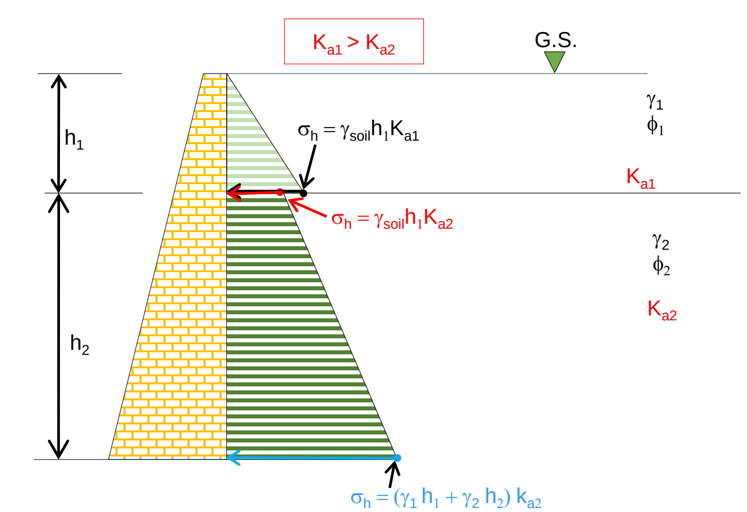



## Effect of Two Soil Layers on Active Earth Pressure




## Effect of Surcharge (q) Load on Active Earth Pressure




*Kamal Tawfiq*

# Active Earth Pressure in $\phi$ – Soil

## Example -1

**Given**:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ($\gamma$) = 115 pcf
- Angle of soil friction ($\phi$) = 30$^o$
- Assume wall to be smooth

**Find**:

- Lateral force $P_a$ acting on the wall

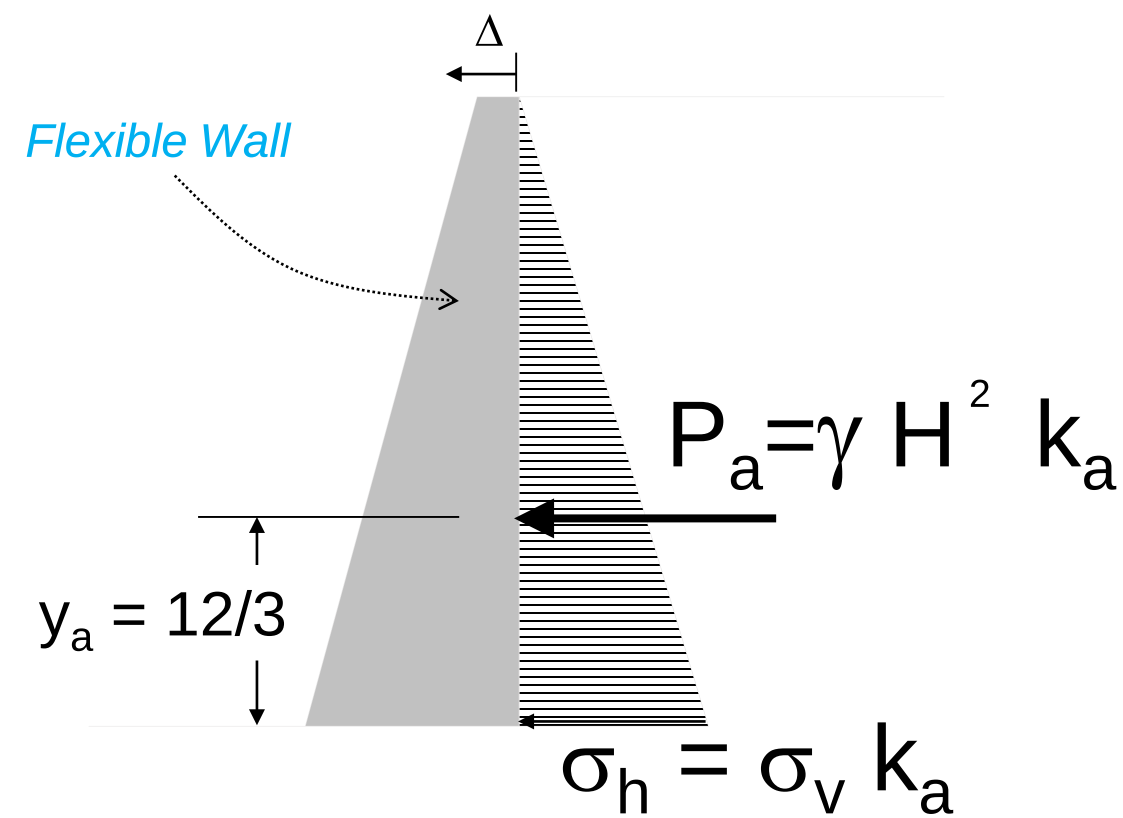
**Solution**:

$\sigma_h = \sigma_v k_a$

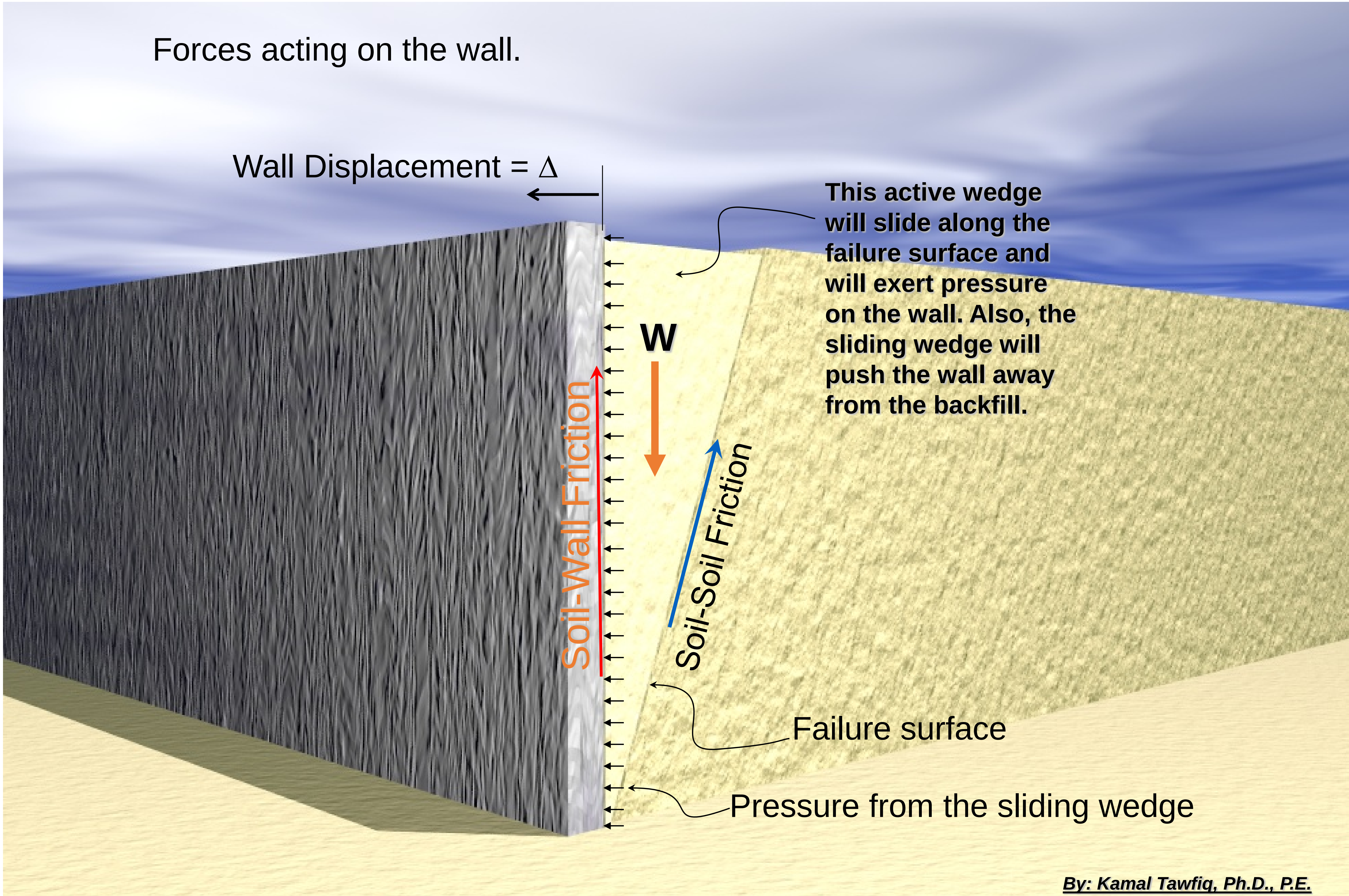
$P_a = \gamma H^2 k_a$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

