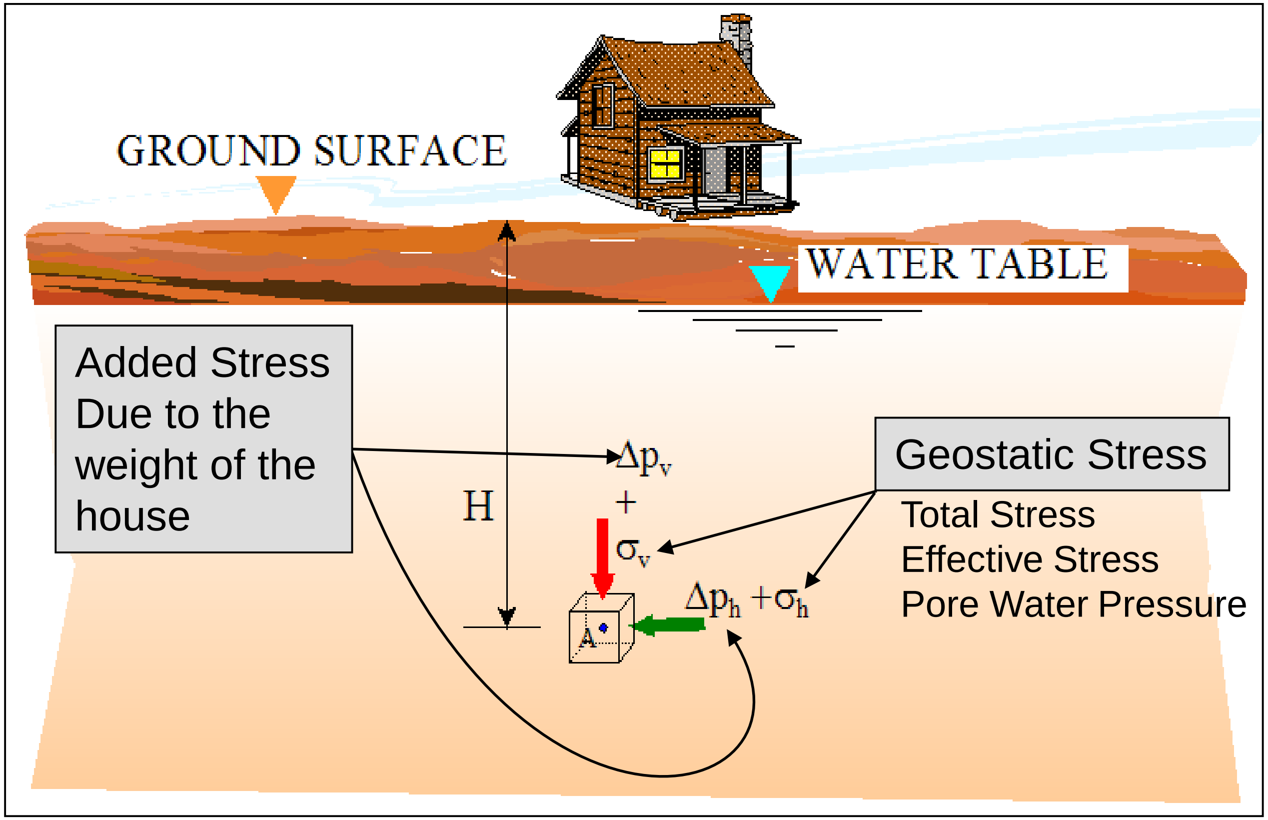
# Lateral Stresses in Soil

By

Kamal Tawfiq, Ph.D., P.E.

Spring 2017

**There are two types of lateral stresses in soil.**



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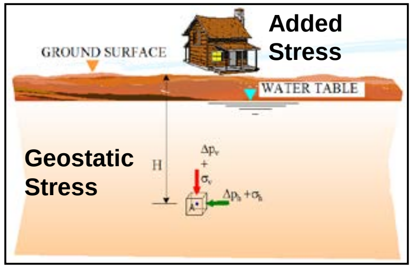


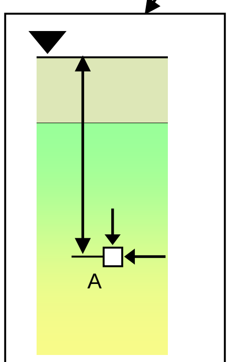

# Stress Distribution in Soils

## Geostatic Stresses

Total Stress
Effective Stress
Pore Water Pressure

Total Stress= Effective Stress+ Pore Water Pressure

$$\sigma_{total} = \sigma_{eff} + u$$

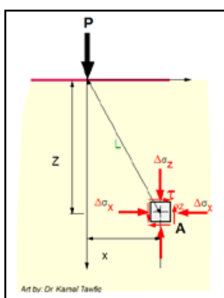



## Added Stresses (Point, line, strip, triangular, circular, rectangular)

**Bossinisque Equations**

1. Point Load
2. Line Load
3. Strip Load
4. Triangular Load
5. Circular Load
6. Rectangular Load

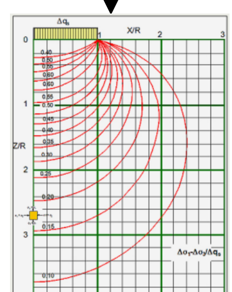
$\sigma_y$
$\sigma_x$
$\tau_{xy}$



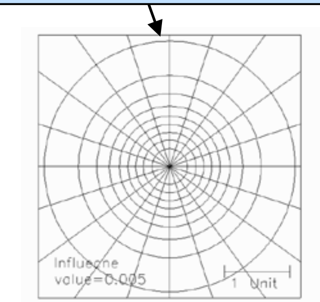

Influence Charts

$$\Delta\sigma_z = I_\sigma \cdot q$$

Stress Bulbs

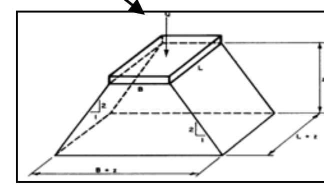


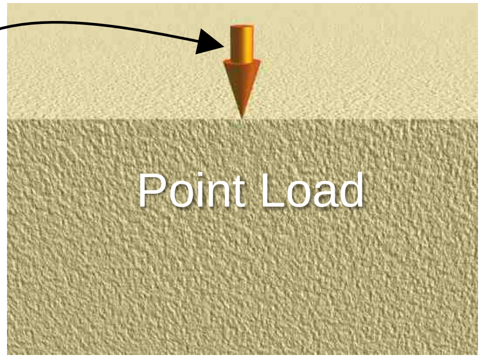
Newmark Charts

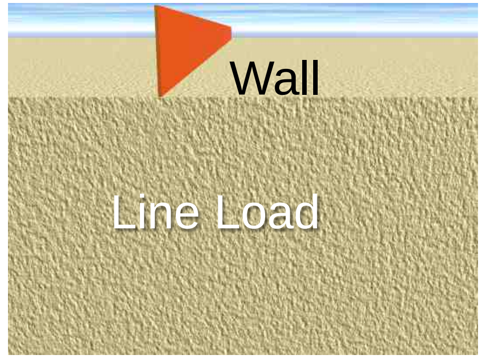



Westergaard's Method
(For Pavement)

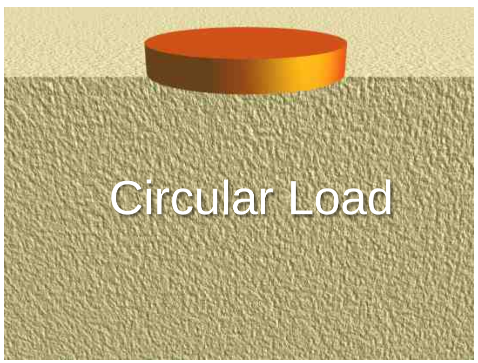
**Approximate Method**
1:2 Method




Point Load

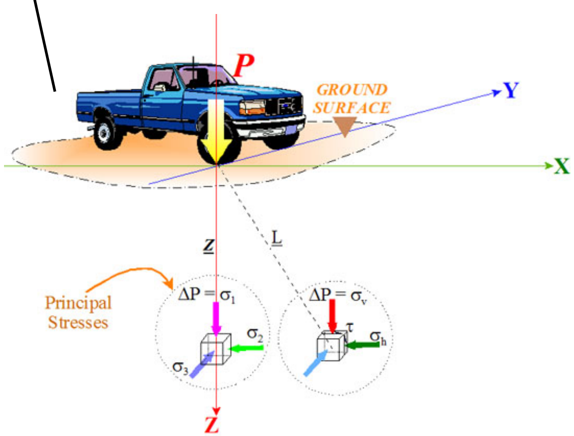

Wall
Line Load


Continuous Load


Circular Load


Triangular Load


Rectangular Load


P
GROUND SURFACE
Y
X
z
L
Principal Stresses
$\Delta P = \sigma_1$
$\sigma_2$
$\sigma_3$
$\Delta P = \sigma_v$
$\tau$
$\sigma_h$
Z