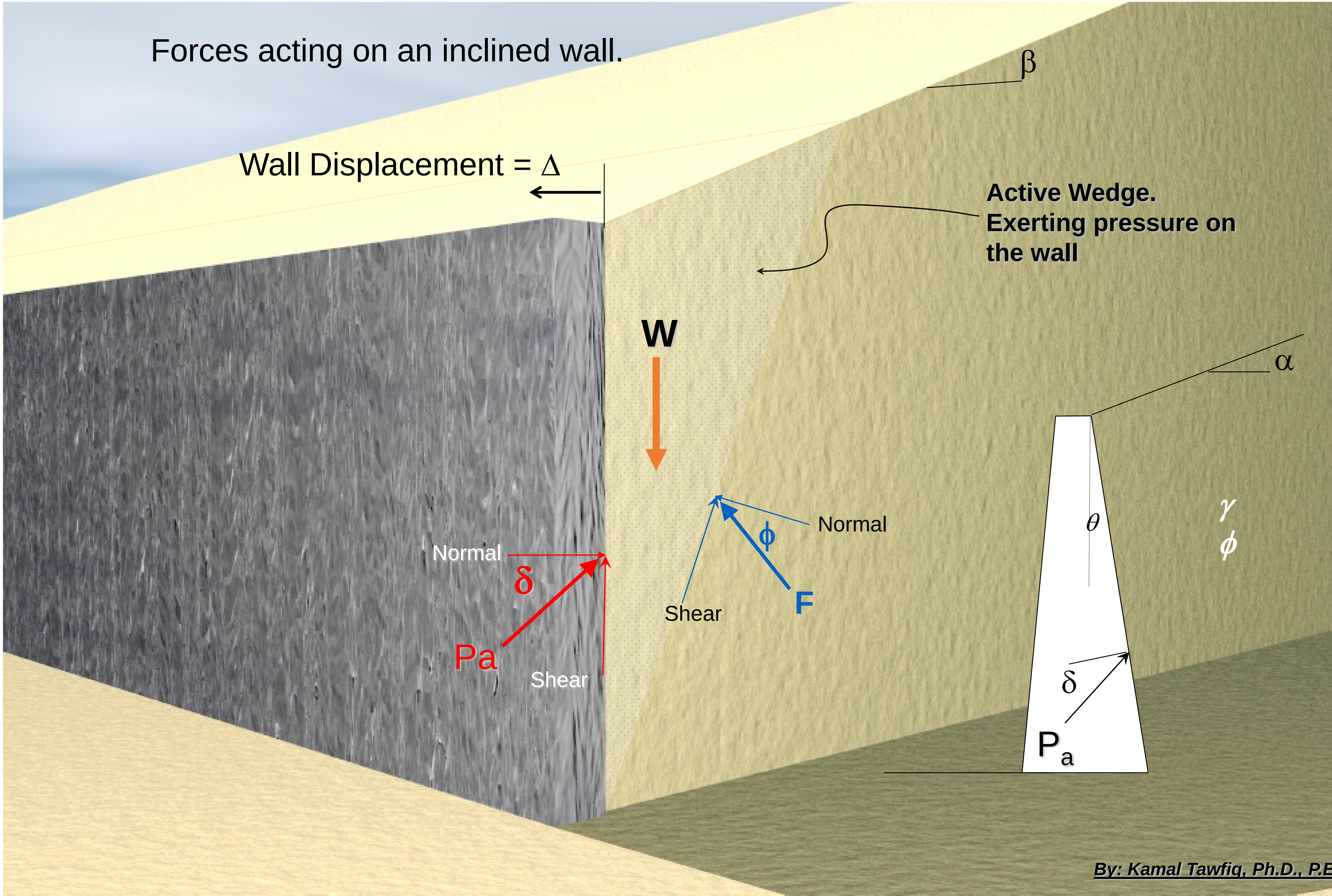
$P_a = 115 \times 12^2 \times 0.5$



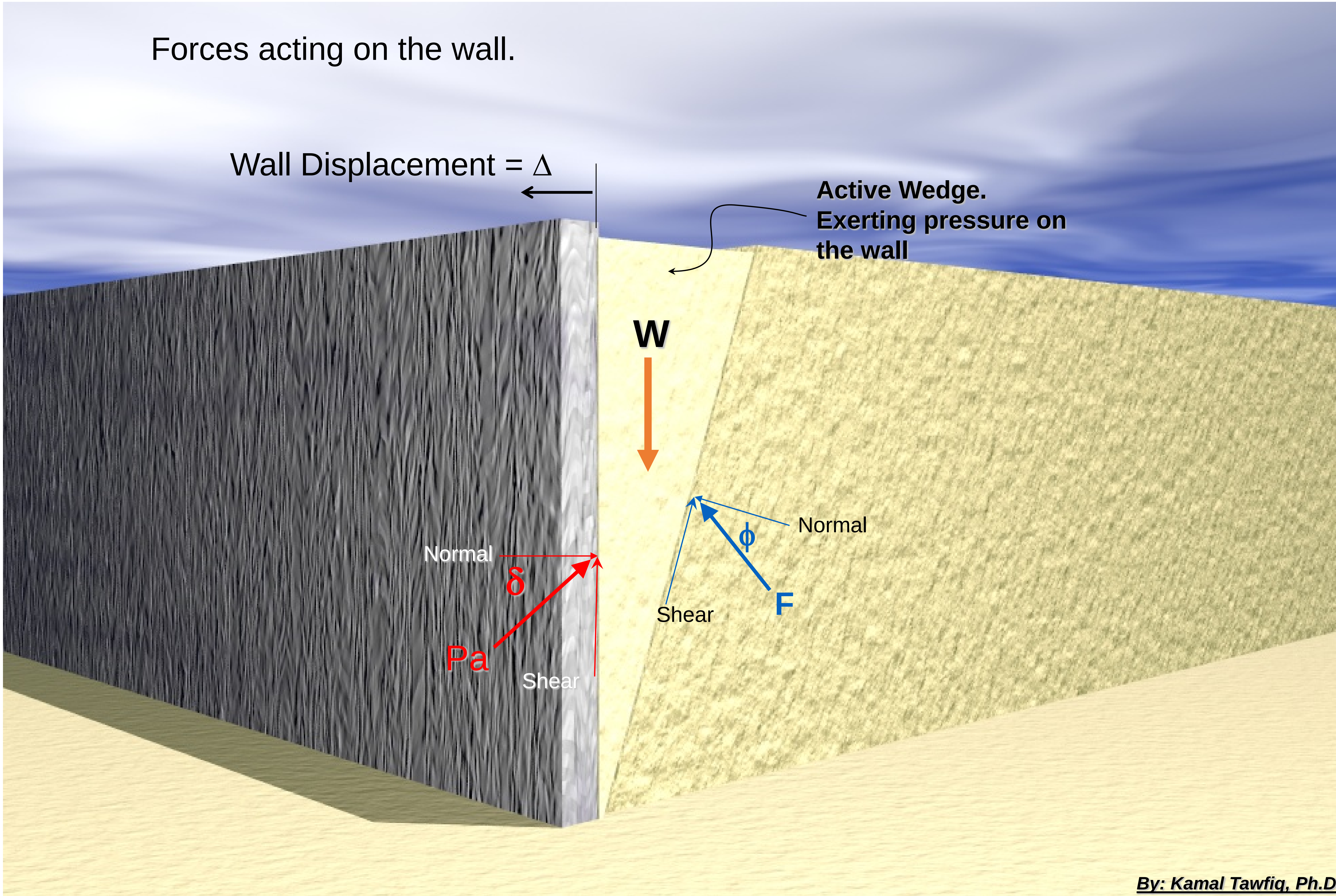
# Coulomb Earth Pressure Method



# Coulomb Earth Pressure Method



# Coulomb Earth Pressure Method

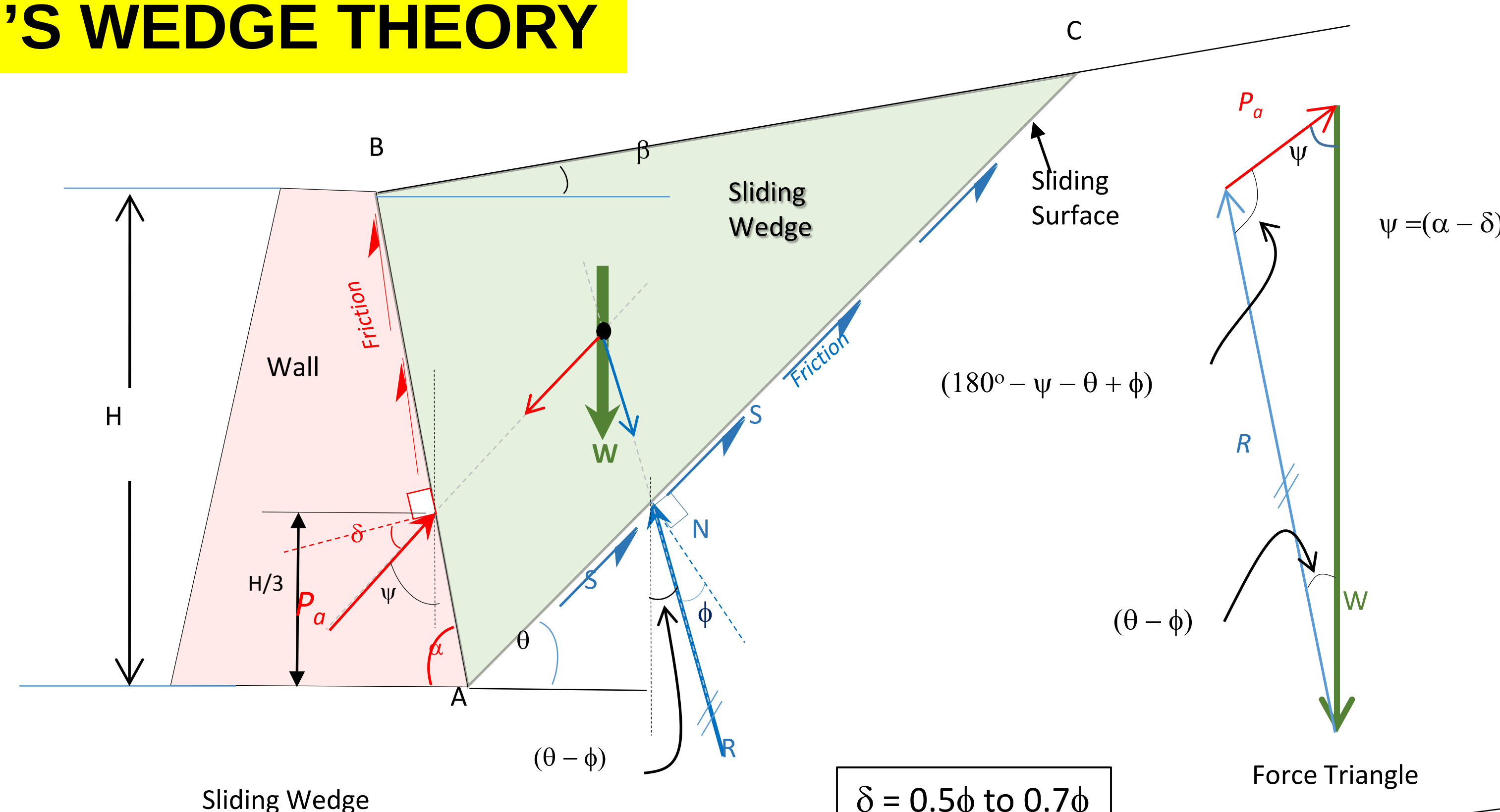



*By: Kamal Tawfiq, Ph.D., P.E.*

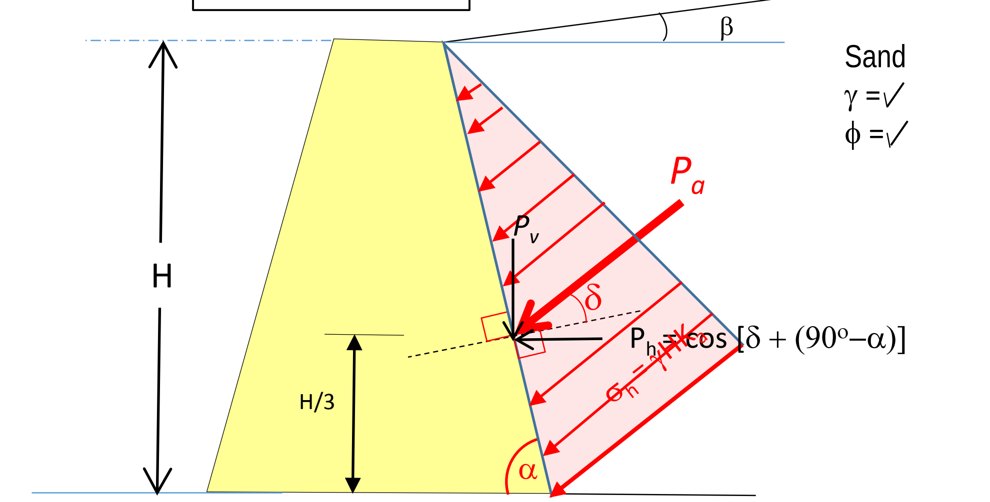
# COULOMB'S WEDGE THEORY

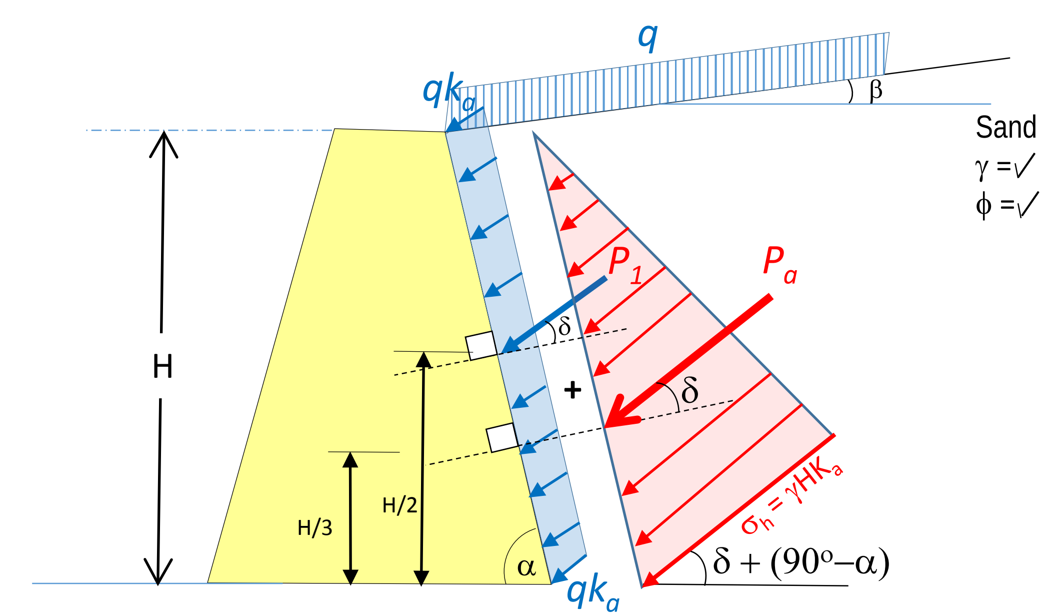
**W** = weight of the soil wedge
**R** = resultant of the shear and normal forces on the failure surface BC
$\mathbf{P_a}$ = the active force per unit length of the wall. The direction of Pa is inclined at an angle δ to the normal drawn and the face of the wall that supports the soil

δ = the angle of friction between the soil and the wall




δ = 0.5ϕ to 0.7ϕ




*W* = g (area of wedge *ABC*)

From the triangles of forces,

$$\frac{P_a}{\sin(\theta-\emptyset)} = \frac{W}{\sin(180^o-\psi-\theta+\phi)}$$

$$P_a = \frac{W\sin(\theta-\emptyset)}{\sin(180^o-\psi-\theta+\phi)}$$

Substituting for *W*,

$$P_a = \frac{1}{2}\cdot\frac{\gamma H^2}{sin^2\,\alpha}\cdot\frac{\sin\,(\theta\;-\emptyset)}{\sin(180^o-\psi-\theta+\phi)}\cdot\frac{\sin(\theta+\alpha)\,.\sin(\alpha+\beta)}{\sin(\theta-\beta)}$$

The maximum value of *Pa* is obtained by equating the first derivative of *Pa* with respect to θ to zero; or

$(\partial P_a)/\partial\theta = 0$, and substituting the corresponding value of θ.

The value of $P_a$ so obtained is written as

$$P_a = \frac{1}{2}\,.\gamma H^2.\frac{sin^2(\alpha+\phi)}{sin^2\alpha\;\;sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$$

This is usually written as

$$P_a = \tfrac{1}{2}\,.\gamma H^2.\;K_a$$

Where $K_a$ being the coefficient of active earth pressur $= \dfrac{sin^2(\alpha+\phi)}{sin^2\alpha\;\;sin(\alpha-\delta)\left[1+\sqrt{\dfrac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\sin(\alpha-\delta)\sin(\alpha+\beta)}}\right]^2}$

Kamal Tawfiq

# Coulomb's Earth Pressure

$\phi = \surd$

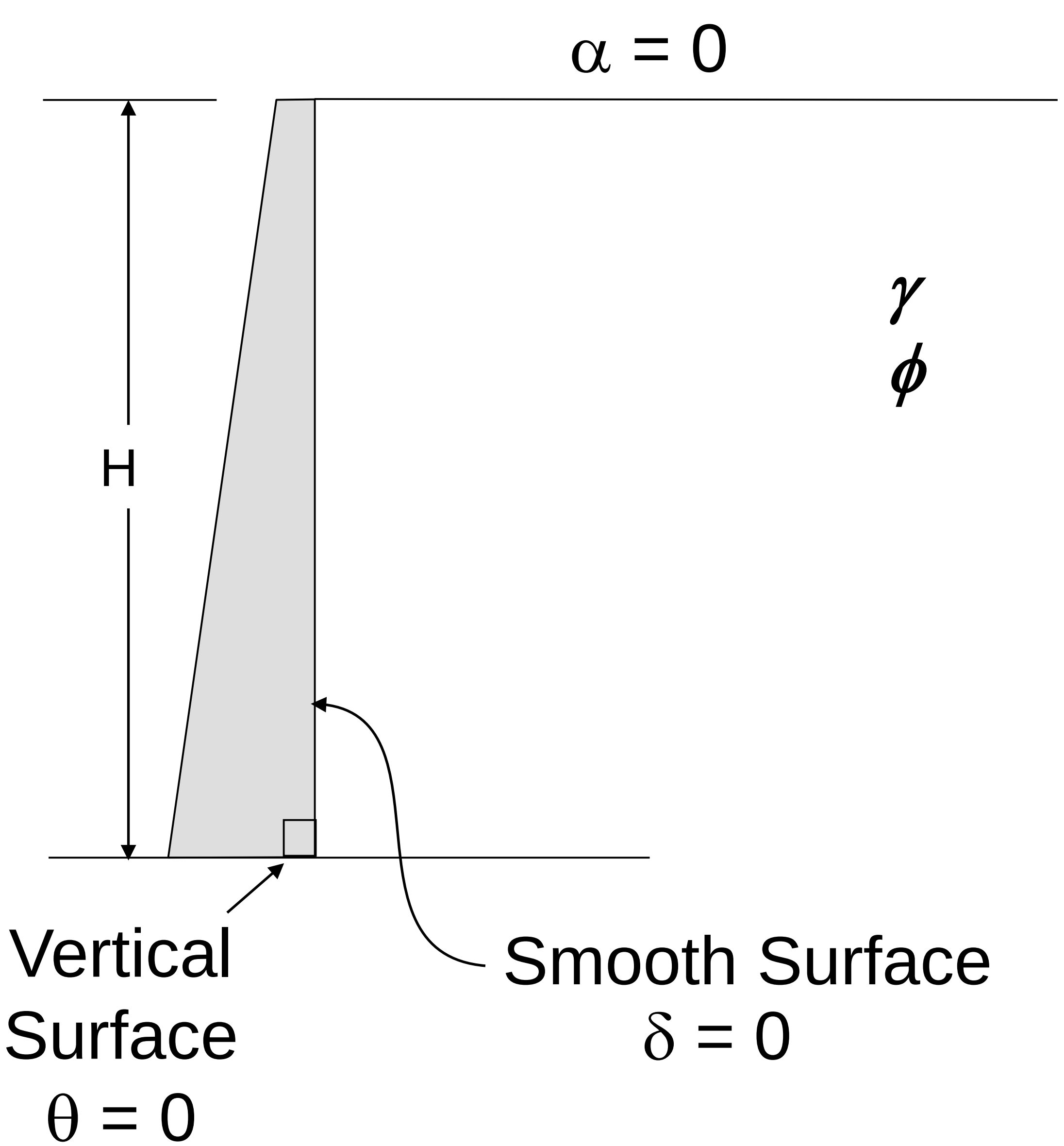
$\theta = 0$
$\delta = 0$
$\alpha = 0$

$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \cos(\delta - \theta)\left[1 + \sqrt{\frac{\sin(\delta + \phi)\sin(\phi - \alpha)}{\cos(\delta + \theta)\cos(\theta - \alpha)}}\right]^2}$$

Under the given wall and backfill conditions, $K_a$ of Coulomb's active earth pressure becomes equivalent to $K_a$ of Rankine's

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$P_a = \tfrac{1}{2} K_a \gamma H^2$$




By: Kamal Tawfiq, Ph.D., P.E.