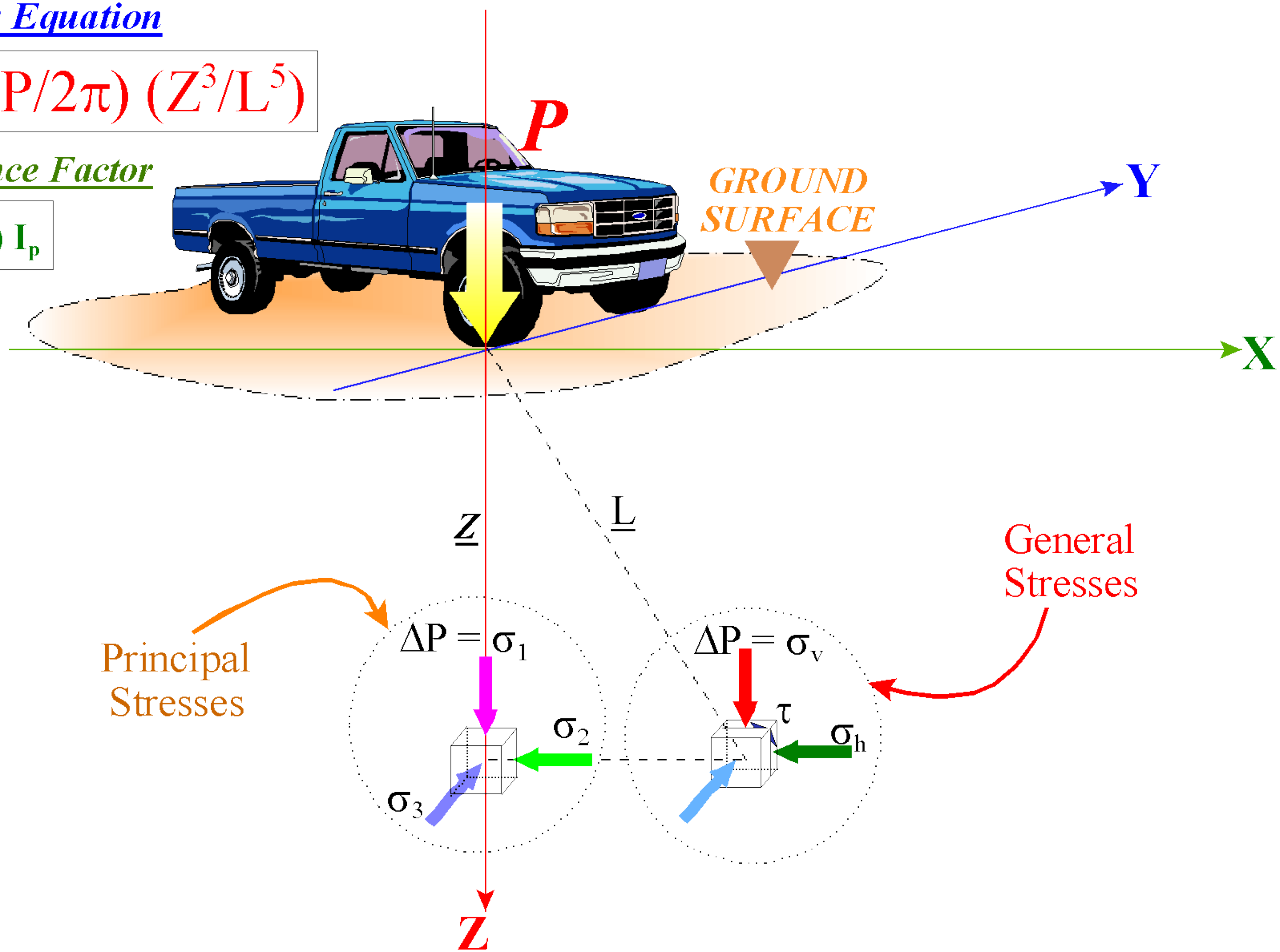
# 1. STRESSES CAUSED BY A POINT LOAD

**Boussinesq's Equation**

$$\Delta P_z = (3P/2\pi)\,(Z^3/L^5)$$

**Using Influence Factor**

$$\Delta P = (P/Z^2)\, I_p$$

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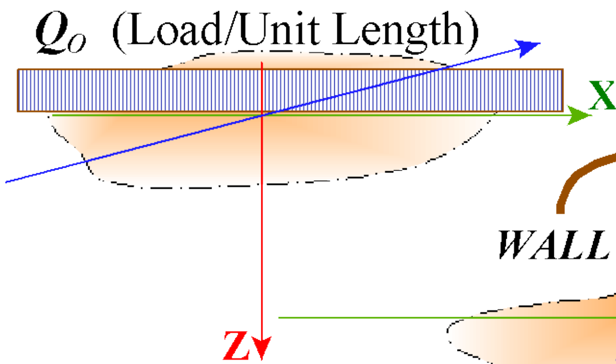
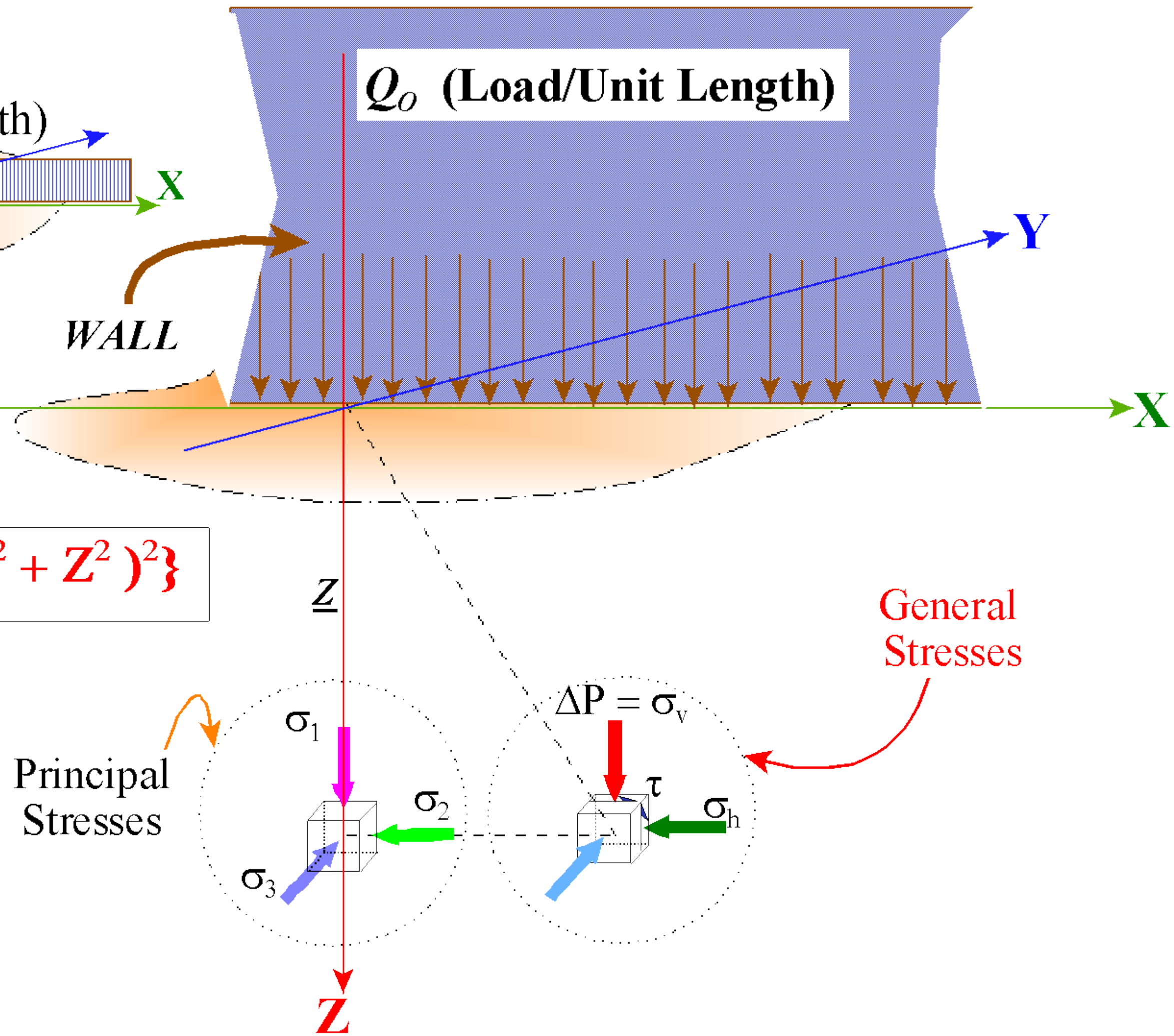
# 2. STRESSES CAUSED BY A LINE LOAD




***Boussinesq's Equation***

$$\Delta P_z = \{2\ Q_o\ Z^3/\pi\ (X^2 + Z^2)^2\}$$

***Using Influence Factor***

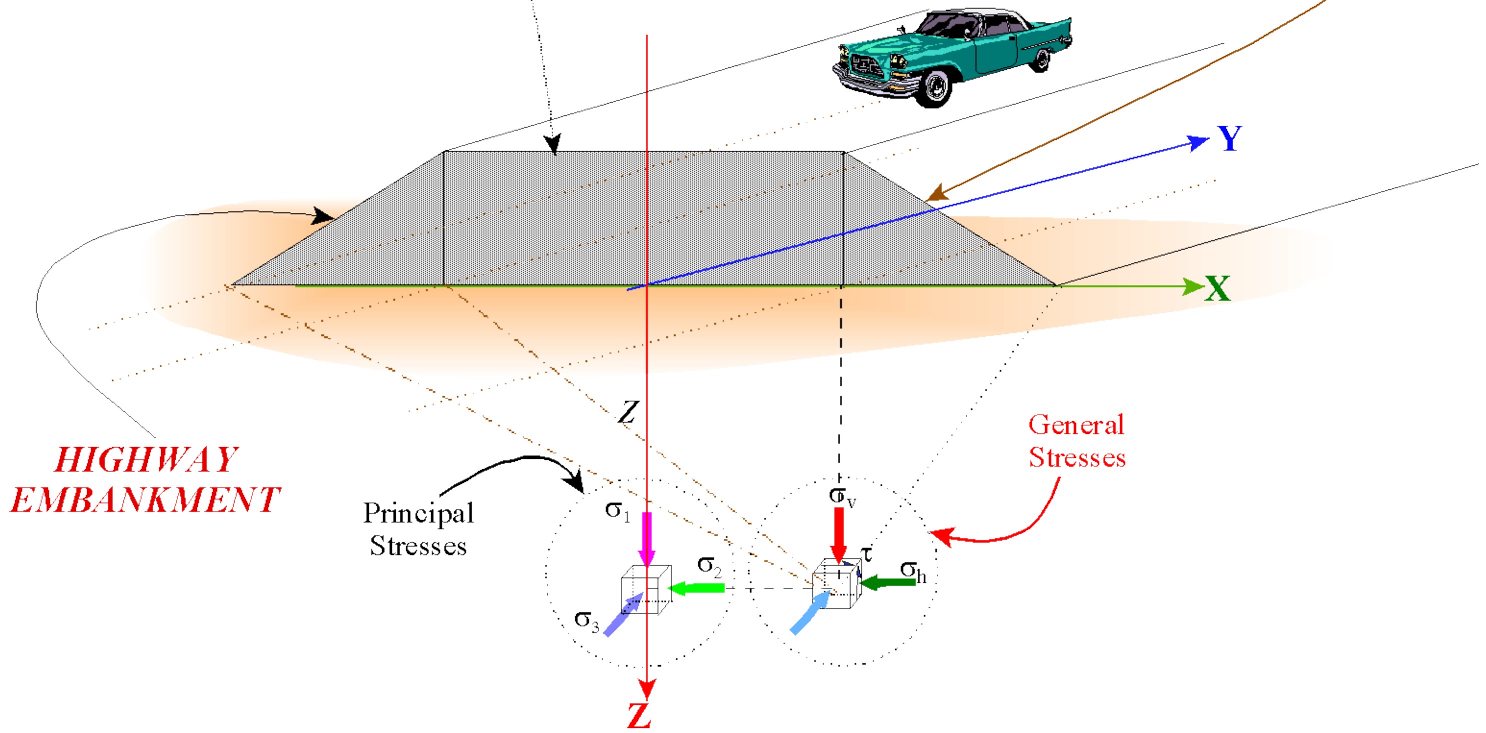
$$\Delta P = (P/Z)\ I_L$$

## 3. CONTINUOUS LOAD

$$\Delta P = q/\pi \left[\beta + \sin\beta \cos(\beta + 2\,\delta)\right]$$

## 4. TRIANGULAR LOAD

$$\Delta P = (q/\pi)\ \{(X/B)\,\alpha - (\sin 2\delta/2)\}$$

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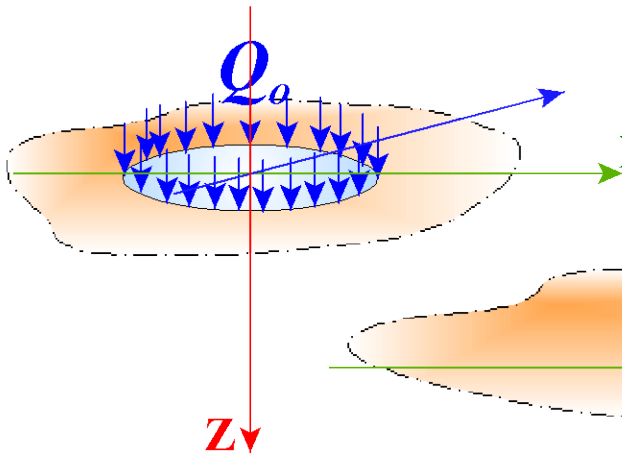
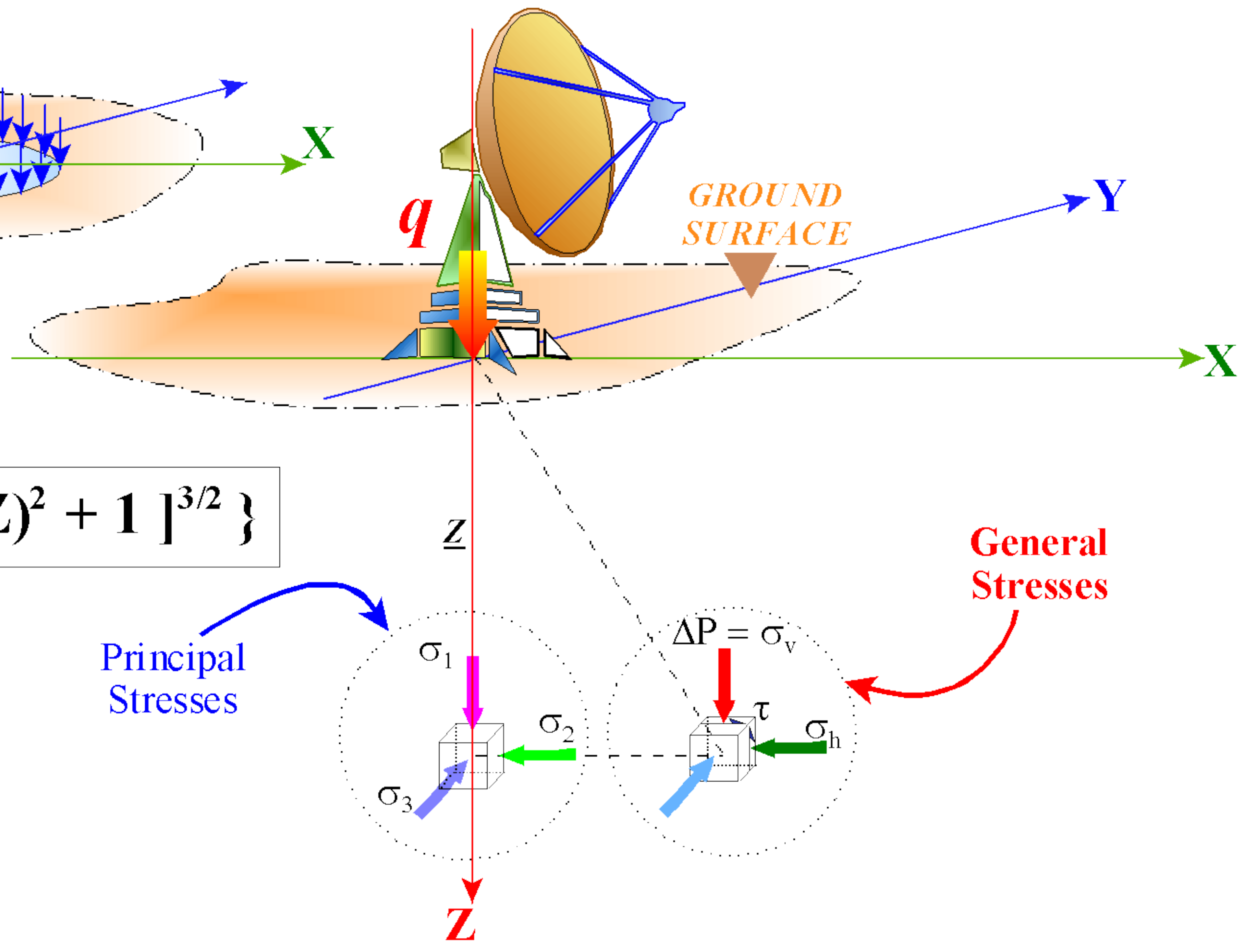
# 5. VERTICAL STRESS DUE TO CIRCULAR LOAD