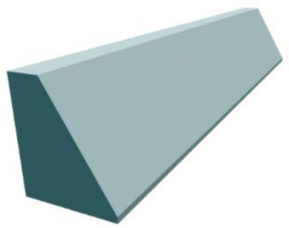
# Earth Retaining Walls

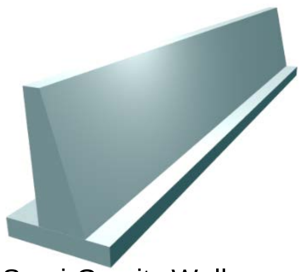


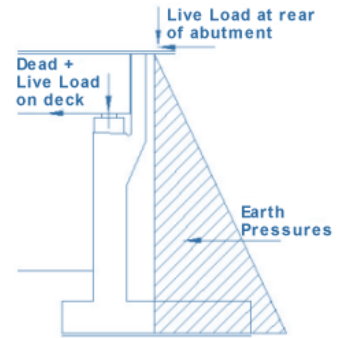

# Design of Retaining Wall

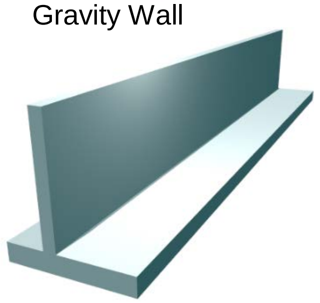
## Types of Earth Retaining Walls

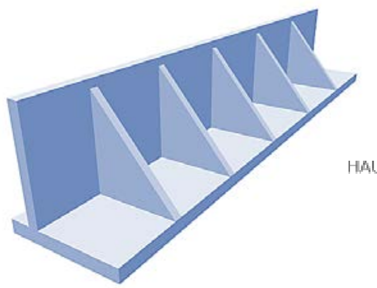
### I- Permanent Walls

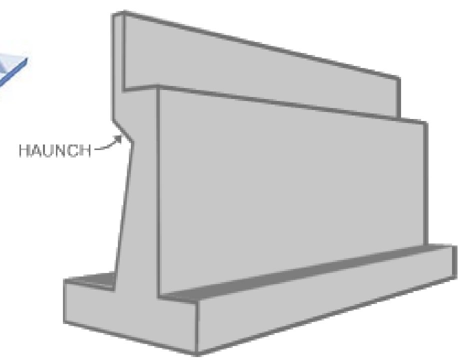

Gravity Wall

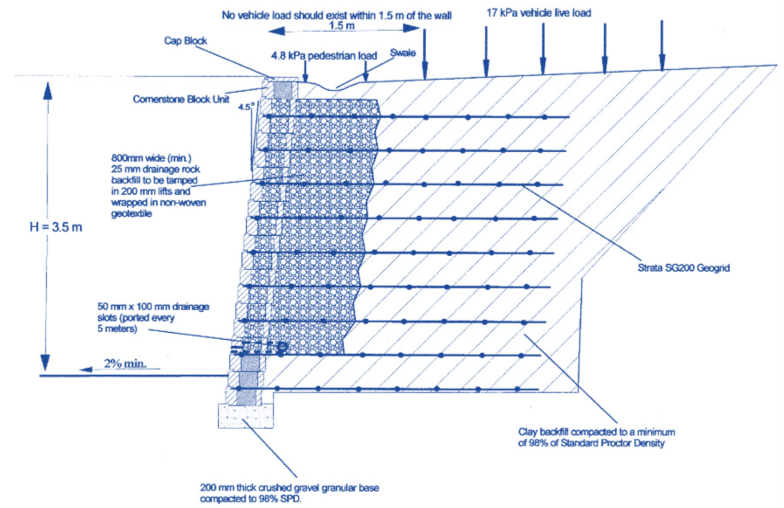

Semi-Gravity Wall





Cantilever Wall


Counterfort wall



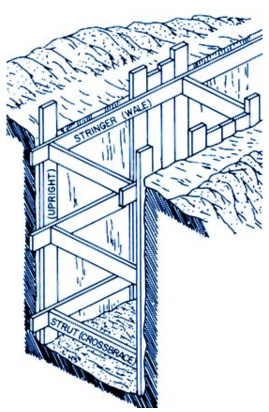



Segmental Walls


MSE Walls



### II- Temporary Walls



Braced cuts


Sheet pile wall


Sheet pile wall

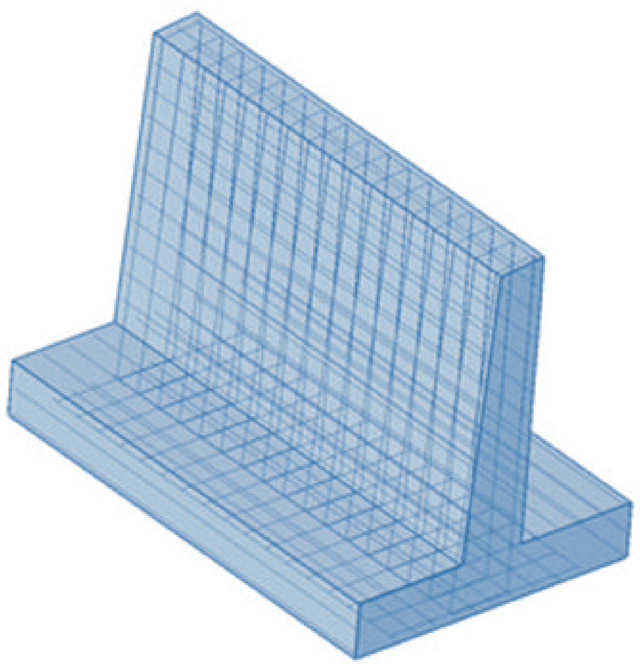
By Kamal Tawfiq, Ph.D., P.E

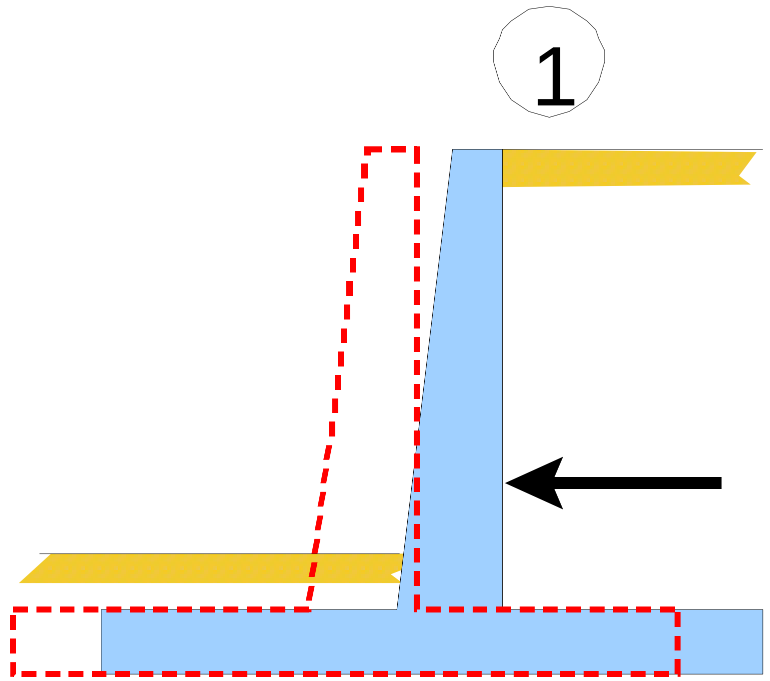
# Design of Retaining Wall

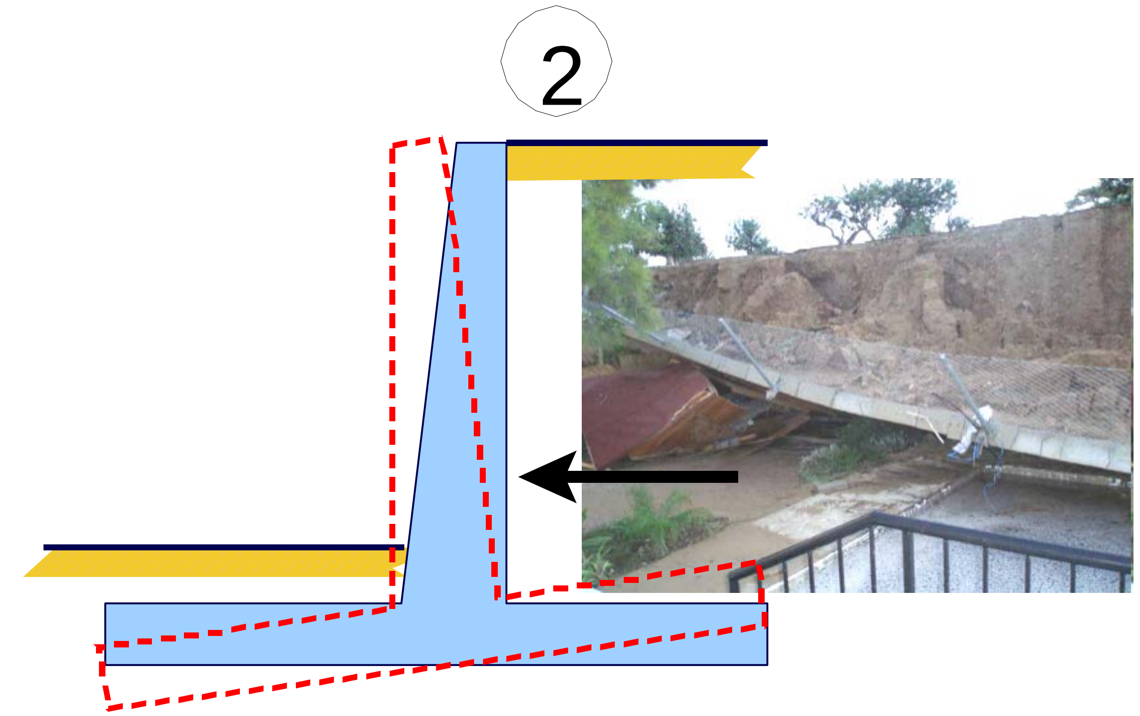
1- External Stability
2- Internal Stability

## 1. External Stability

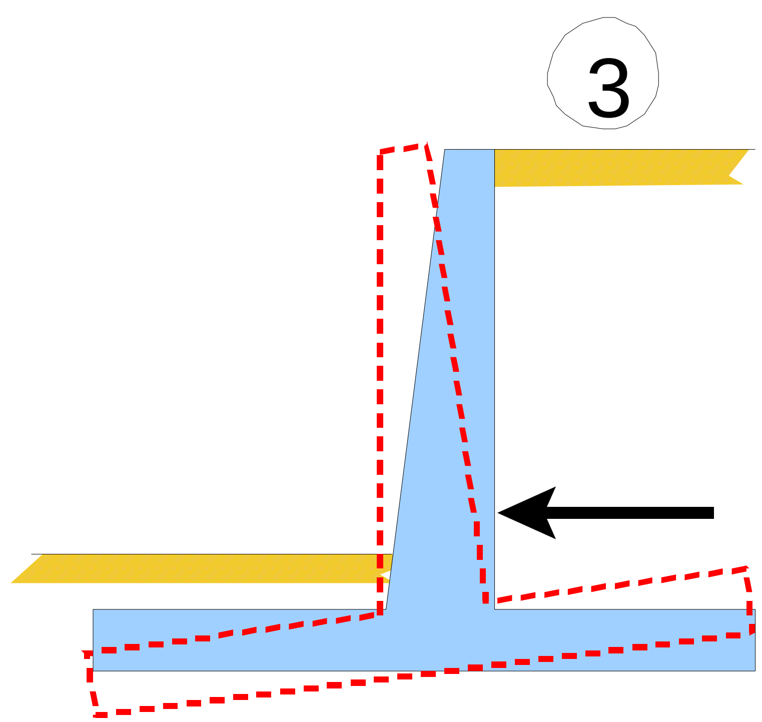
*1- Sliding*
*2- Overturning*
*3- Settlement*
*4- Overall Failure*