$$\Delta P = q \{ 1 - 1/[(R/Z)^2 + 1]^{3/2} \}$$

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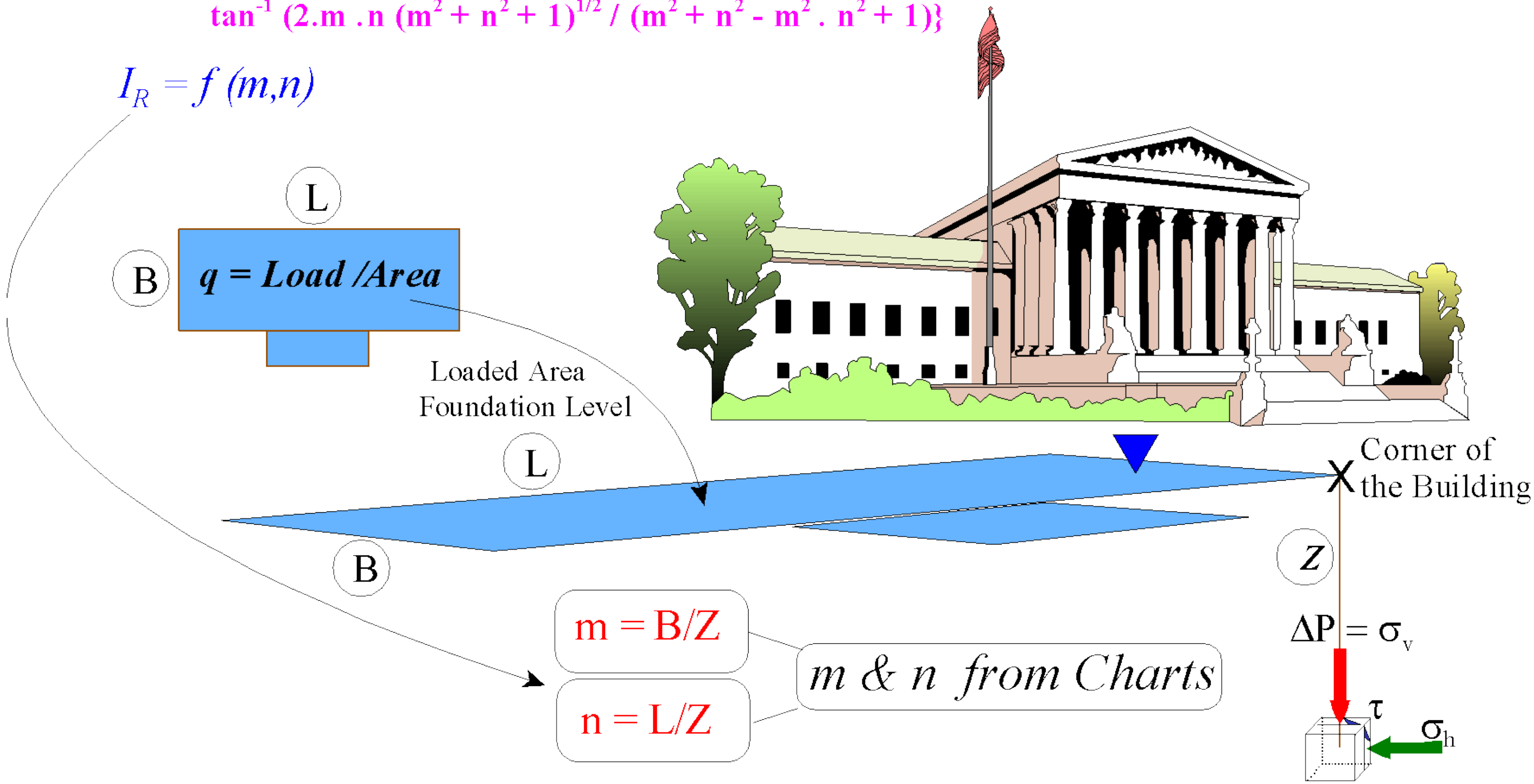
# 6. VERTICAL STRESS DUE TO RECTANGULAR LOAD

$$\Delta P = q\, I_R$$

$$I_R = 1/4\pi \{[ (2.\ m.n\ (m^2 + n^2 + 1)^{1/2} ) / (m^2 + n^2 + m^2.\ n^2 + 1) ]\ [(m^2 + n^2 + 2)/(m^2 + n^2 + 1)] + \tan^{-1} (2.m\ .n\ (m^2 + n^2 + 1)^{1/2} / (m^2 + n^2 - m^2\ .\ n^2 + 1)\}$$

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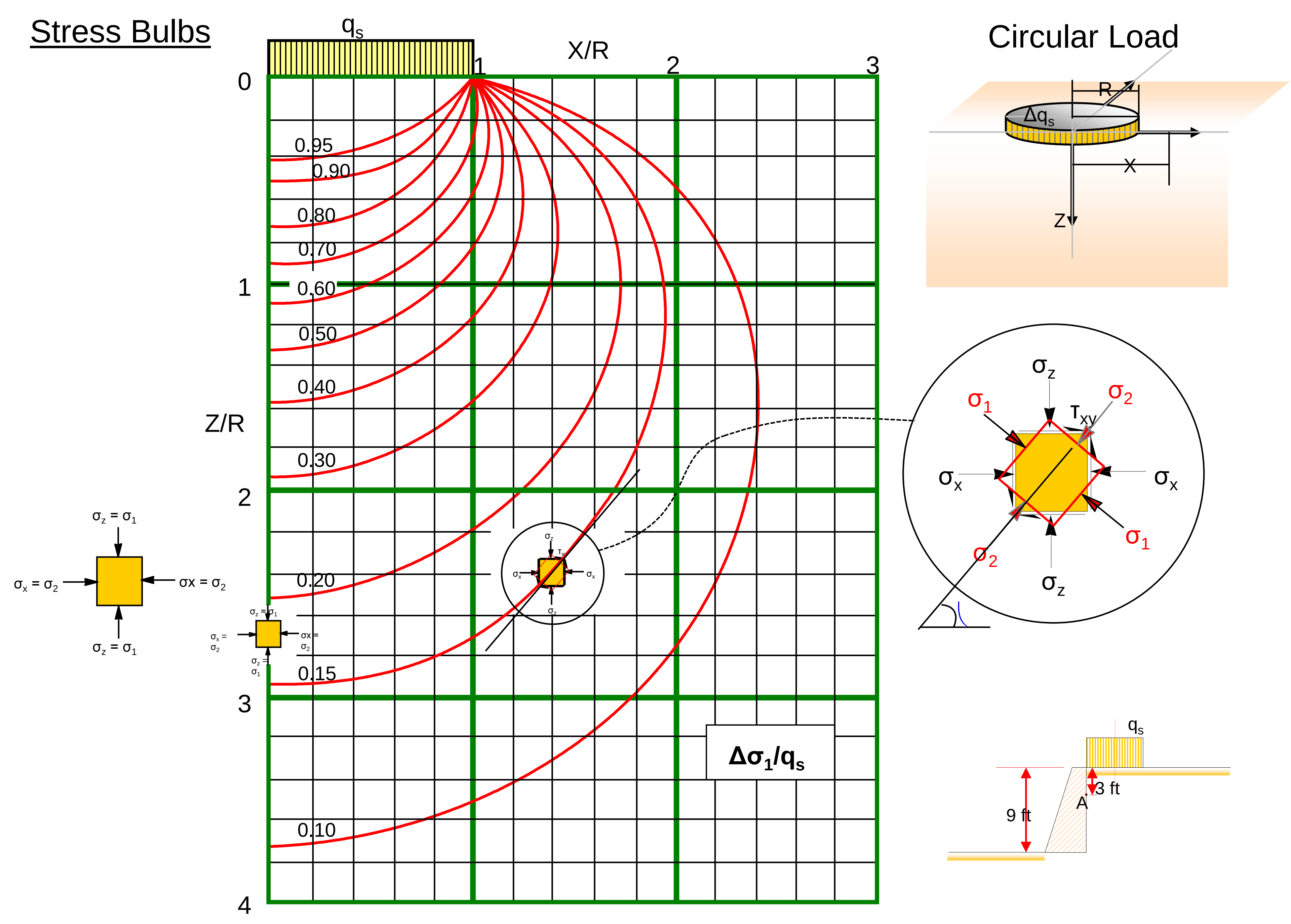
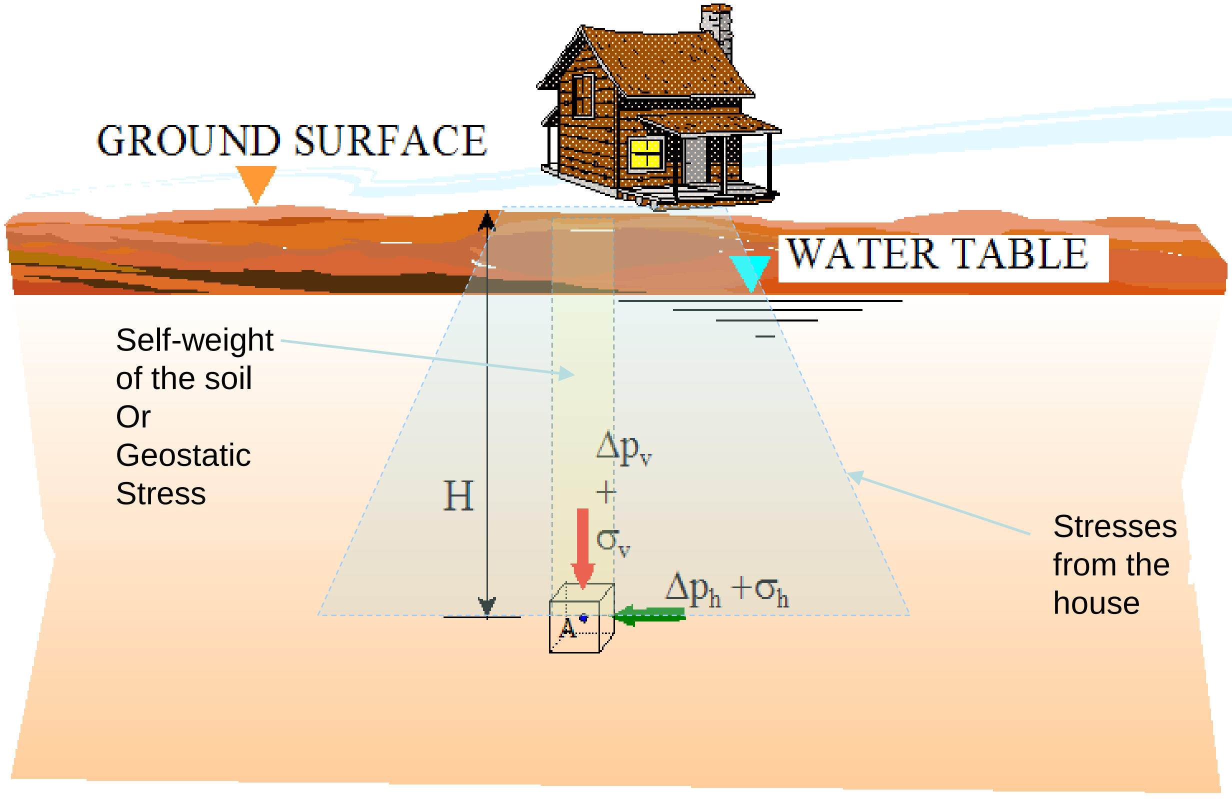


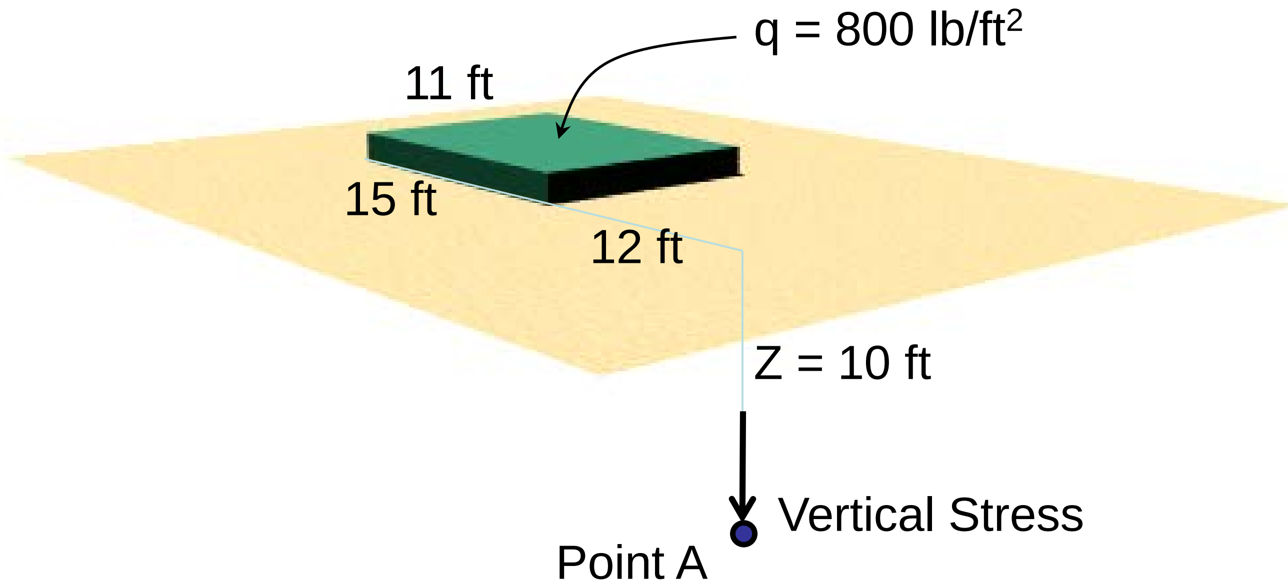

Circular Load: (Major Principal Stress)/(Surface Stress)

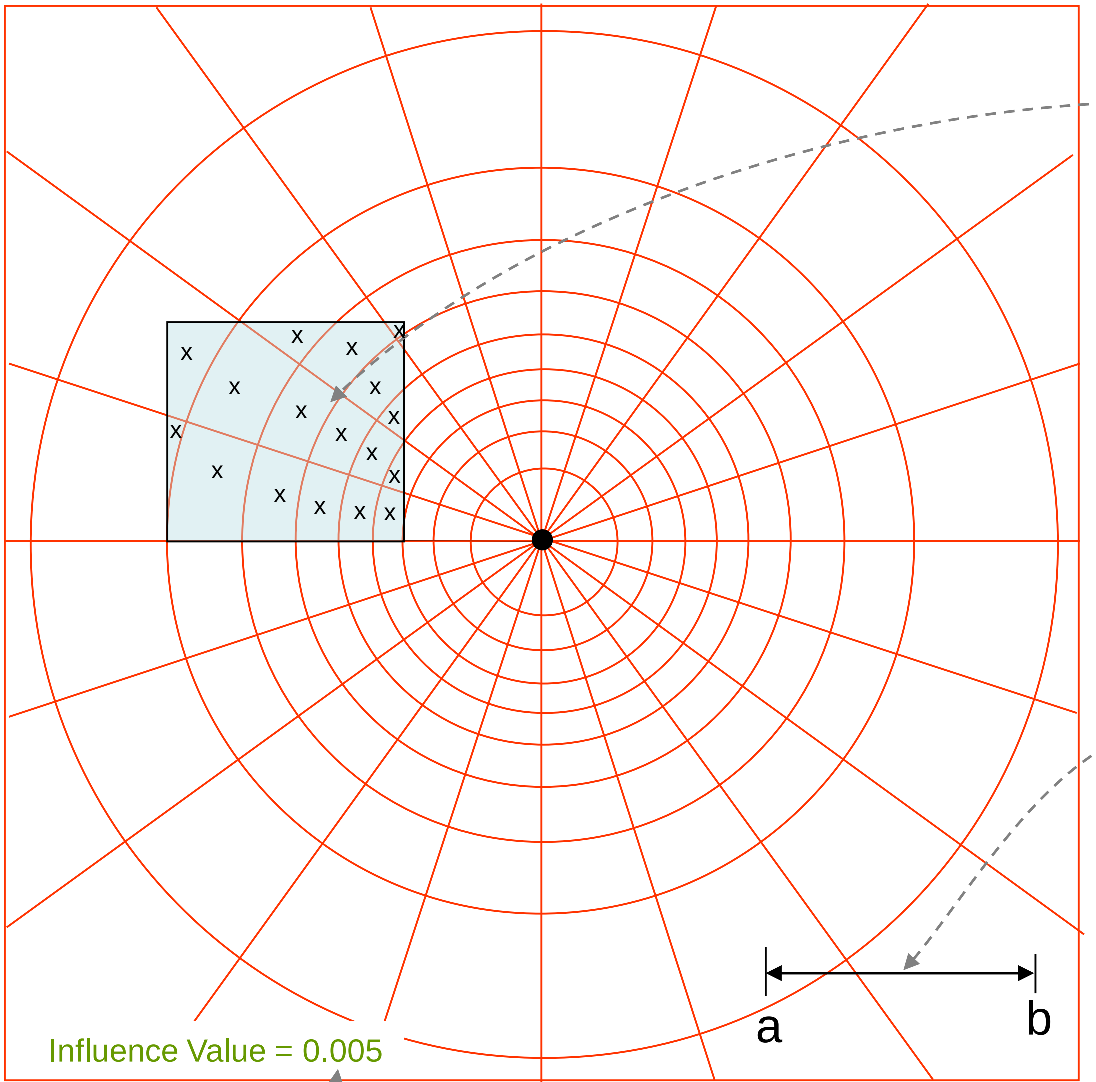
# Geostatic Stresses
# &
# Stress Distribution in Soils

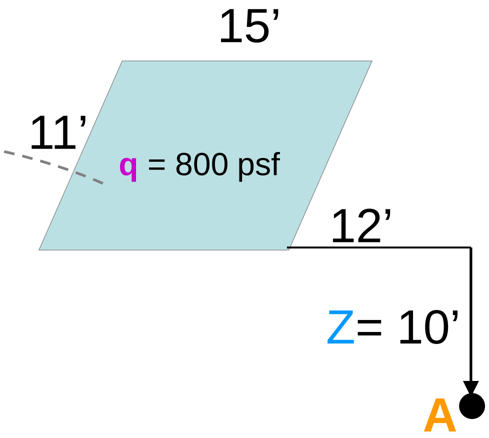


# Newmark's Charts

Find:
Vertical Stress at Point A






**Steps to determine Vertical Stress Using Newmark’s Charts:**

1- Select a scale to draw the shape shown above

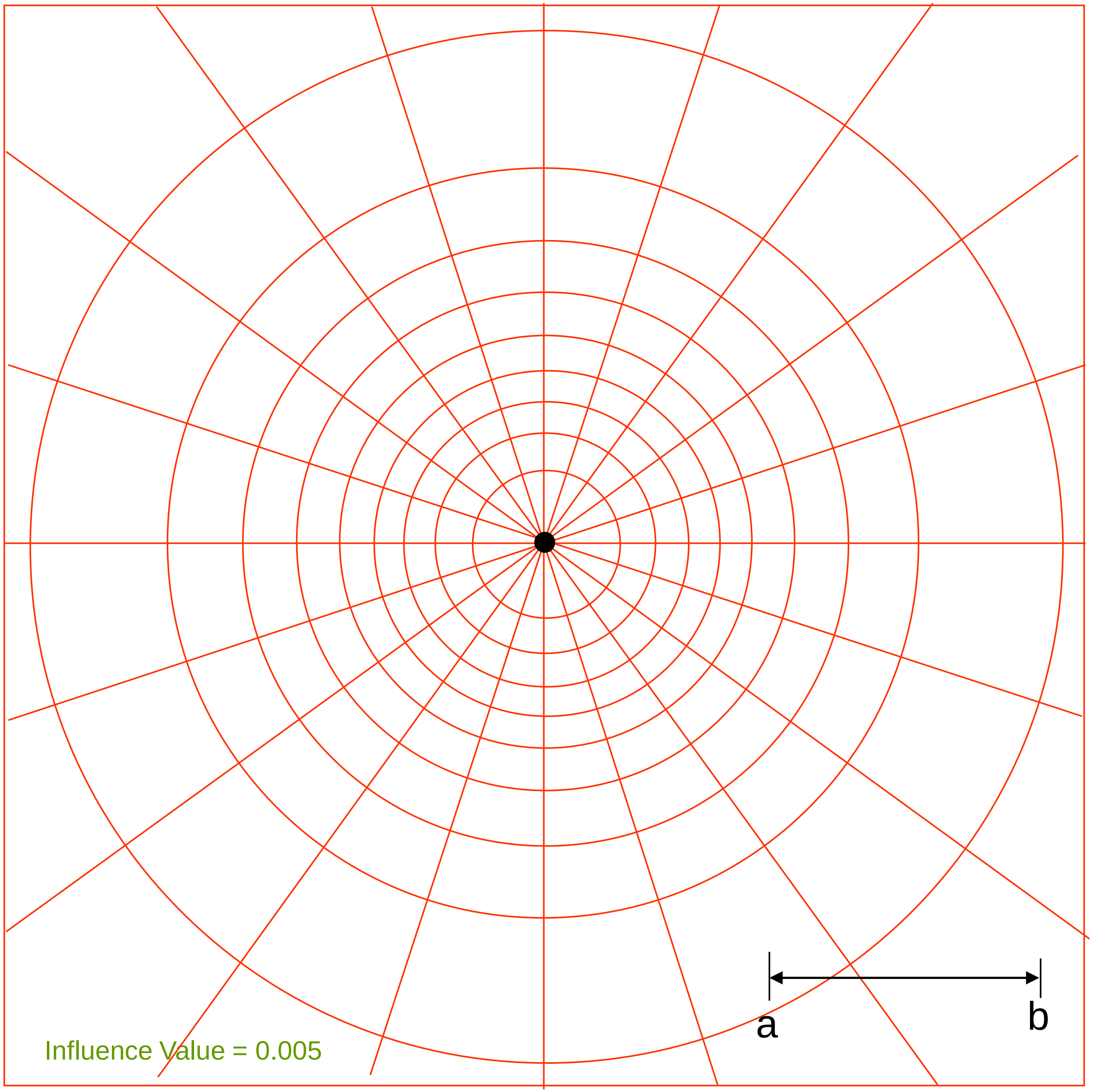
2- The scale is
Z = length of line **ab**

3- Draw the shape using the selected scale

4- Place the shape on Newmark chart so that point A is at the center

5- Count the stress fields **n**

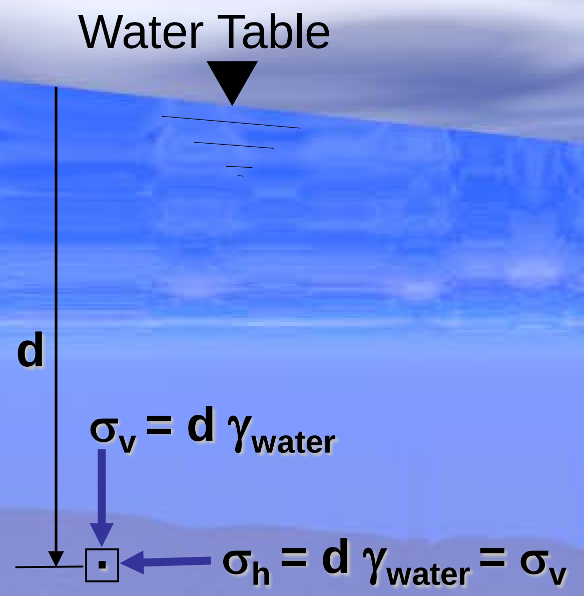
6- The stress at point **A** = **n** x **q** x 0.005


a
b
Influence Value = 0.005

# Lateral stresses due to self-weight of soil

# Vertical and Horizontal Stress in Water

Hydrostatic Stress or Pressure
Isotropic Stress

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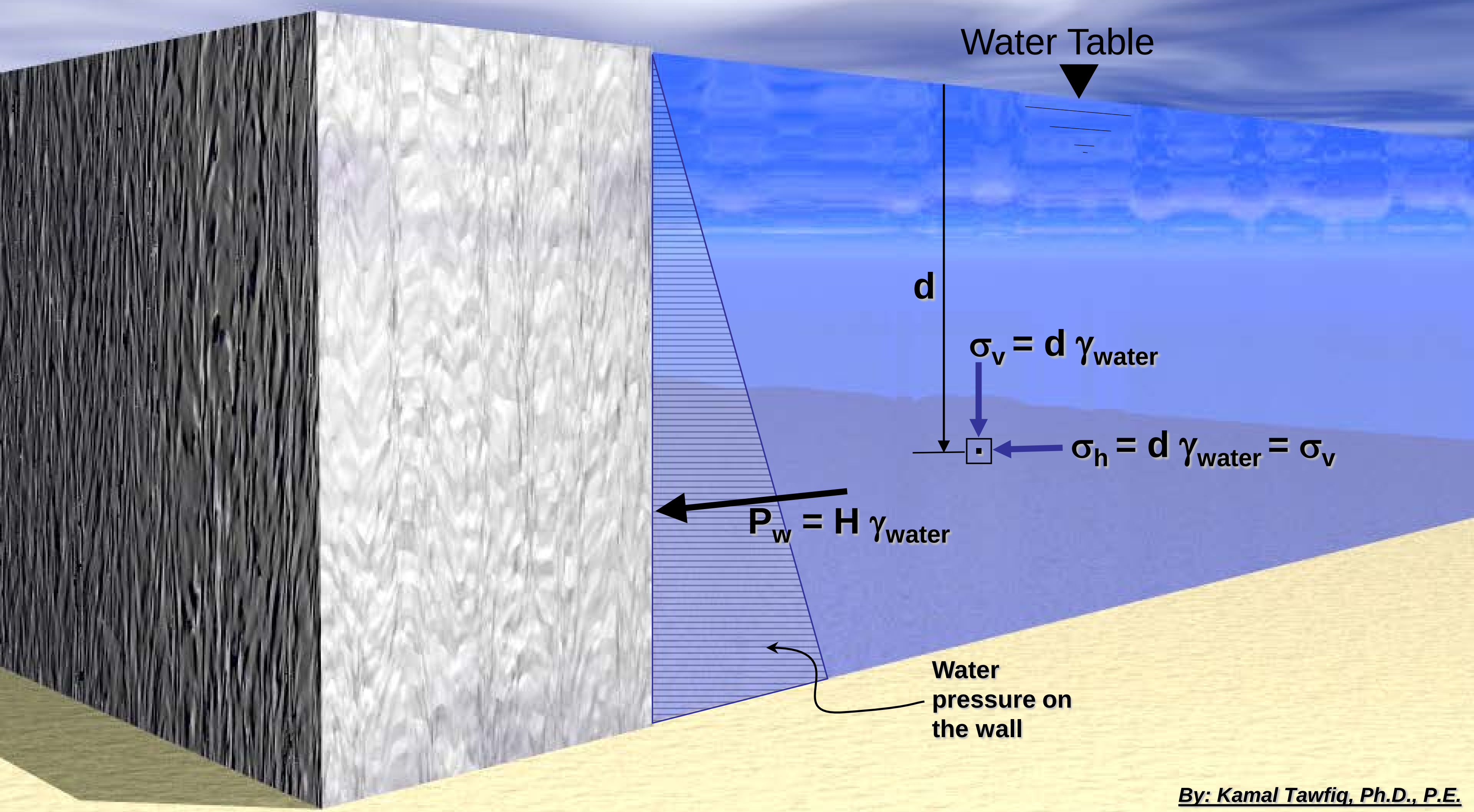
# Vertical and Horizontal Stress in Water

Assume a massive wall retaining water. What is the pressure on this wall??