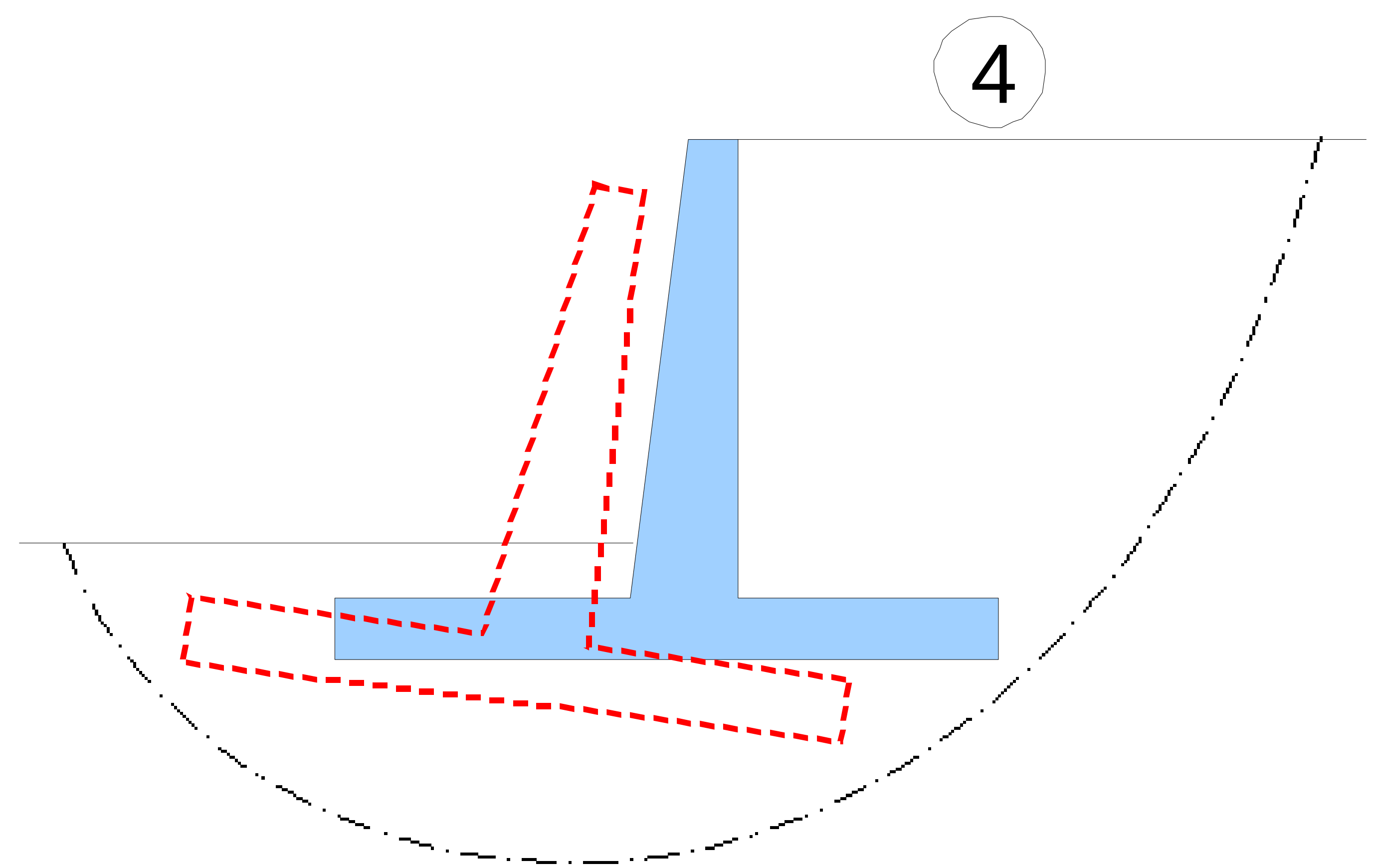




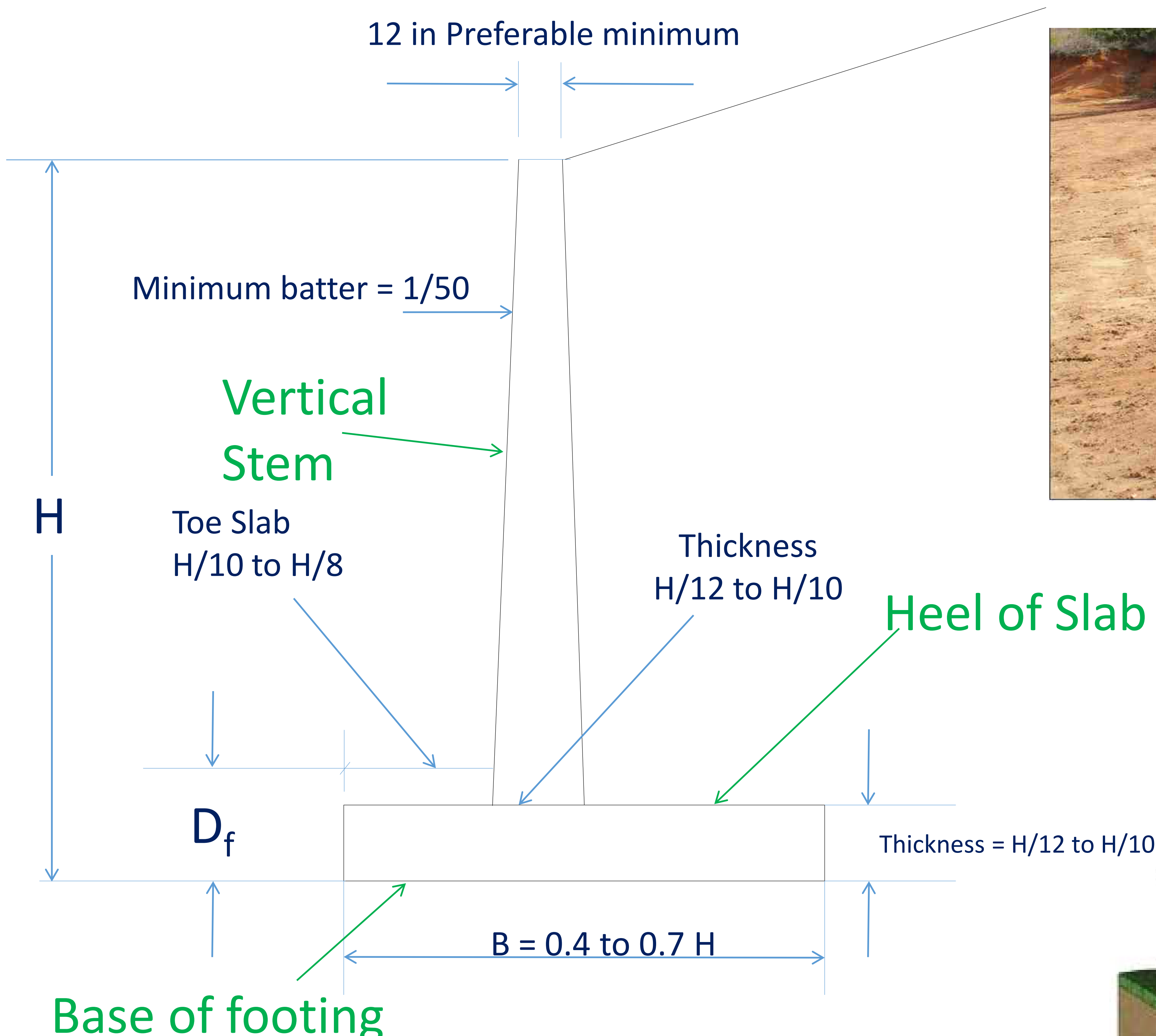



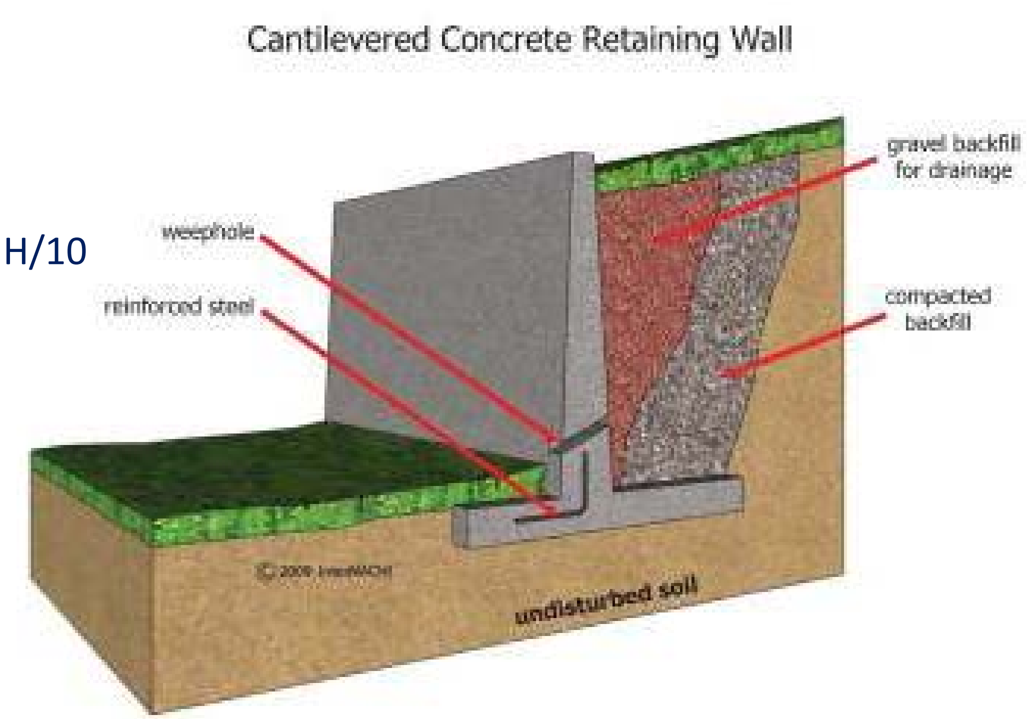

Internal Stability
Steel Reinforcement
and Thicknesses



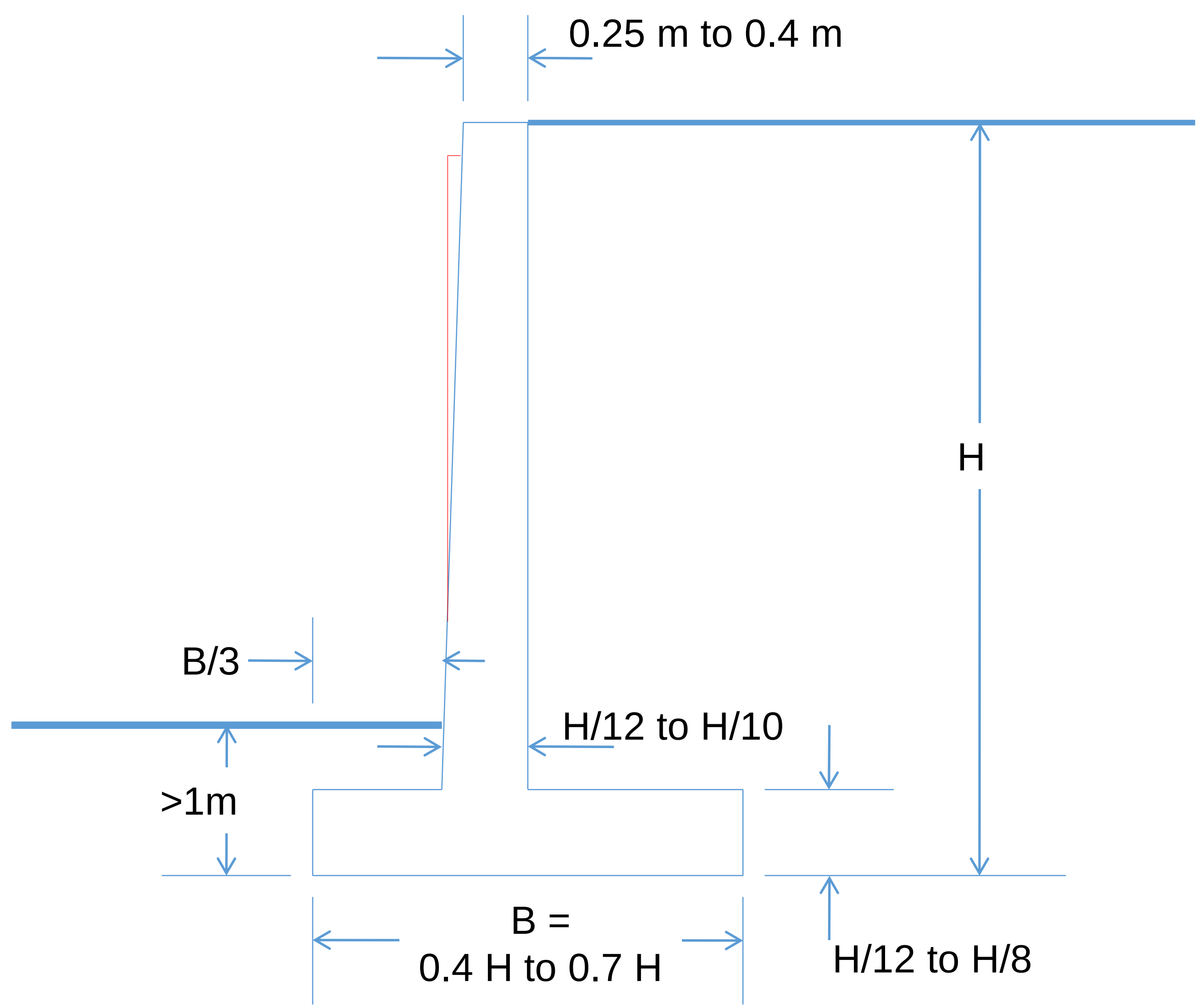



# Common Proportions  of Cantilever Wall

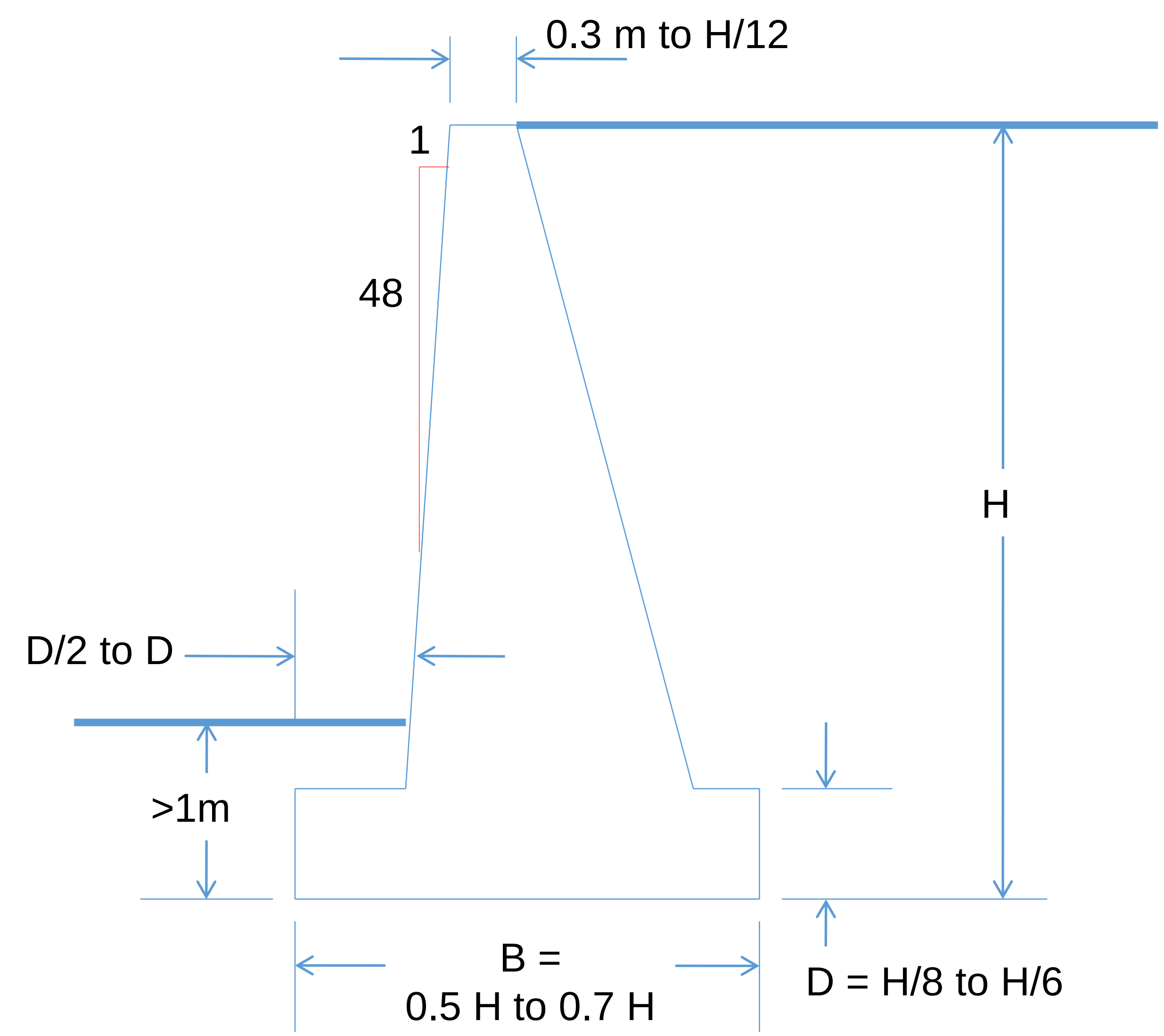


# Approximate Dimensions



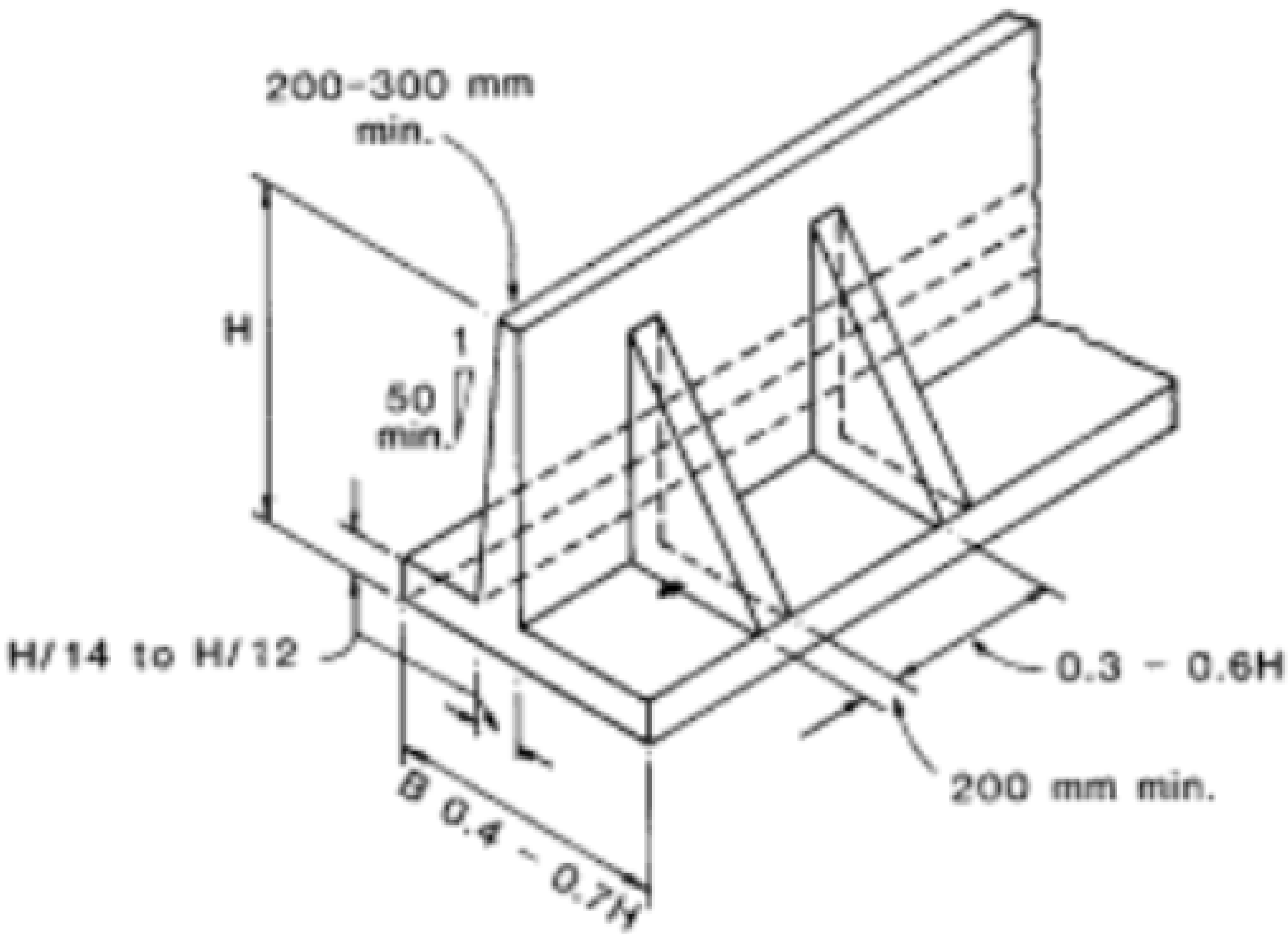

Cantilever Retaining Wall




Gravity Retaining Wall

Kamal Tawfiq, 2011

# Counterfort Retaining Wall




# Internal Stability

**Structural Design**
Steel Reinforcement and Thicknesses — Structural Design

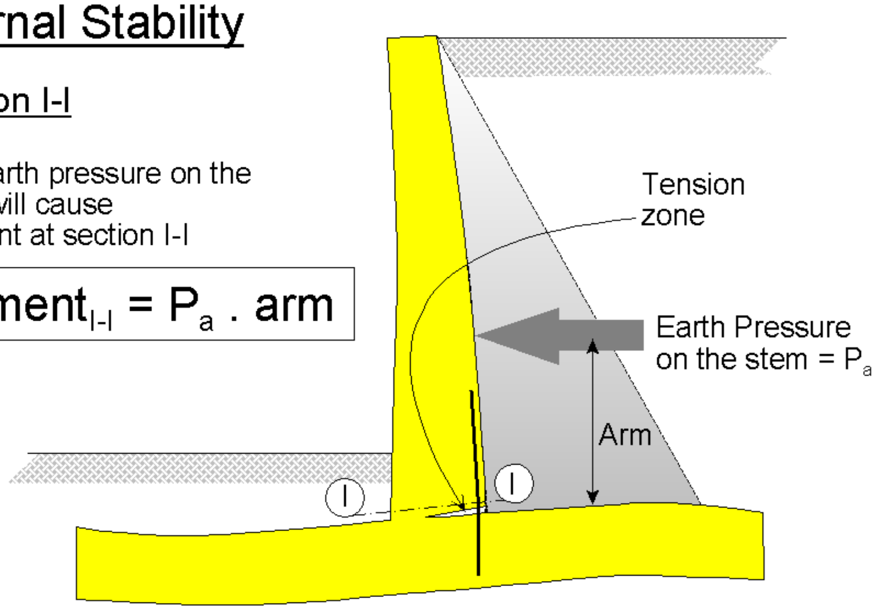
2-

## Internal Stability

Section I-I

The earth pressure on the stem will cause moment at section I-I

$\text{Moment}_{I\text{-}I} = P_a \cdot \text{arm}$

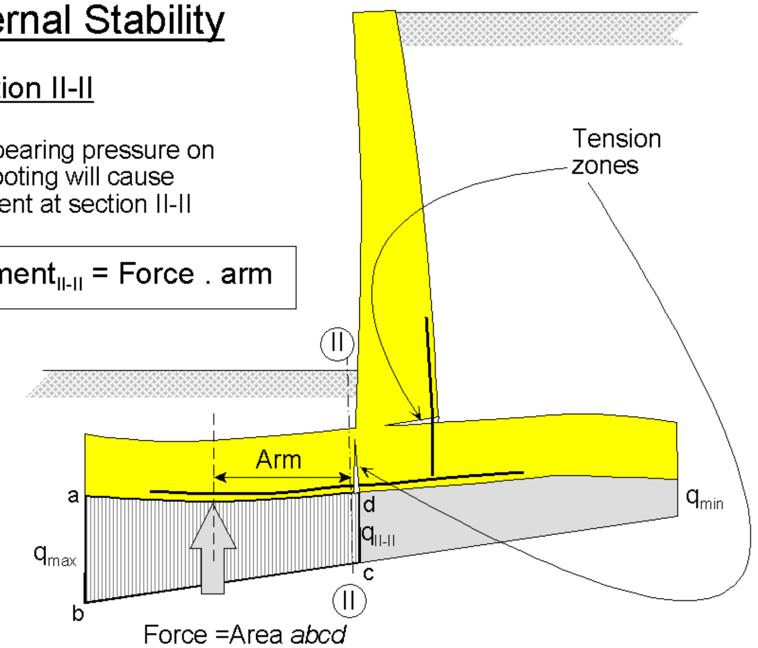



## Internal Stability

Section II-II

The bearing pressure on the footing will cause moment at section II-II

$\text{Moment}_{II\text{-}II} = \text{Force} \cdot \text{arm}$