Is the wall stable???

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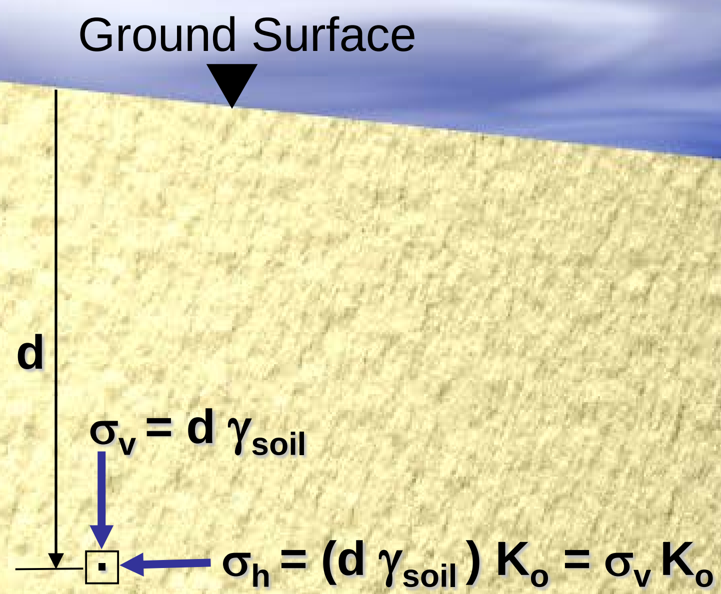
# Vertical and Horizontal Stress in Soil

Geostatic Stress
Anisotropic Stress

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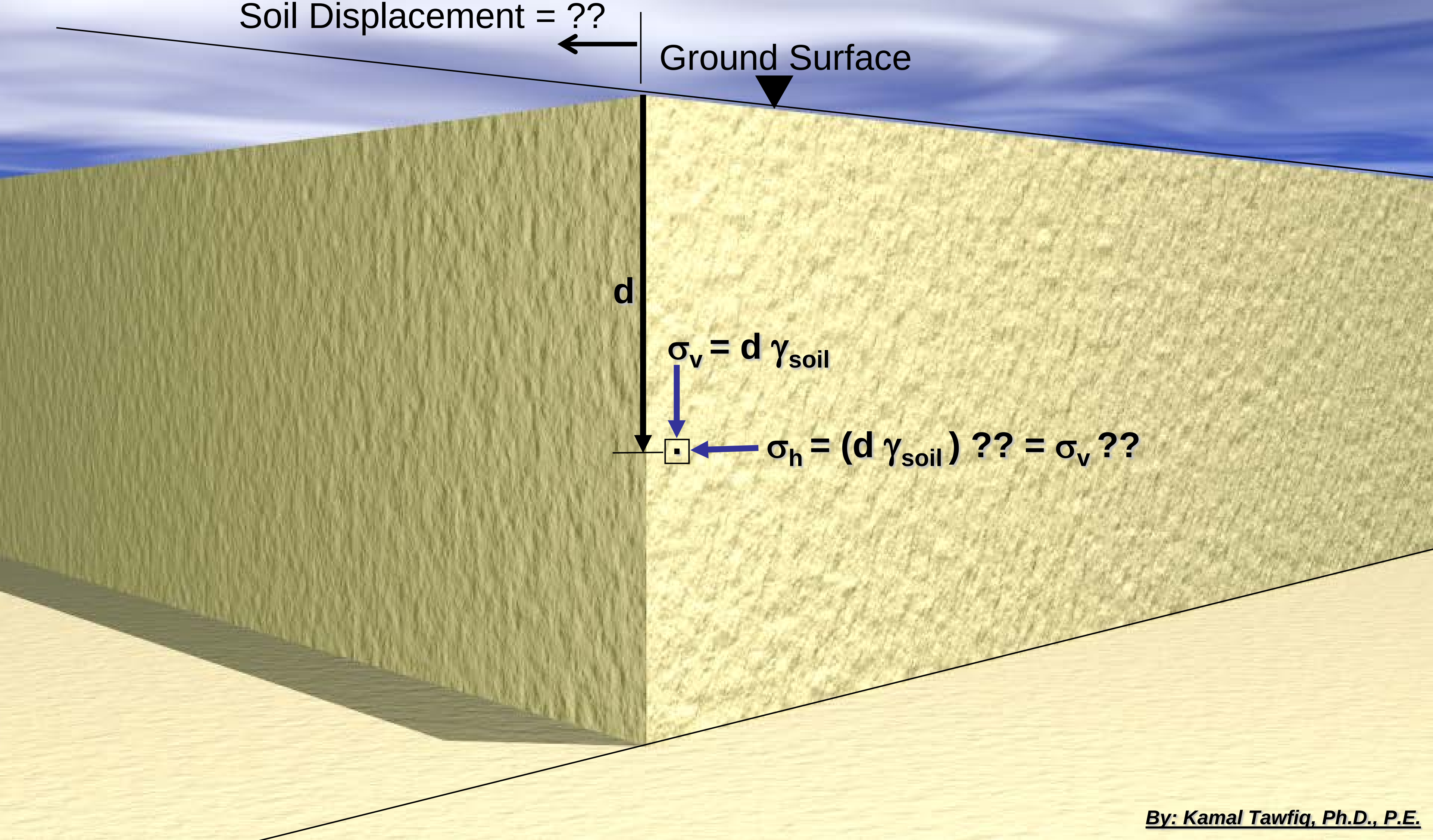
# Vertical and Horizontal Stress in Soil

Assume a vertical cut in sand soil.  Is this cut safe????

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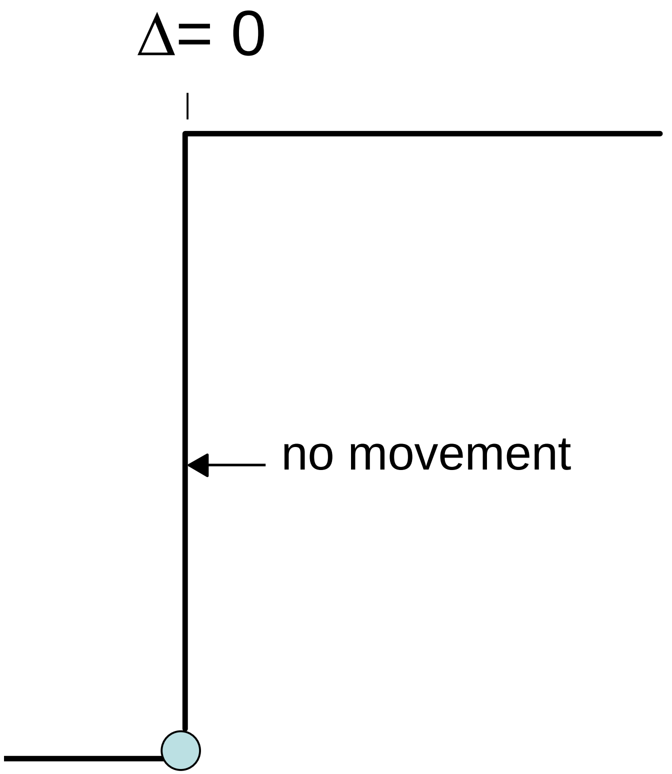
# AT REST CONDITIONS

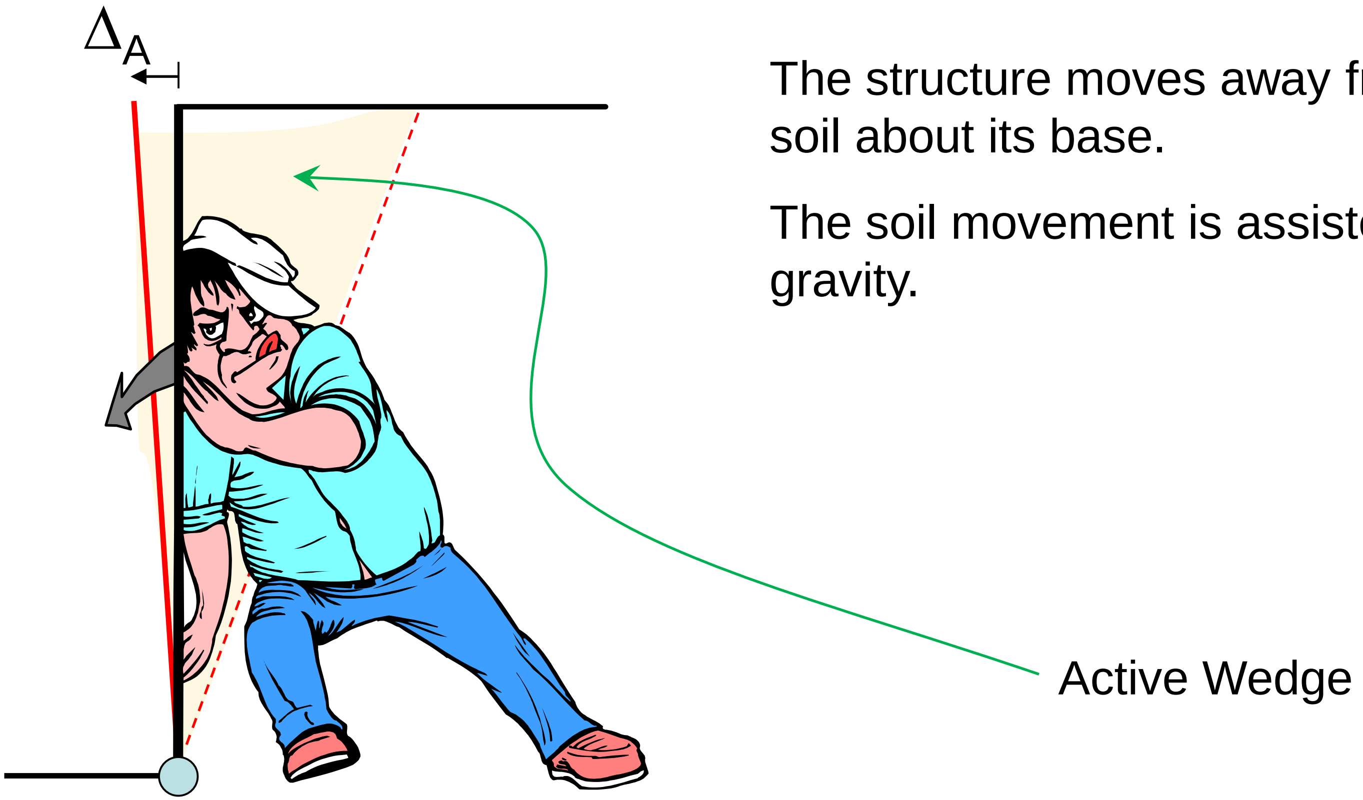



The soil remains undisturbed.

Structure is rigid, does not move and can be placed in the soil without allowing any lateral soil movement.

Lateral pressures existing in the soil before installations are still prevailing

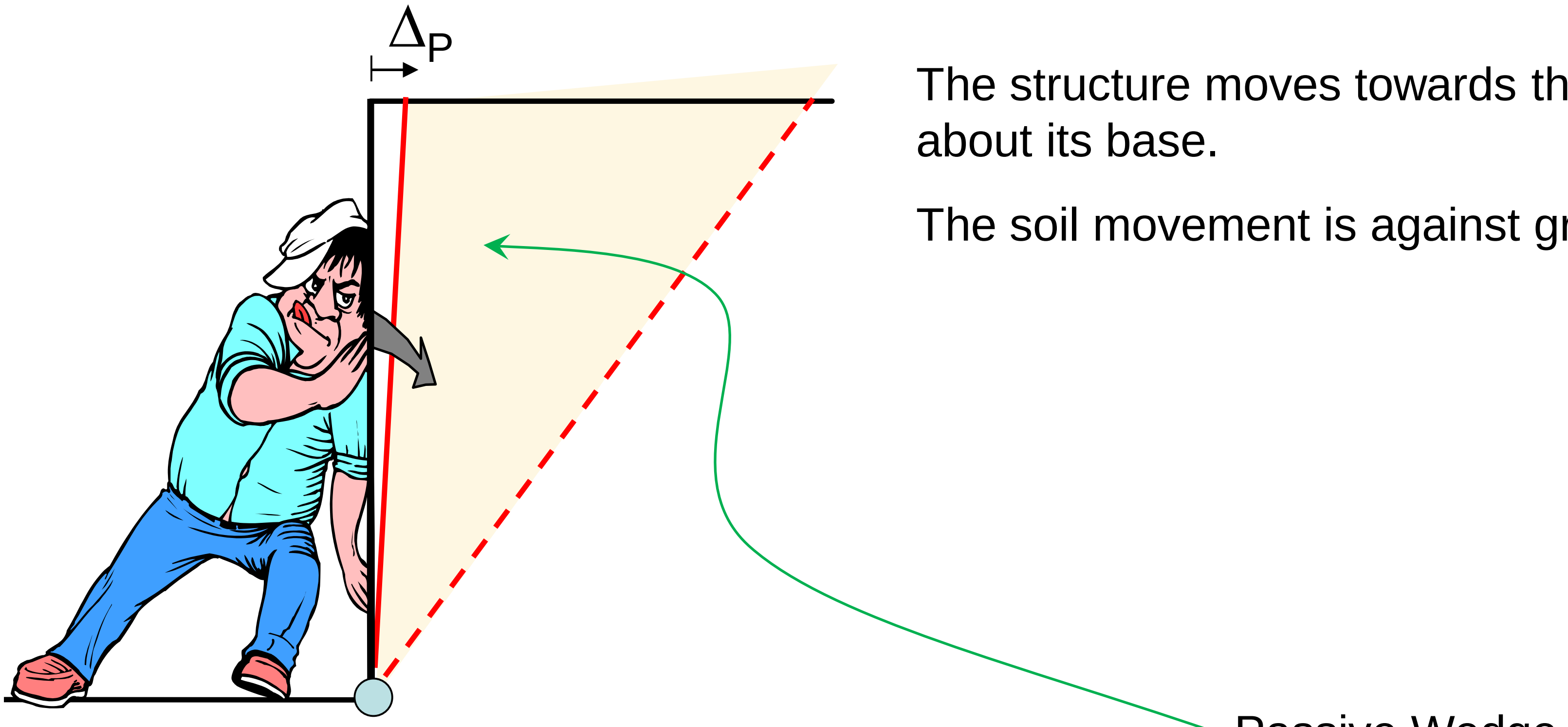
# ACTIVE CASE




The structure moves away from the soil about its base.

The soil movement is assisted by gravity.

# PASSIVE CASE

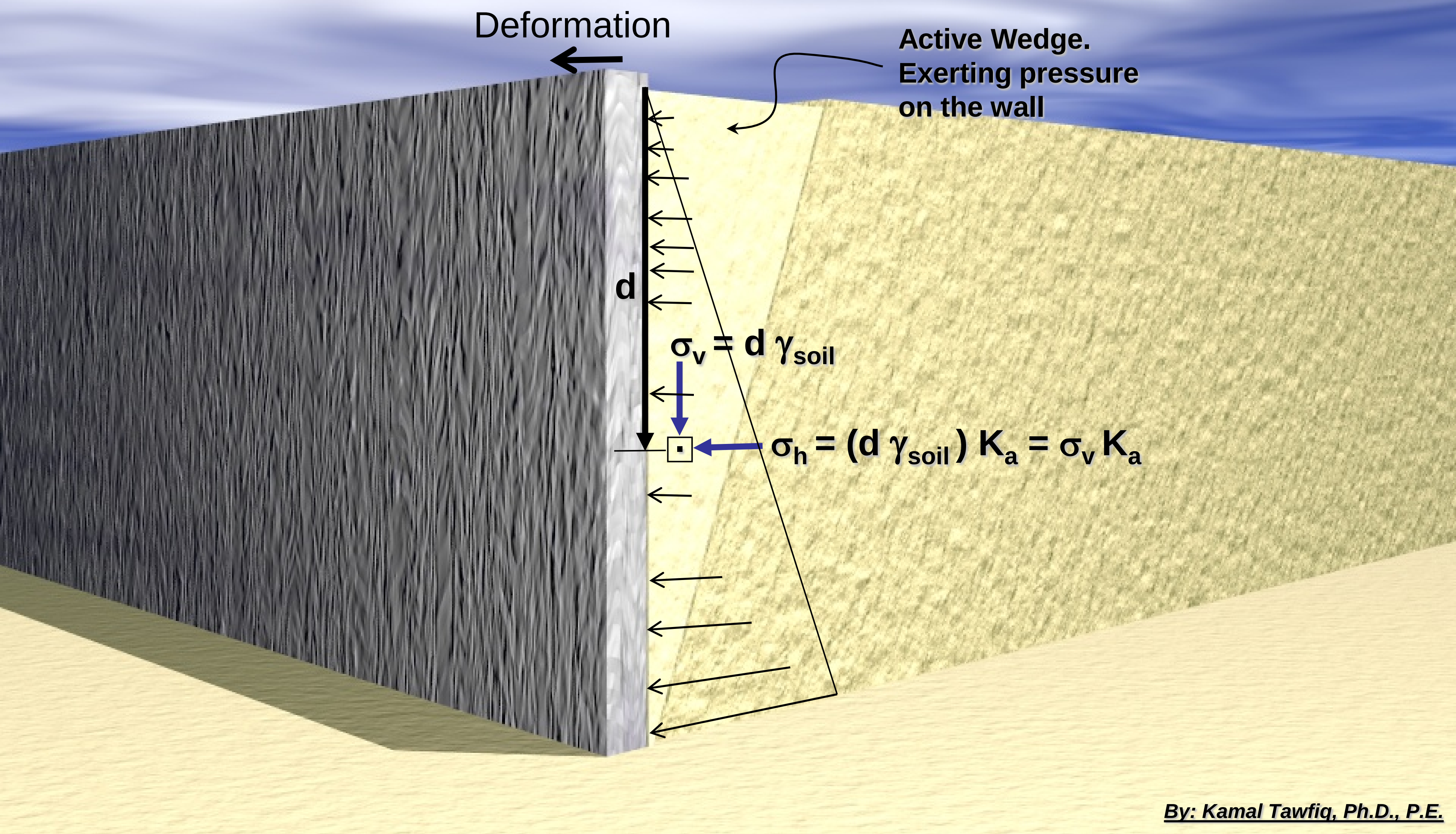



The structure moves towards the soil about its base.

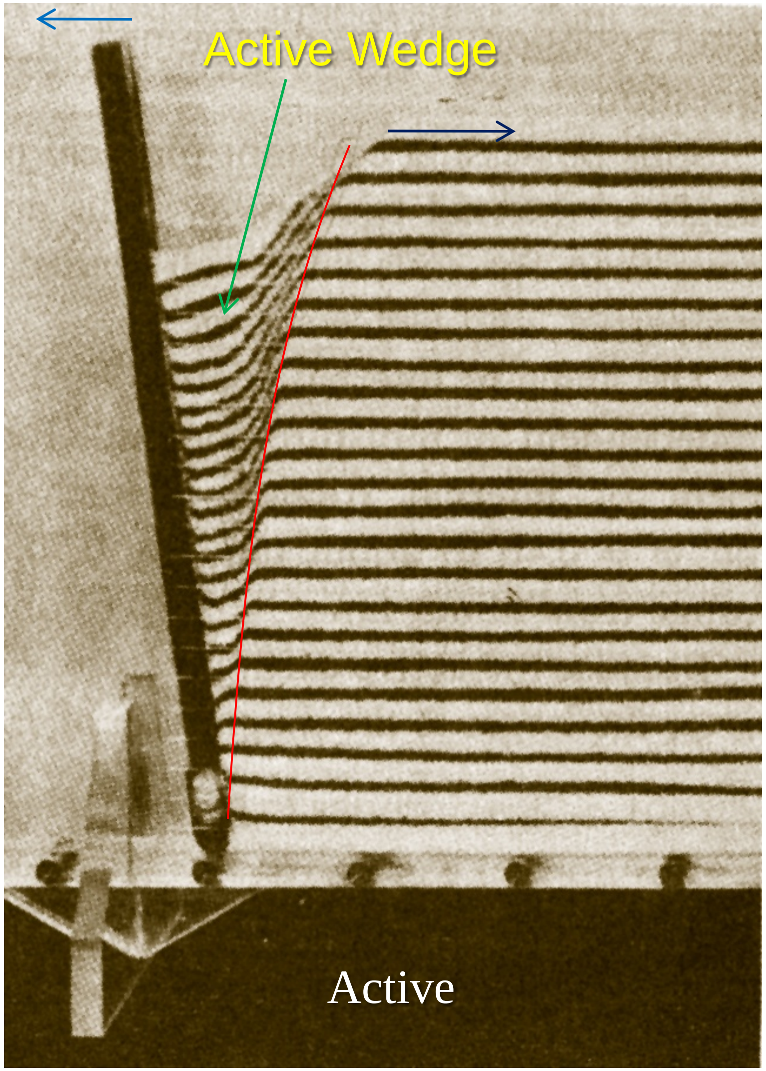
The soil movement is against gravity

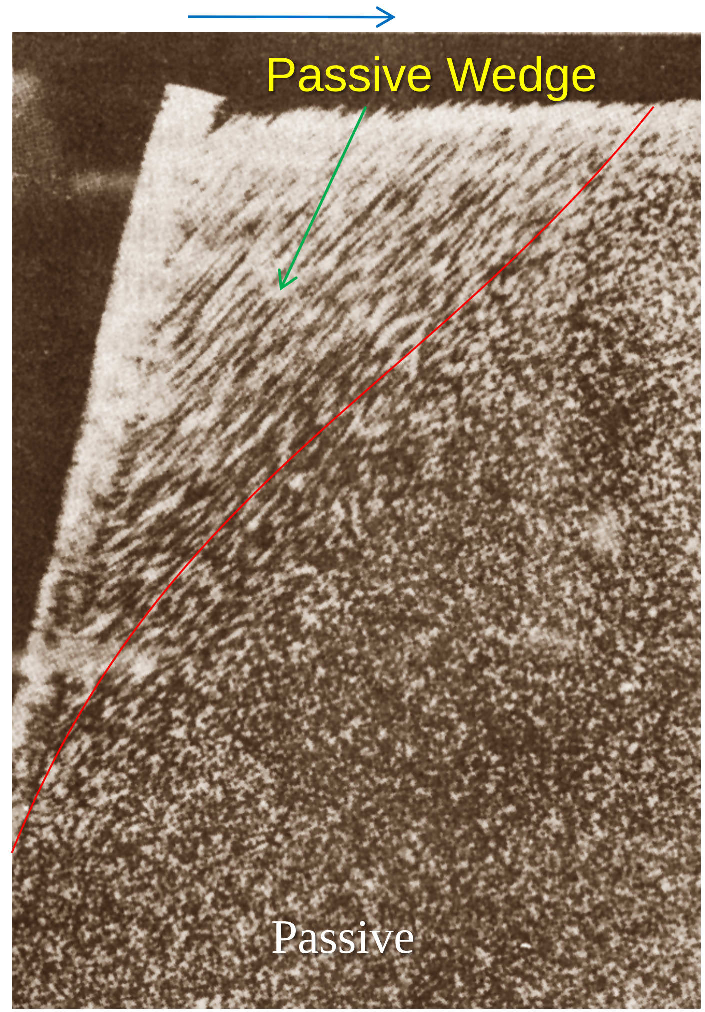
# Vertical and Horizontal Stress in Soil

Stability of a retaining wall.



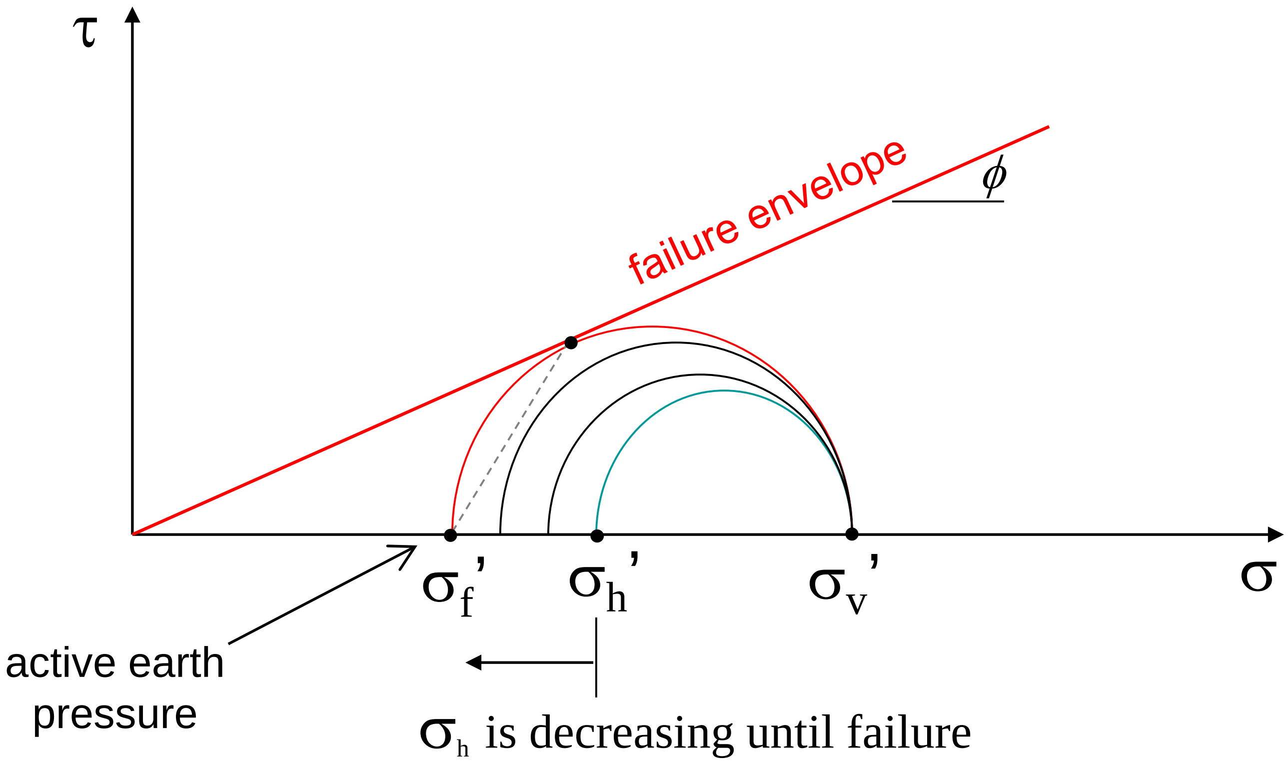
Geotechnical Design
CEG 4801
Spring 2014
By: Dr. Kamal Tawfiq