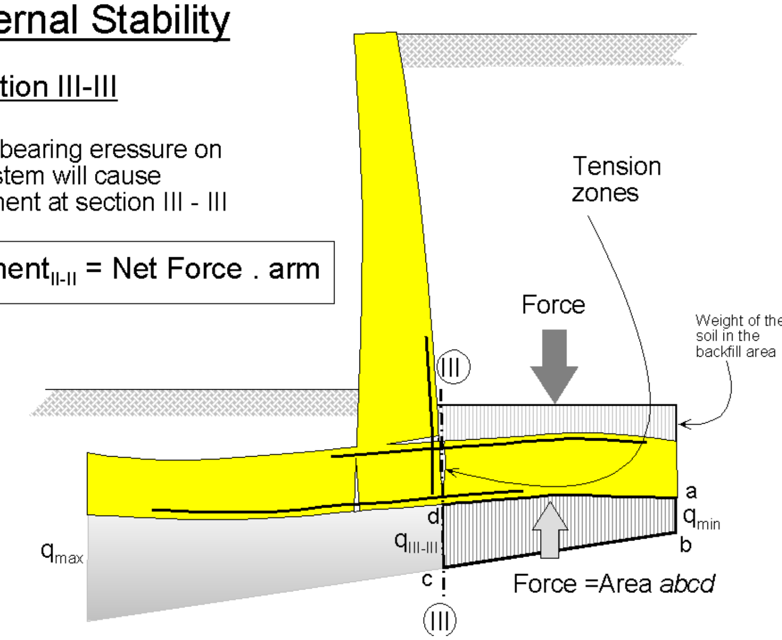


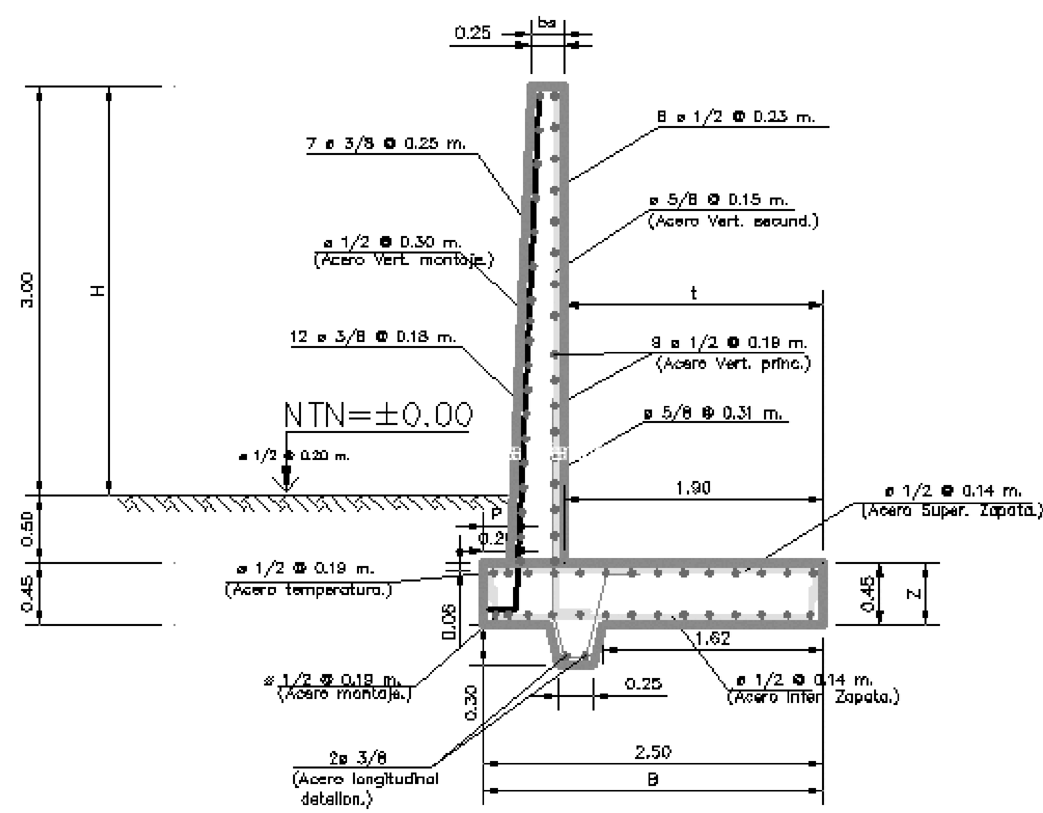

## Internal Stability

Section III-III

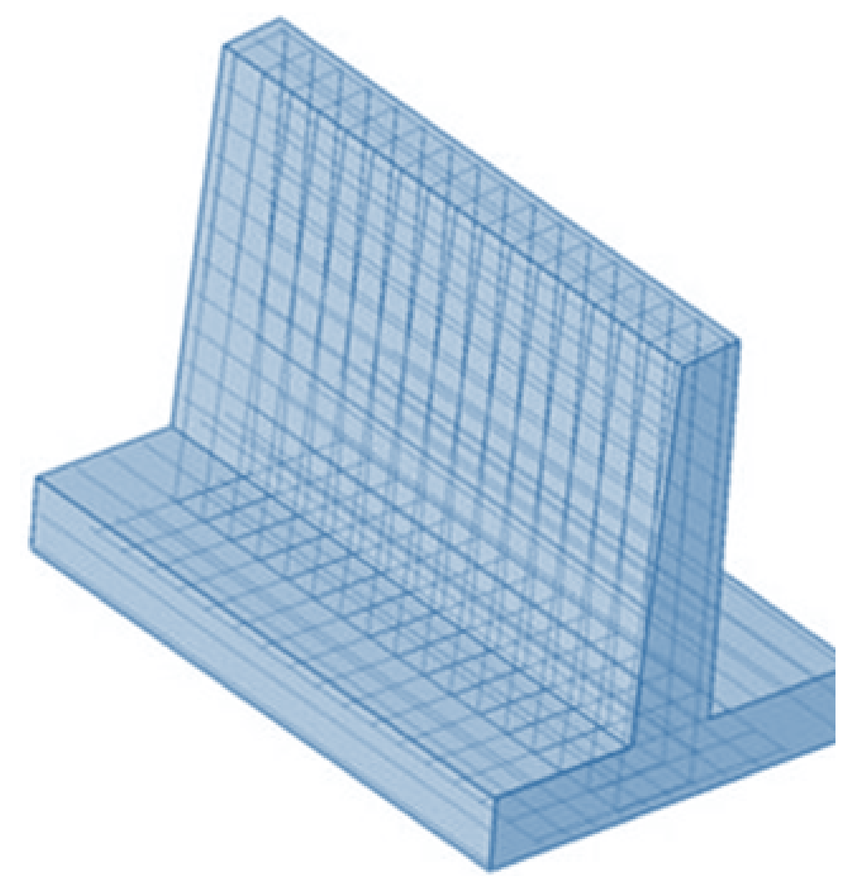
The bearing eressure on the stem will cause moment at section III - III

$\text{Moment}_{II\text{-}II} = \text{Net Force} \cdot \text{arm}$



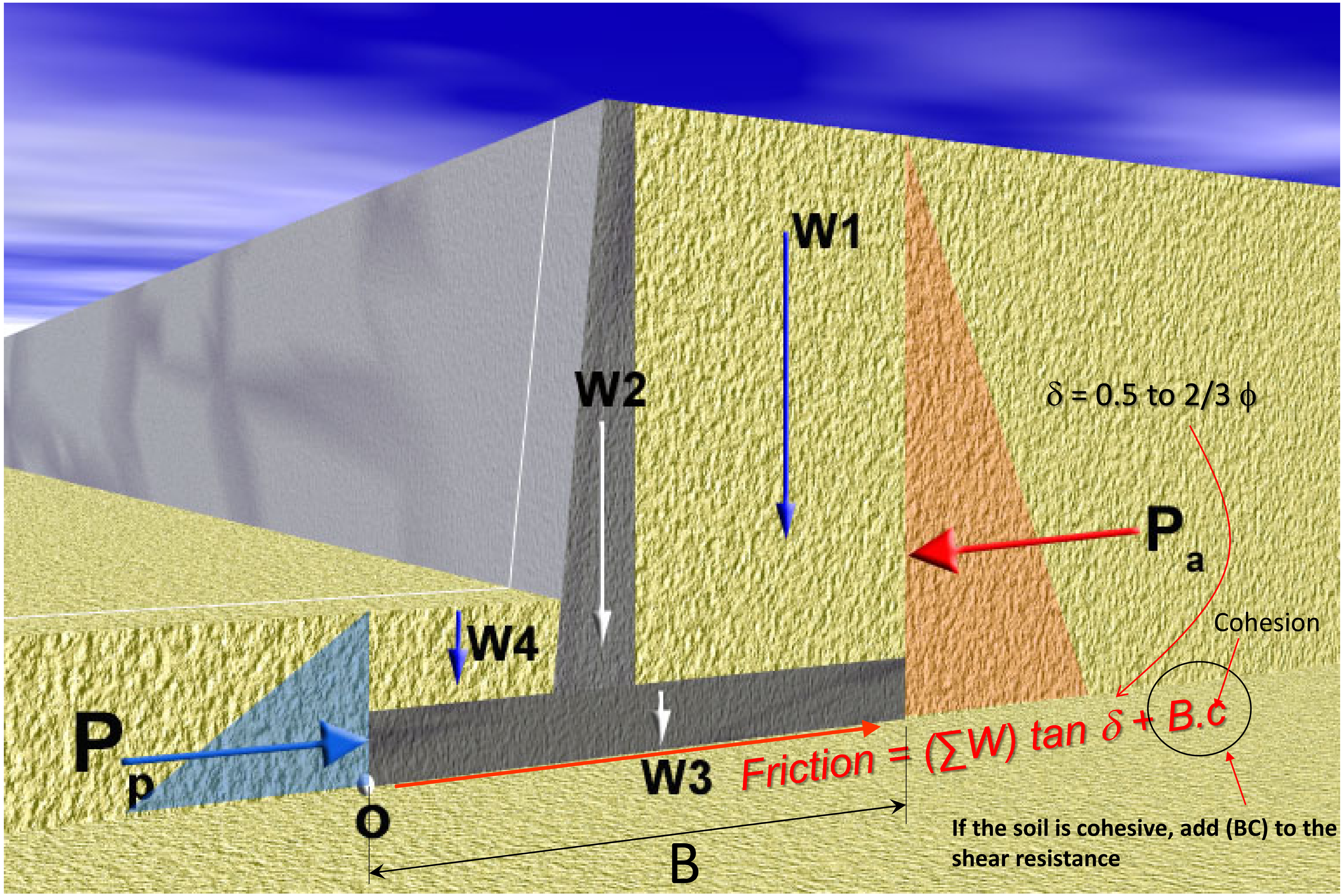

DETALLES DE MURO DE CONTENCION



# I. External Stability

*1- Sliding*

$$\text{Factor of Safety Against Sliding} = \frac{\text{Resisting Force}}{\text{Driving Force}} = \frac{F_R}{F_D}$$



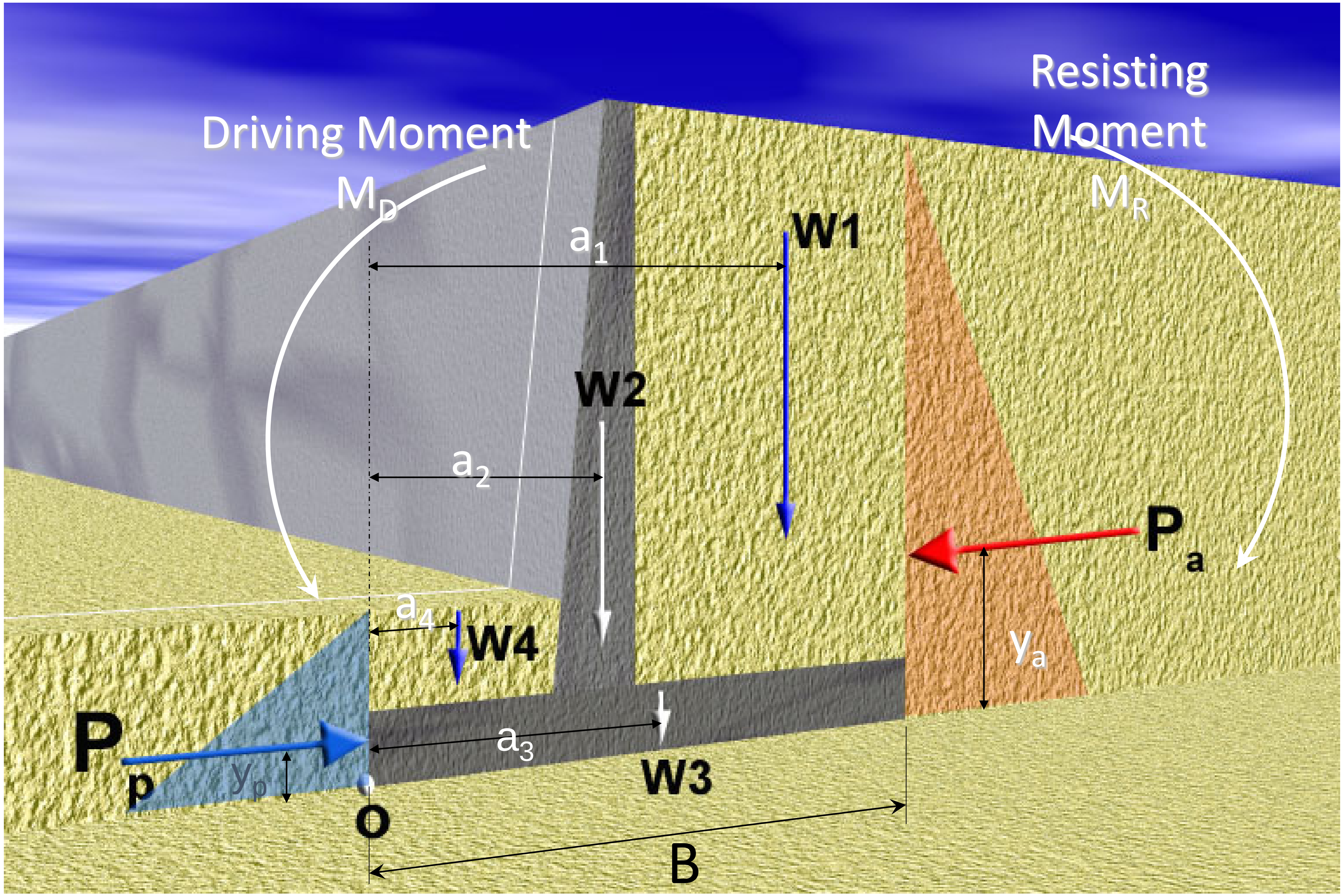

$$F_D = P_a \qquad F_R = P_p + \text{Friction}$$

By: Kamal Tawfiq

# I. External Stability

*2- Overturning*

$$\text{Factor of Safety Against Sliding} = \frac{\text{Resisting Moment}}{\text{Driving Moment}} = \frac{M_R}{M_D}$$

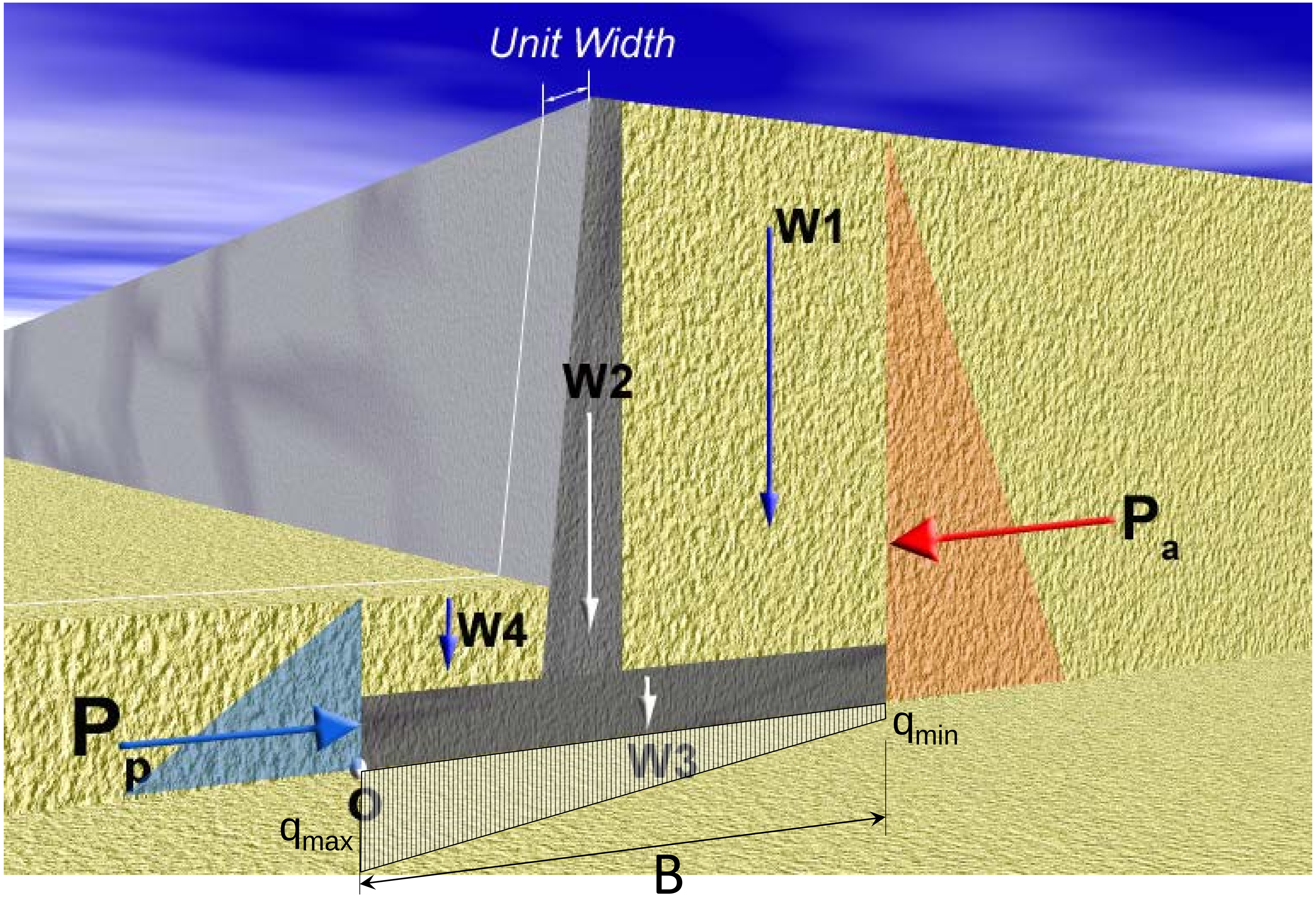



Moment About o $\quad M_D = P_a \cdot y_a \quad M_R = P_p . y_p + W_1\, a_1 + W_2\, a_2 + W_3\, a_3 + W_4\, a_4$