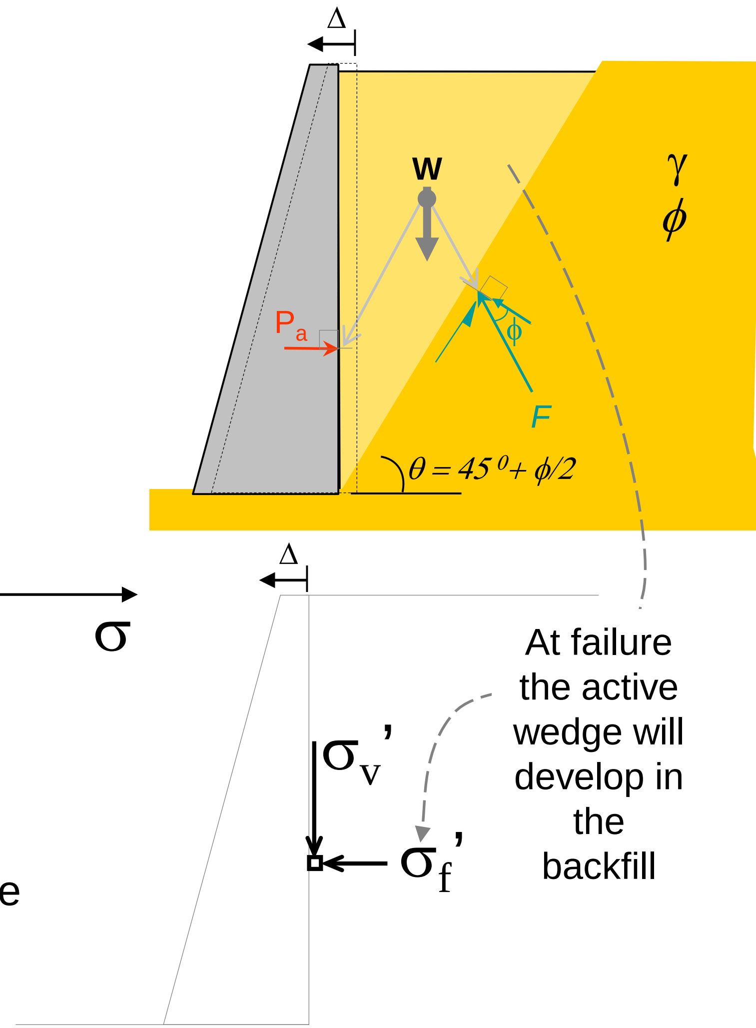



Earth Pressure Behind Retaining Wall

# Rankine's Active Earth Pressure in $\phi$ – Soil




Coefficient of active earth pressure

$$\sigma_f' = \sigma_v' \tan^2 (45 - \phi/2)$$

$$K_a = \tan^2 (45 - \phi/2) = \frac{1 - \sin\phi}{1 + \sin\phi}$$

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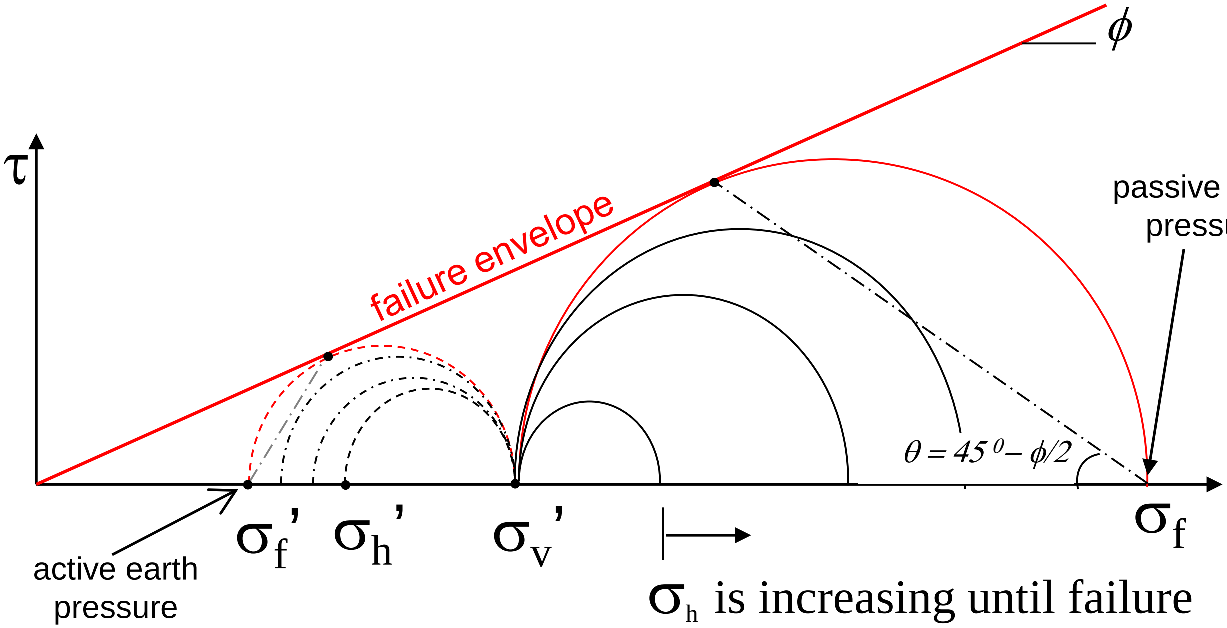
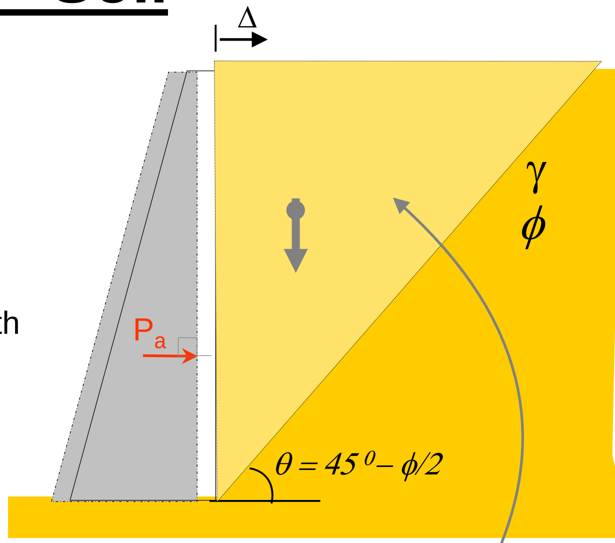
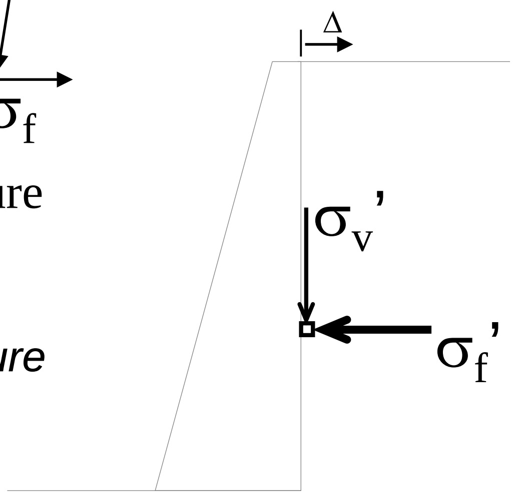
# Rankine's Passive Earth Pressure in $\phi$ – Soil