By: Kamal Tawfiq

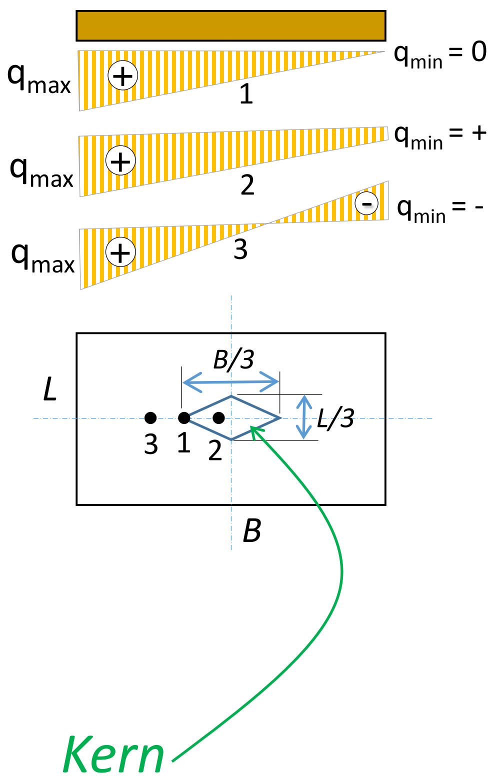
# I. External Stability

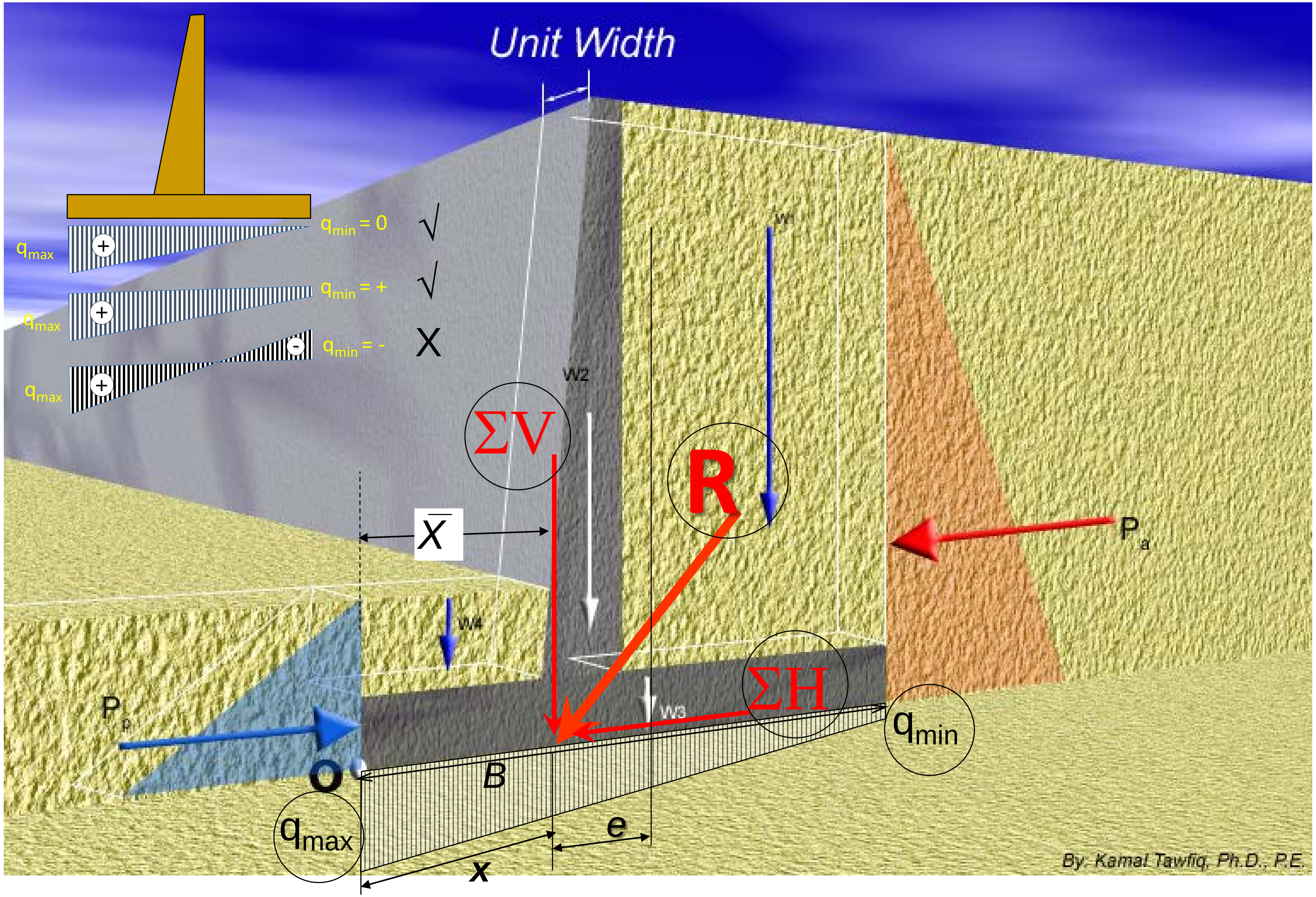
*3- Bearing Capacity Failure*



# 3- Check for Bearing Capacity Failure

Factor of Safety Against Bearing Capacity Failure = $\dfrac{q_{all}}{q_{max}}$




ΣV = sum of all vertical loads
ΣH = sum of all horizontal loads

$$R = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$$

$$M_{net} = \Sigma M_R - \Sigma M_D$$

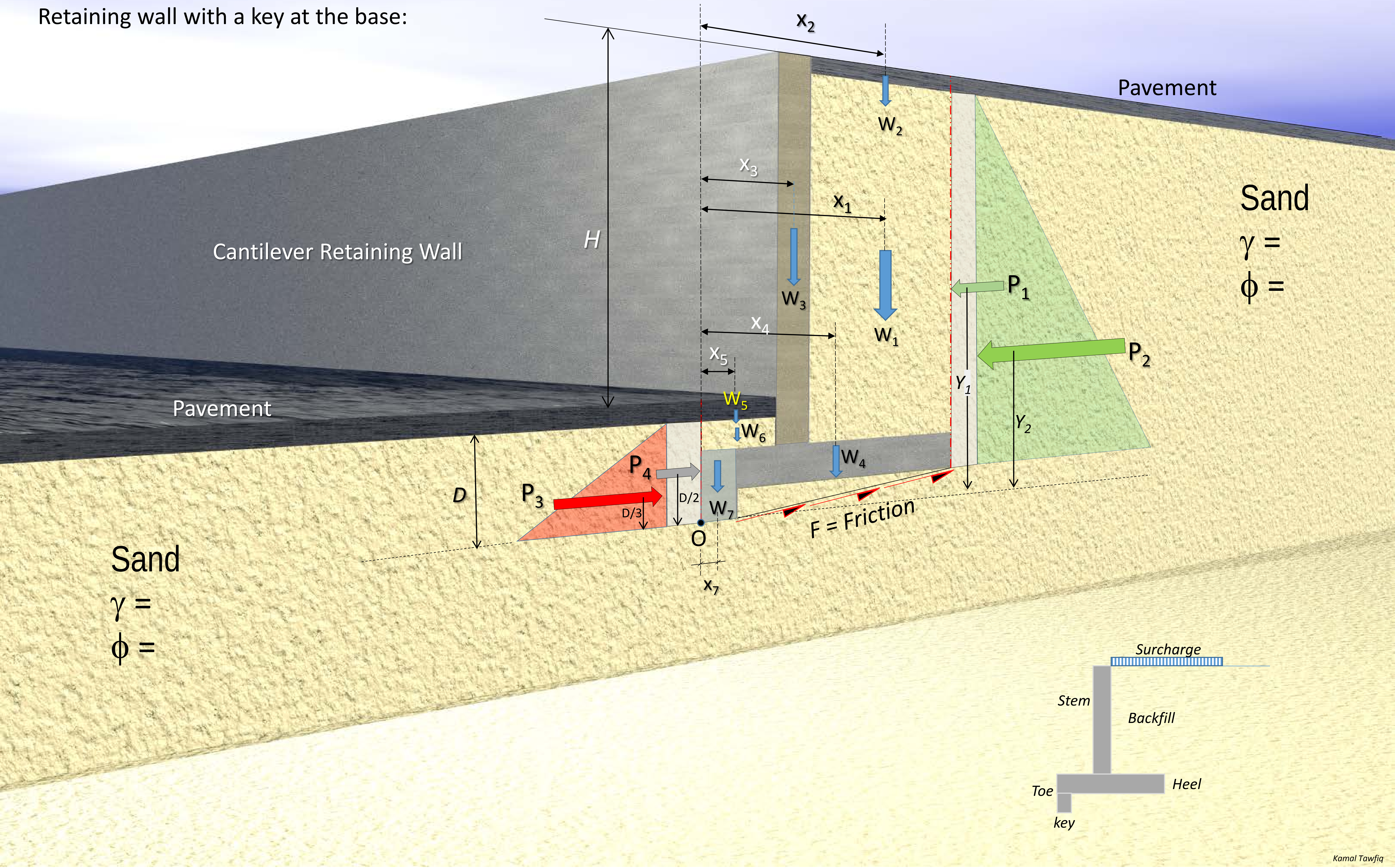
$$\Sigma V \, \bar{X} = M_{net}$$

$$e = \frac{B}{2} - \bar{X}$$

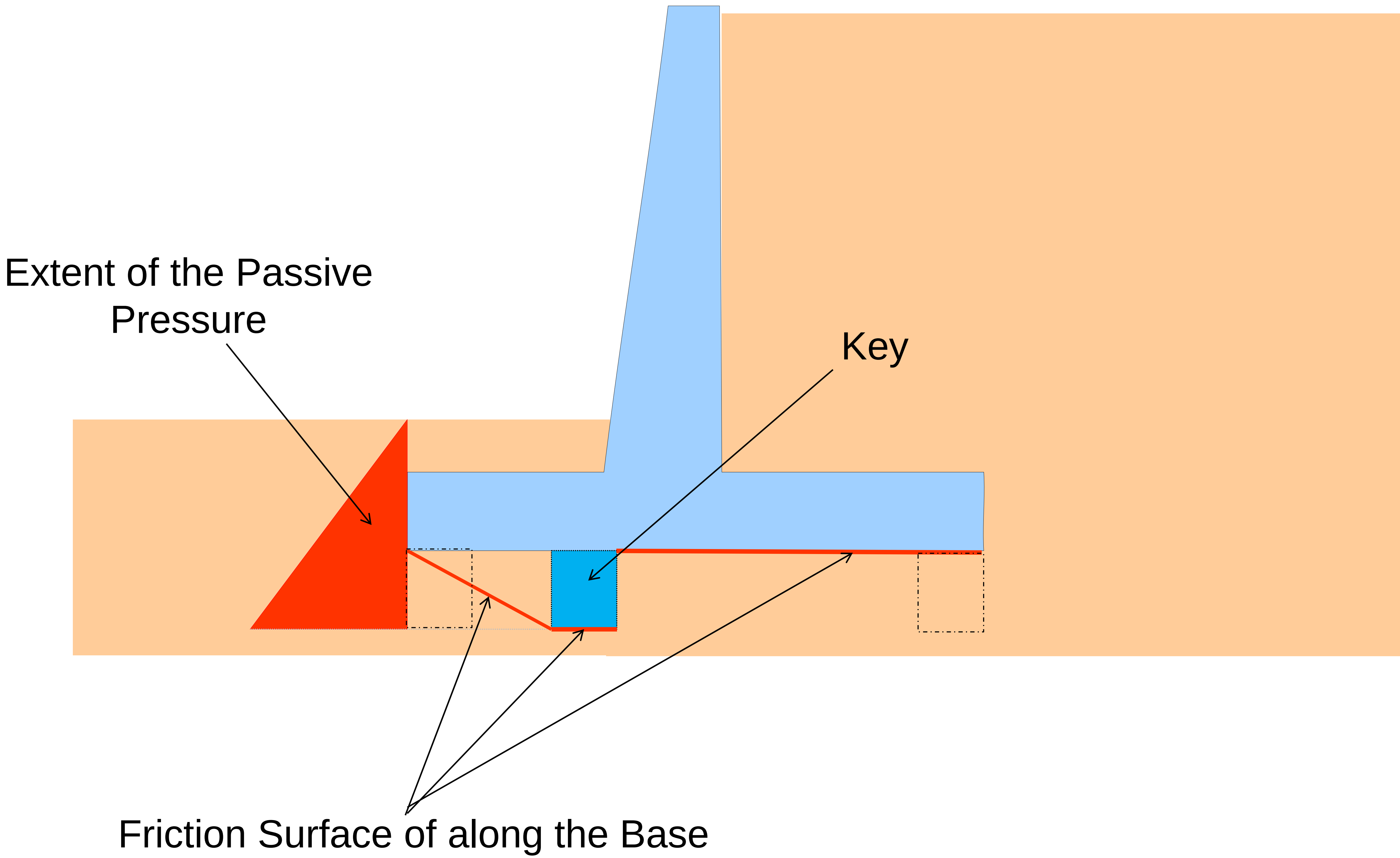
$$q = \frac{\Sigma V}{A} \pm \frac{M_{net}\, y}{I}$$

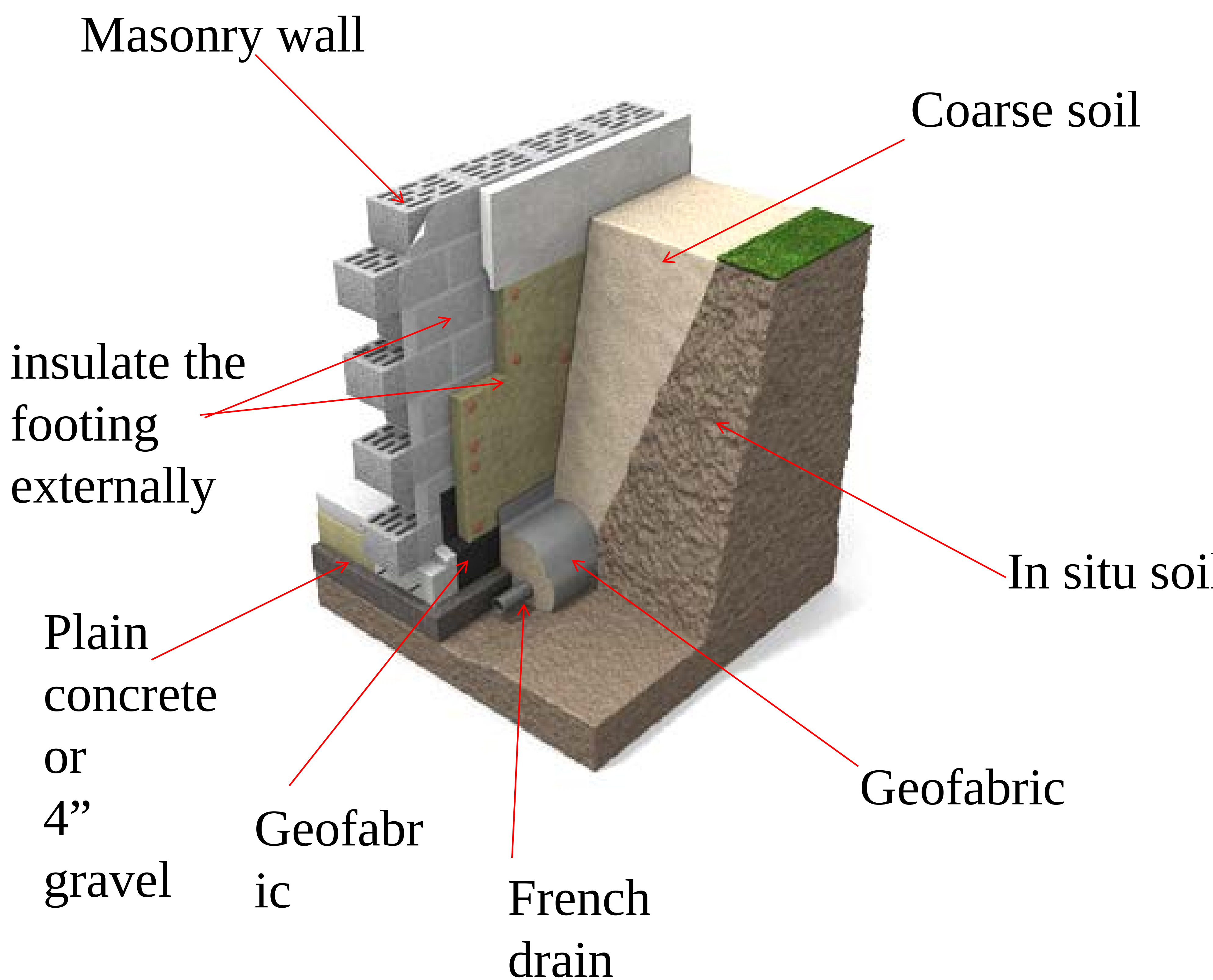
$$q_{\substack{max\\min}} = \frac{\Sigma V}{B}\left(1 \pm \frac{6e}{B}\right)$$

Retaining wall with a key at the base:

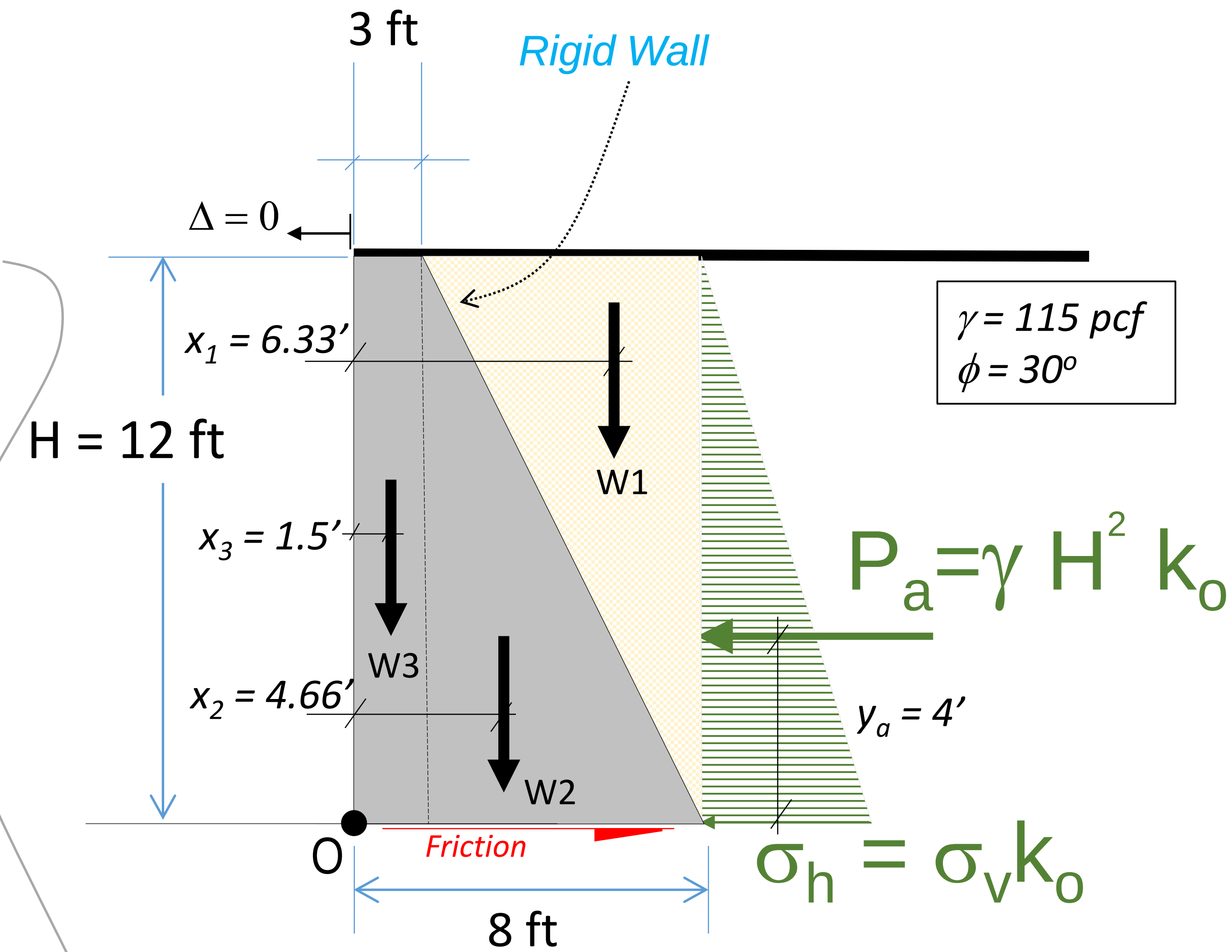


Using Key at the Base to Improve Sliding Resistance



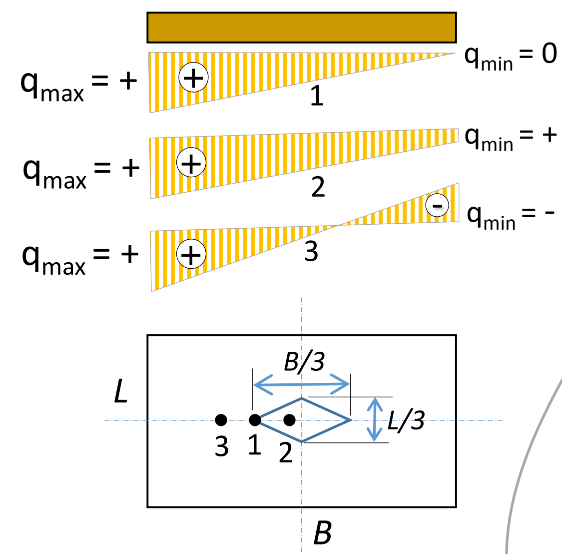

Masonry wall
Coarse soil
insulate the footing externally
In situ soil
Plain concrete or 4” gravel
Geofabric
Geofabr ic
French drain

# Active Earth Pressure in $\phi$ – Soil




**Example -1**

**Given**:
- Vertical retaining wall (Rigid)
- Wall height (H) = 12 ft
- Backfill unit weight ($\gamma$) = 115 pcf
- Angle of soil friction ($\phi$) = 30º
- Assume wall to be smooth
- Angle of friction between the base and the soil $\delta$ = 20º




**Determine**:
The stability of the wall

**Solution**:

$\sigma_h = \sigma_v k_o$ For Rigid Wall use $k_o$

$P_o = 0.5\,\gamma\, H^2 k_o$

$K_o = 1 - \sin\phi$

$P_o = 0.5 x 115 x 12^2 x 0.5 = 4{,}140$ lb/ft

**1- Factor of Safety Against Sliding**

$FS_{(sliding)} = \dfrac{\sum V \tan(20) + Pp}{P_a}$ $P_p = 0$

$= \dfrac{[(13350)\tan 20^o]}{4140} = 1.2 > 1.5$ *Not OK*

**2- Factor of Safety Against Overturning**

$FS_{(overturning)} = \dfrac{\sum M_R}{M_D} = \dfrac{50908.5}{16560} = 3.1 > 2$ *OK*

| X - Force (lb)/ft | Vertical Distance (ft) | $F_y$ (lb)/ft | Moment Arm X (ft) | Driving Moment (ft.lb)/ft | Resisting Moment (ft.lb)/ft |
|---|---|---|---|---|---|
| $P_a$ = 4,140 | 4 | | | 4,140 x 4 = 16,560 | |
| | | $W_1$= 0.5x5x12x115 = 3,450 | 6.33 | | 3,450x6.33=21,838.5 |
| | | $W_2$ =0.5x5x12x150 = 4,500 | 4.66 | | 4,500x4.66 = 20,970 |
| | | $W_3$ = 3x12x150 = 5400 | 1.5 | | 5,400 x 1.5 = 8,100 |
| $P_p$ = 0 | 0 | | | | o |
| | | **13,350** | | **16,560** | **50,908.5** |

**3- Factor of Safety Bearing Capacity Failure = $FS_{(BC)}$**

$M_{net} = \sum M_R - \sum M_D = 50908.5 - 13350 = 37558.5$ ft.lb/ft

$M_{net} = 37{,}558.5 = \sum F_y(X) = 13350\,(X)$

$X = (M_{net} / \sum F_y) = 2.81$ ft

e = (8/2) – 2.81 = 1.18 ft < B/6 or 8/6 = 1.33 (Full contact)

$$q_{max} = \frac{\sum F_y}{B}\left(1 + \frac{6\,e}{B}\right) = \frac{13350}{8}\left(1 + \frac{(6)(1.18)}{8}\right) = 3145.6\,\frac{lb}{ft^2} > q_{all} = 3{,}000\,\frac{lb}{ft^2}$$

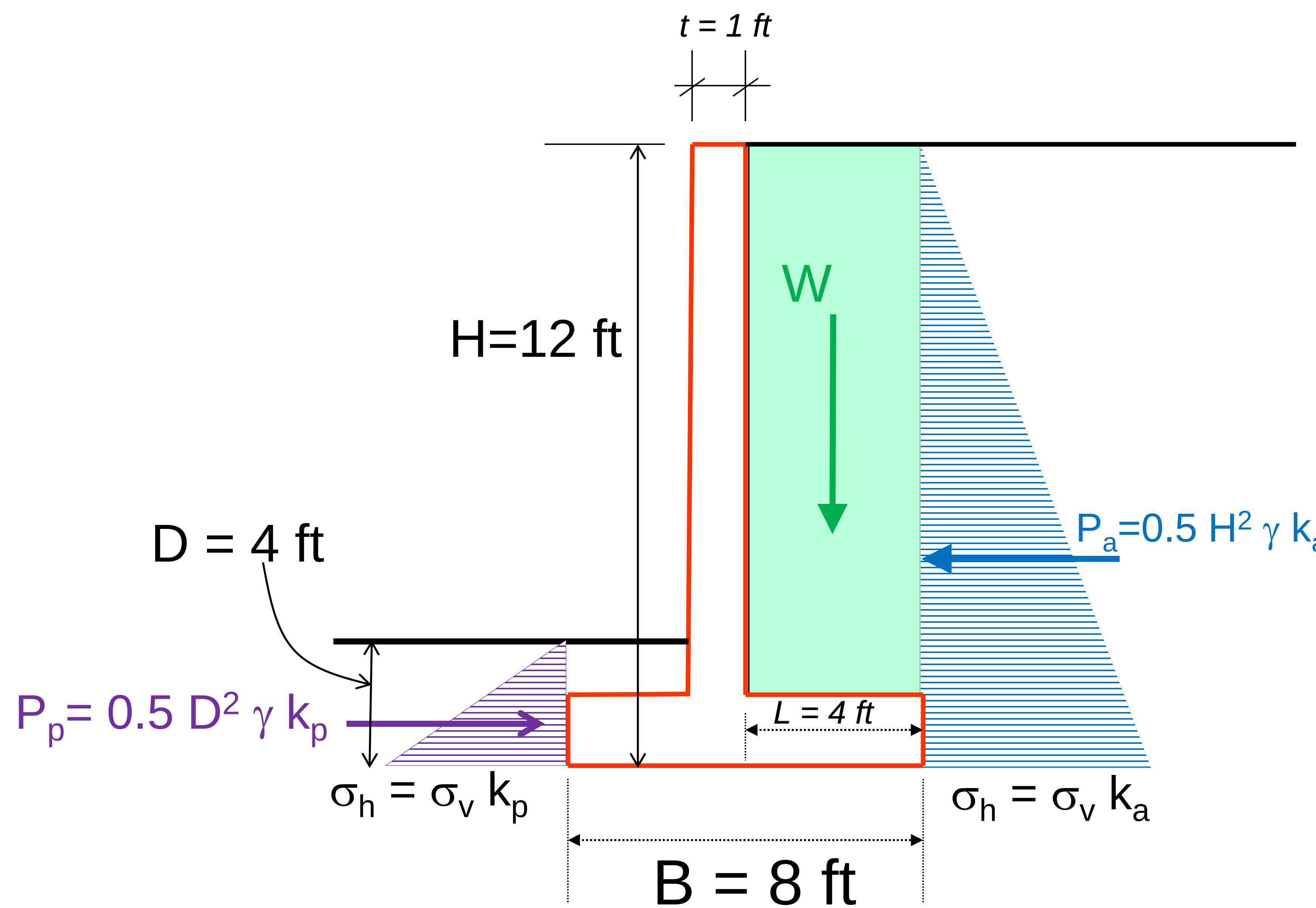
$$q_{min} = \frac{\sum F_y}{B}\left(1 - \frac{6\,e}{B}\right) = \frac{13350}{8}\left(1 - \frac{(6)(1.18)}{8}\right) = 192\,\frac{lb}{ft^2}$$

# Active & Passive Earth Pressure in ϕ – Soil

## Example -2

**Given**:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ($\gamma$) = 115 pcf
- Angle of soil friction ($\phi$) = 30°
- Assume wall to be smooth
- $\gamma_{concrete}$ = 150 lb/ft³
- D = 4 ft




**Find**:

- Resultant Force of the Wall

**Solution**:

$\sigma_h = \sigma_v k_a$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi} \qquad K_p = \frac{1 + \sin\phi}{1 - \sin\phi}$$

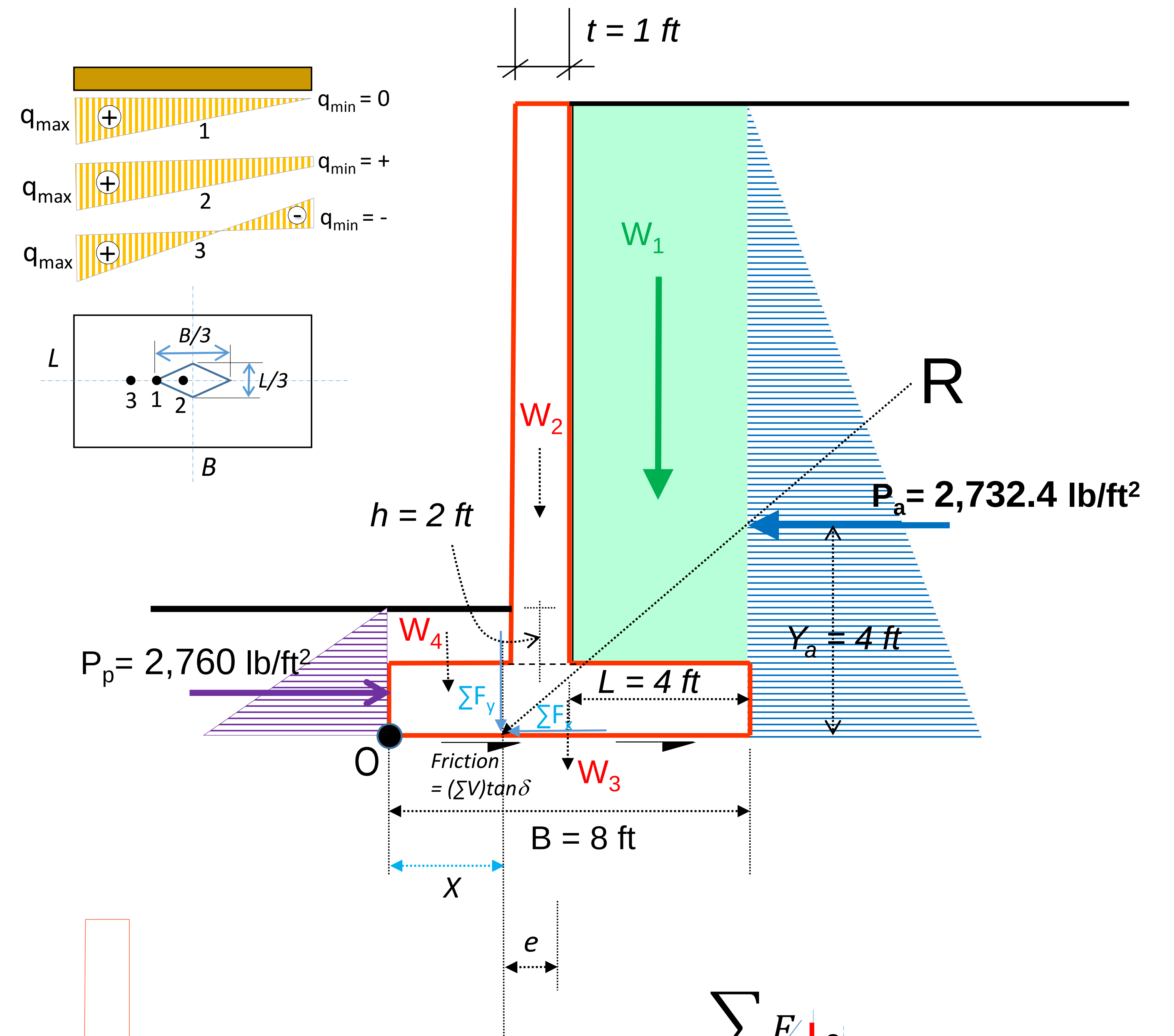
$P_a = 0.5 \gamma \; H^2 k_a$

$P_a = 0.5 \text{ x } 12^2 \text{ x}115\text{x } 0.33 = 2{,}732.4 \text{ lb/ft}^2$

$P_p = 0.5 D^2 \gamma k_p$

$P_p = 0.5\text{x } 4^2 \text{ x}115\text{x}3 = 2{,}760 \text{ lb/ft}^2$

| $F_x$ (lb)/ft | Y (ft) | $F_y$ (lb)/ft | Moment Arm X (ft) | Driving Moment (ft.lb)/ft | Resisting M (ft.lb)/ft |
|---|---|---|---|---|---|
| $P_a$= 2,732.4 | 4 | | | 2,732.4x4 = 10,929.6 | |
| | | $W_1$= 4x10x115 = 4,600 | 6 | | 4600x6=27,600 |
| | | $W_2$ =1x10x150 = 1,500 | 3.5 | | 1,500x3.5=5,250 |
| | | $W_3$ = 8x1x150=1,200 | 4 | | 1,200x4=4,800 |
| | | $W_4$ =2x3x115 = 690 | 1.5 | | 690x1.5= 1,035 |
| $P_p$= 2,760 | 1.33 | | | | 2,760x1.33= 3,680 |
| | | **7,990** | | **10,929.6** | **42,365** |




1- Factor of Safety Against Sliding = $FS_{(sliding)} = \frac{\sum F_y \tan(20)+Pp}{P_a} = \frac{[(7{,}990)\tan 20^o]+2{,}760}{2{,}732.4} = 2.1 > 1.5\ OK$

2- Factor of Safety Against Overturning = $FS_{(overturning)} = \frac{\sum M_{Resisting}}{M_{Driving}} = \frac{42{,}365}{10{,}929.6} = 3.8 > 2\ OK$

3- Factor of Safety Bearing Capacity Failure = $FS_{(BC)}$

$M_{net} = \sum M_r - \sum M_o$ = 42,365 -10,929.6 = 31,435.4 ft.lb/ft

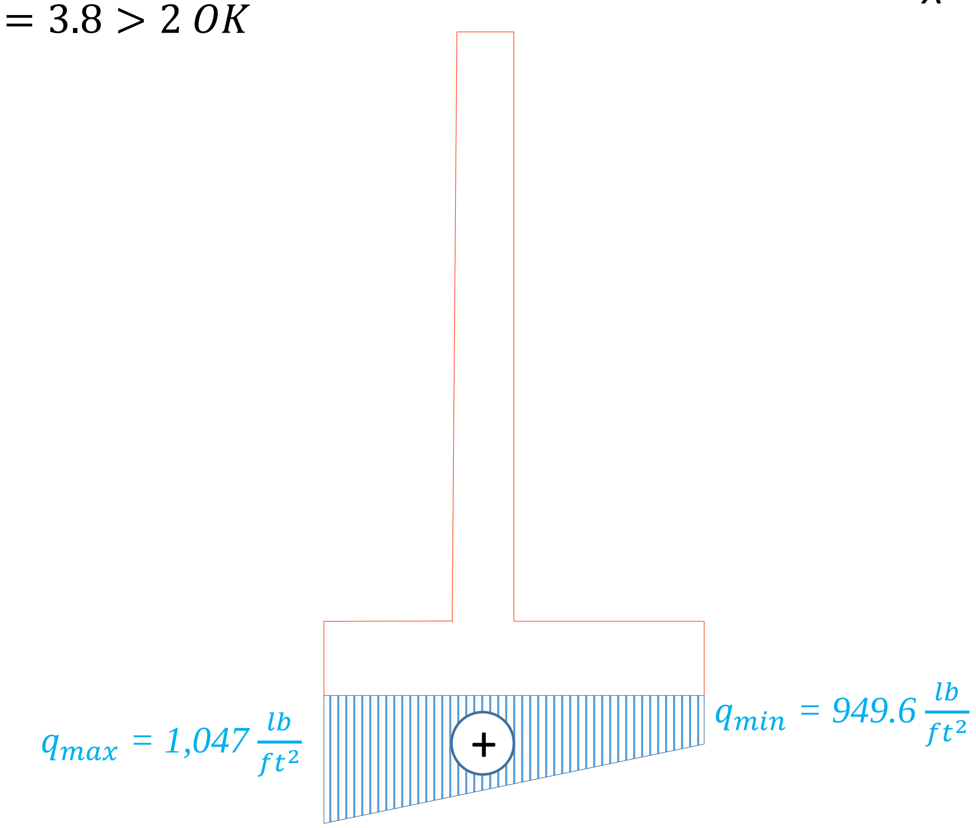
$M_{net}$ = 31,435.4 = $\sum F_y(X)$ = 7,990.($X$)

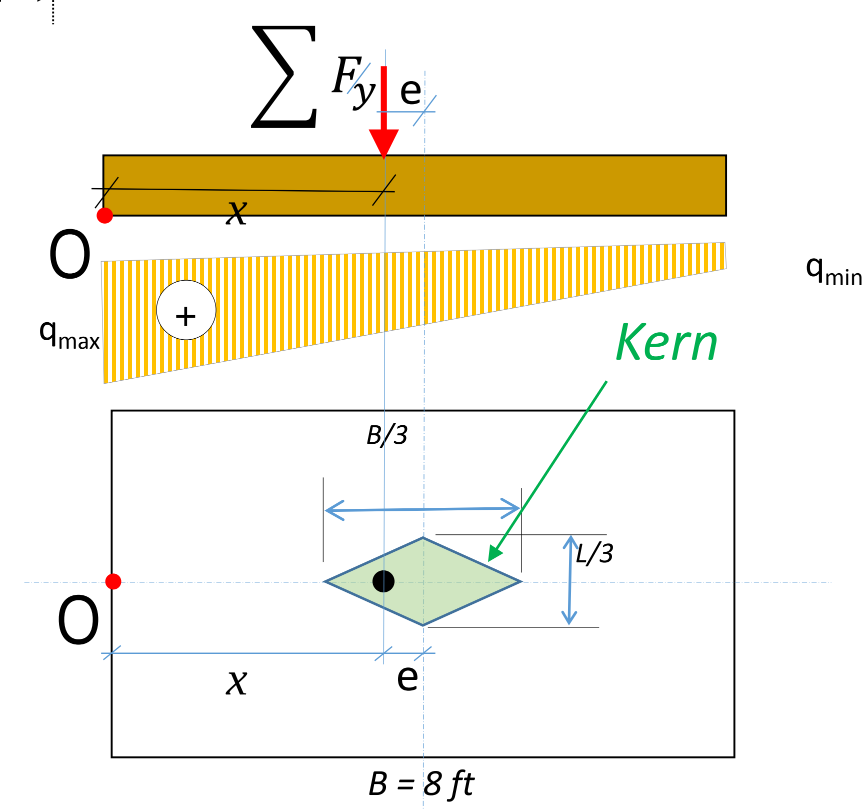
$X = (M_{net} / \sum F_y) = 3.93$ ft

e = (8/2) – 3.93 = 0.0656 ft < B/6 or 8/6 = 1.33 (Full contact)

$$q_{max} = \frac{\sum F_y}{B}\left(1+\frac{6\,e}{B}\right) = \frac{7{,}990}{8}\left(1+\frac{(6)(0.0656)}{8}\right) = 1{,}047.8\,\frac{lb}{ft^2} < \quad q_{all} = 3{,}000\,\frac{lb}{ft^2}$$

$$q_{min} = \frac{\sum F_y}{B}\left(1-\frac{6\,e}{B}\right) = \frac{7{,}990}{8}\left(1-\frac{(6)(0.0656)}{8}\right) = 949.6\,\frac{lb}{ft^2}$$



# Example 1

## Given

The cross section of a cantilever retaining wall is shown in Figure 1. Calculate the factors of safety with respect to overturning, sliding, and bearing capacity.

## Solution

From the figure,

$H^* = H_1 + H_2 + T_1 = 8 \tan 10° + 19.5 + 2 = 22.91$ ft

The Rankine active force per unit length of wall =

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}} \qquad K_a = \cos 10 \frac{\cos 10 - \sqrt{\cos^2 10 - \cos^2 30}}{\cos 10 + \sqrt{\cos^2 10 - \cos^2 30}} = 0.34$$

$$P_a = \text{Lateral Pressure from Surcharge} + \text{Lateral Pressure from Soil}$$

$$P_{a1} = q H^* k_a$$

$$P_{a2} = \frac{1}{2}\gamma H^{*2} k_a$$

$$P_{a1} = 120x22.9x0.34 = 934.32 \, lb/ft$$

$$P_{v1} = 934.32 \sin(10^o) = 162.24 \, lb/ft$$

$$P_{h1} = 934.32 \cos(10^o) = 920.13 \, lb/ft$$

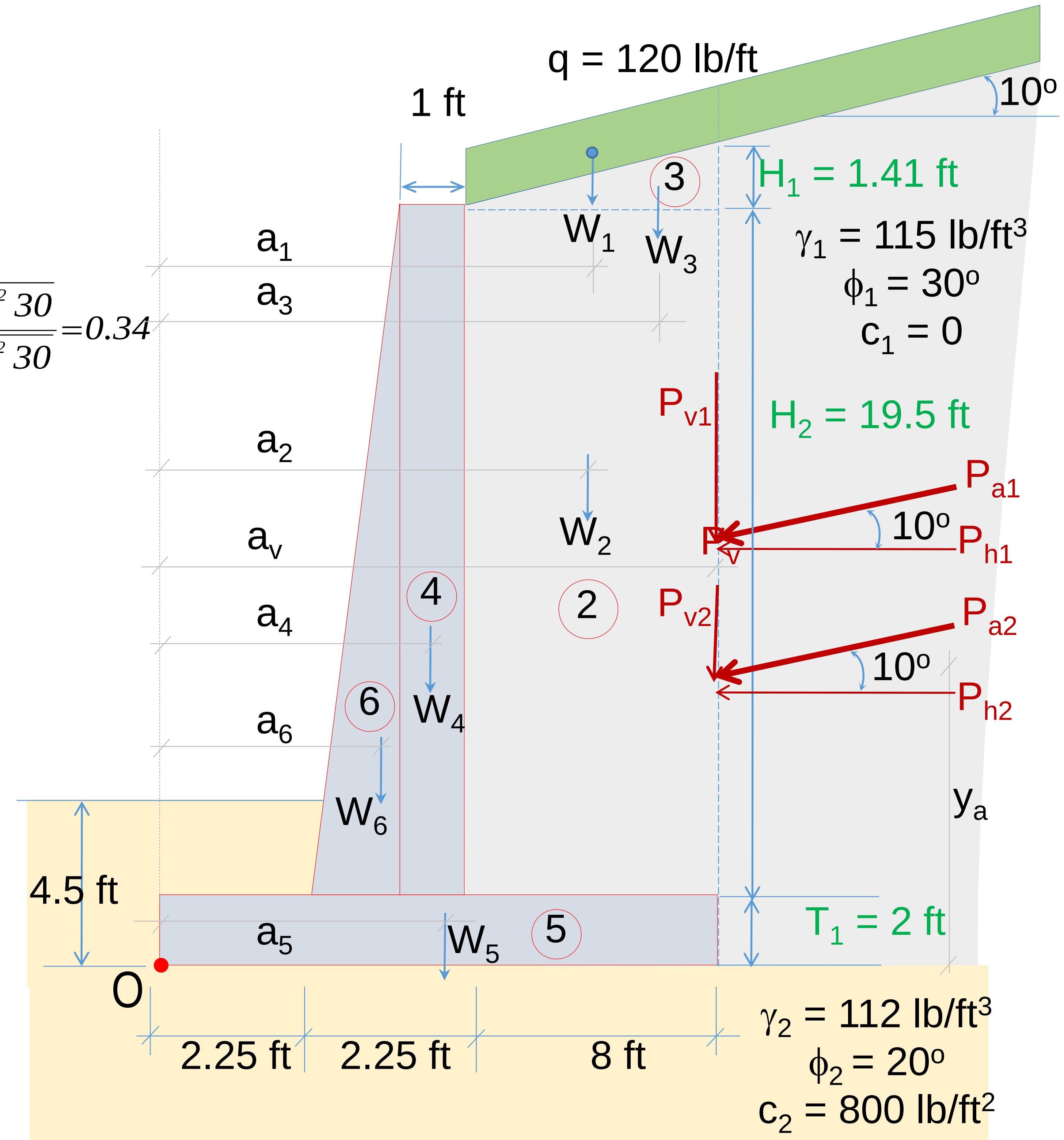
$$Y_{a1} = \frac{22.9}{2} = 11.45 \, ft$$

$$P_{a2} = 0.5x115x22.9^2 x0.34 = 10252.22 = 11186.54 \, lb/ft$$

$$P_{v2} = 10{,}252.2 \sin(10^o) = 1780.28 \, lb/ft$$

$$P_{h2} = 10{,}252.2 \cos(10^o) = 10096.45 \, lb/ft$$

$$Y_{a1} = \frac{22.9}{3} = 7.63 \, ft$$



| Driving Pressure $(lb/ft^2)/ft$ | Resisting Pressure $(lb/ft^2)/ft$ | Weight/Unit Length (lb/ft) | Moment Arm from Point O (ft) | Driving Moment (ft.lb/ft) | Resisting Moment (ft.lb/ft) |
|---|---|---|---|---|---|
| $P_{h1}$ = 920.13 | | | 11.45 | 920.13x11.45 = 10535.49 | |
| $P_{h2}$ = 10096.45 | | | 7.64 | 10096.45x7.64 = 77136.88 | |
| | | $W_1$ = 120 x 8 = 960 | 8.5 | | 960x8.5 = 8,160 |
| | | $W_2$ = 8X19.2X112 = 17,203.2 | 8.5 | | 17,203.2x8.5 = 146,227.2 |
| | | $W_3$ = 0.5X1.41X8X112 = 631.7 | 9.83 | | 631.7x9.83 = 6,209.6 |
| | | $W_4$ = 1X19.5X150 = 2,925 | 4.0 | | 2925x4 = 11,700 |
| | | $W_5$ = 12.5X2X150 = 3,750 | 6.25 | | 3750x6.25 = 23,437.5 |
| | | $W_6$ = 0.5X1.25X19.2X150 = 1,800 | 3.08 | | 1800x3.08 = 5,544 |
| | | ∑Pv = 162.24+1780.28 = 1,942.52 | 12.5 | | 1942.52x12.5 = 24,281.5 |
| | | | | | |
| $\sum P_h$ = 11,016.58 | 0 | $\sum F_y$ = 29,212.4 | | $\sum M_D$ = 87,672.37 | $\sum M_R$ = 225,559.86 |

1- Factor of Safety Against Sliding = $FS_{(sliding)} = \frac{\sum F_y \tan(20)+Pp}{P_a} = \frac{[(29,212.4)\tan 20^o]+0}{11,016.58} = 0.96 < 1.5$ *Not OK*

**Add Passive Resistance**

$P_p = 0.5 \times 4.5^2 \times 115 \times 3 = 3493.13$ lb/ft

2- Factor of Safety Against Sliding = $FS_{(sliding)} = \frac{\sum F_y \tan(20)+Pp}{P_a} = \frac{[(29,212.4)\tan 20^o]+3493.13}{11,016.58} = 1.28 < 1.5$ *Not OK*

3- Factor of Safety Against Overturning = $FS_{(overturning)} = \frac{\sum M_R}{\sum M_D} = \frac{225,559.86}{87,672.37} = 2.57 > 20$ *OK*

4- Factor of Safety Bearing Capacity Failure = $FS_{(BC)}$ =

$M_{net} = \sum M_R - \sum M_D = 225{,}559.86 - 87{,}672.37 = 137{,}887.5$ ft.lb/ft

$M_{net} = 137{,}887.5 = \sum F_y(X) = 29{,}212.4\ (X)$

$X = (M_{net} / \sum F_y) = 4.72$ ft

e = (12.5/2) – 4.72 = 1.53 ft < B/6 or 12.5/6 = 2.083 (Full contact)

$$q_{max} = \frac{\sum F_y}{B}\left(1 + \frac{6\,e}{B}\right) = \frac{29{,}212.4}{12.5}\left(1 + \frac{(6)(1.53)}{12.5}\right) = 4053.28\,\frac{lb}{ft^2} > \quad q_{all} = 3{,}000\,\frac{lb}{ft^2}$$ No good

$$q_{min} = \frac{\sum F_y}{B}\left(1 - \frac{6\,e}{B}\right) = \frac{29{,}212.4}{12.5}\left(1 - \frac{(6)(1.53)}{12.5}\right) = 621\,\frac{lb}{ft^2}$$