At failure the passive wedge will develop in the backfill

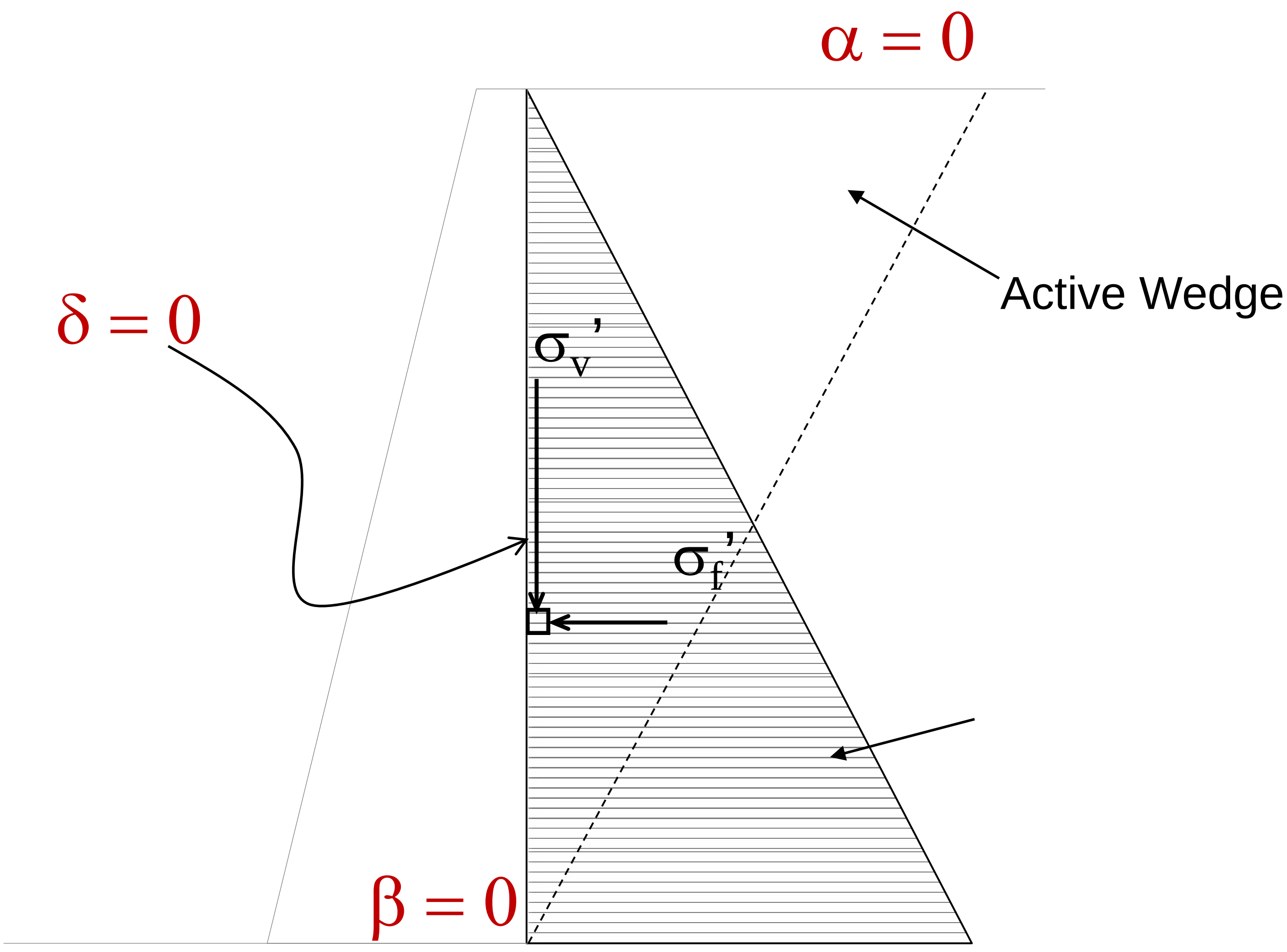
*Coefficient of passive earth pressure*

$$\sigma_f' = \sigma_v' \tan^2 (45 + \phi/2)$$

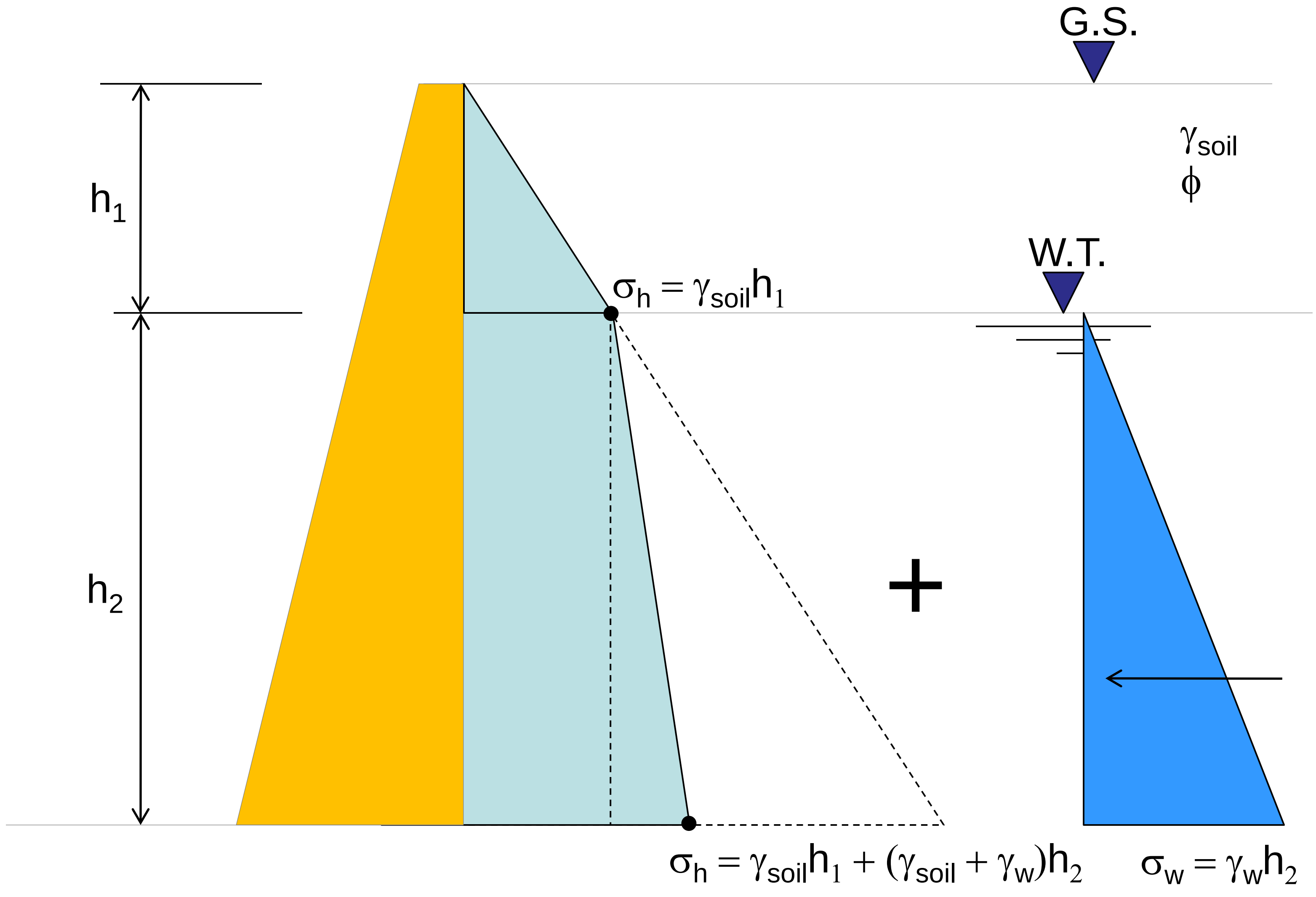
$$K_p = \tan^2 (45 + \phi/2) = \frac{1 + \sin\phi}{1 - \sin\phi}$$

***By: Kamal Tawfiq, Ph.D., P.E.***

# Rankine's Active Earth Pressure in $\phi$ – Soil

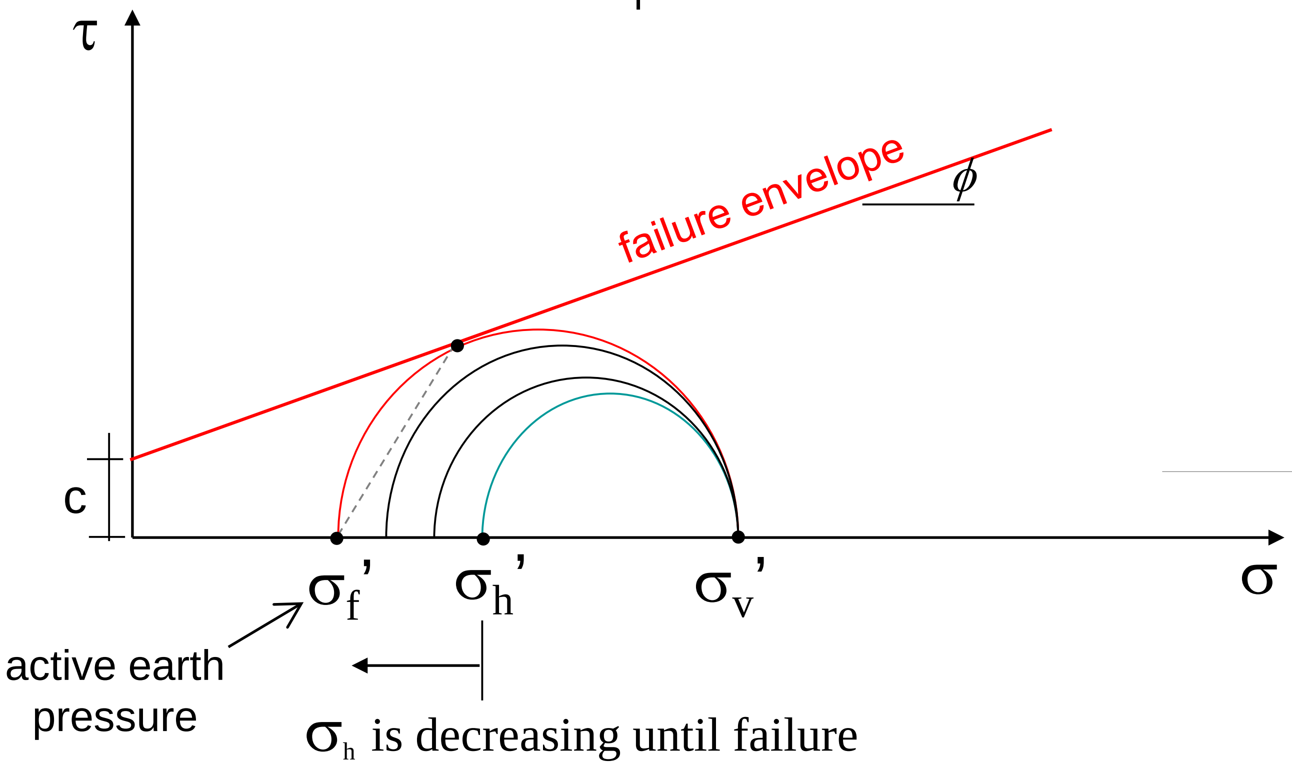


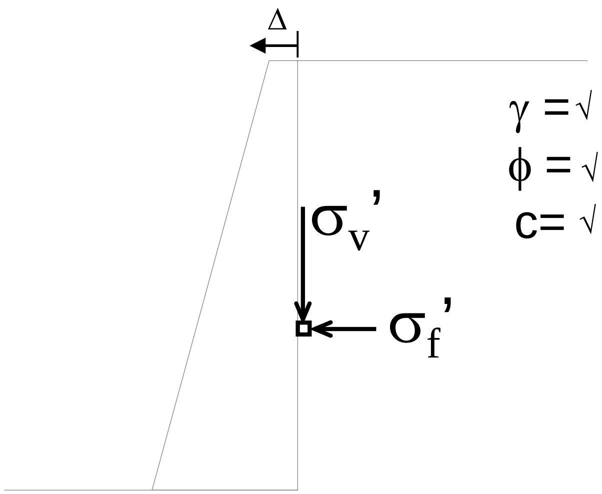

$$K_a = \tan^2(45 - \phi/2) = \frac{1 - \sin\phi}{1 + \sin\phi}$$

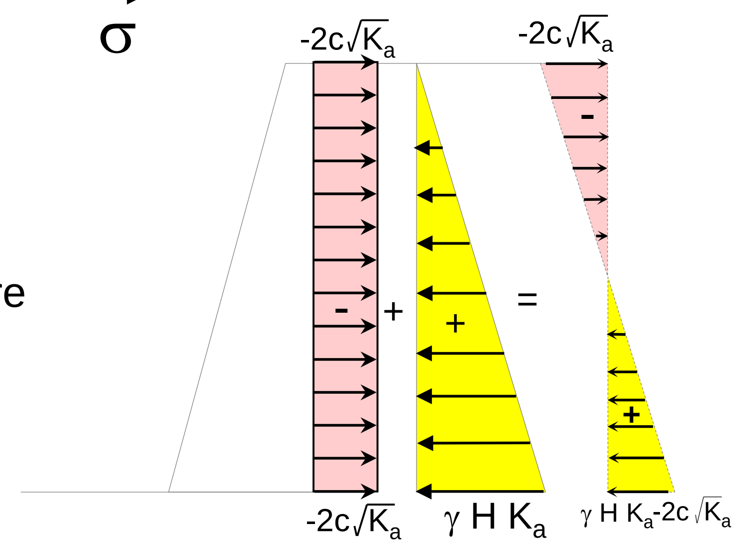

G.S.
$\gamma_{soil}$
$\phi$
W.T.
$h_1$
$h_2$
$\sigma_h = \gamma_{soil} h_1$
+
$\sigma_h = \gamma_{soil} h_1 + (\gamma_{soil} + \gamma_w) h_2$
$\sigma_w = \gamma_w h_2$

# Rankine's Active Earth Pressure

## Case 2: c- ϕ Soil




Coefficient of active earth pressure

$$\sigma_f' = \sigma_v' \tan^2(45 - \phi/2) - 2\,c\,\tan(45 - \phi/2)$$

$$\textit{where } K_a = \tan^2(45 - \phi/2) = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$\sigma_f' = \sigma_v' K_a - 2\,c\,\sqrt{K_a}$$

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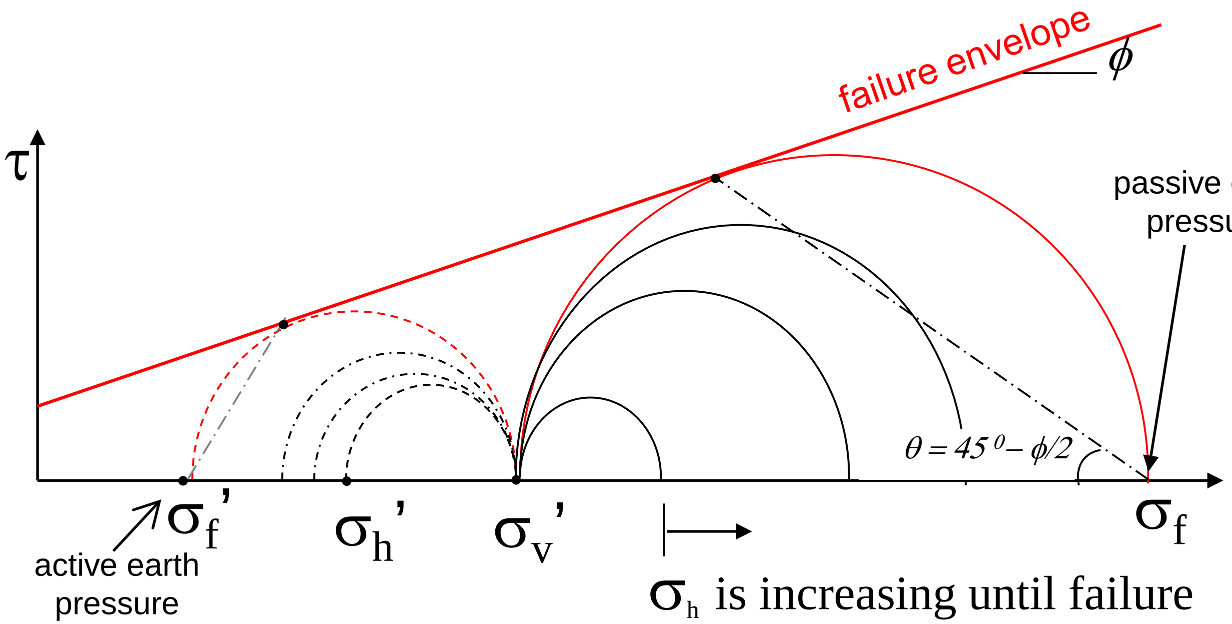
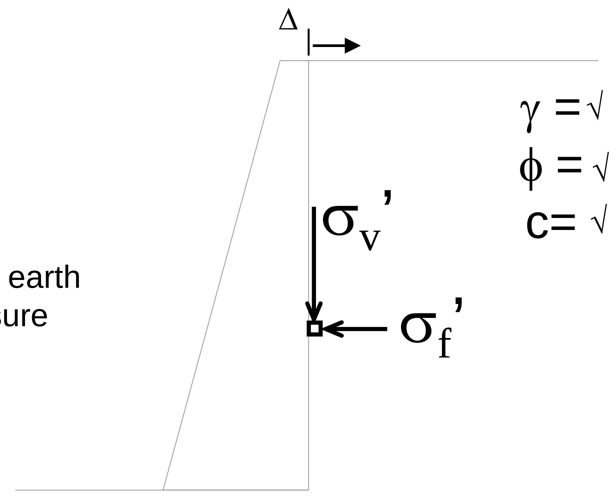
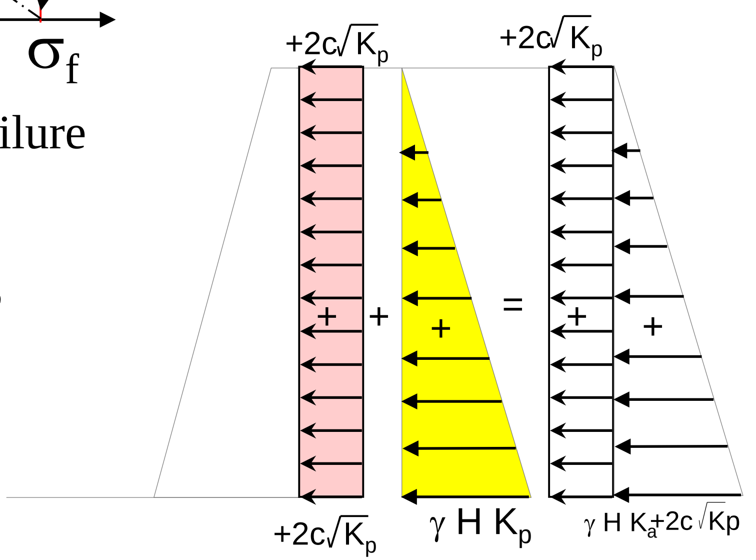
# Rankine's Passive Earth Pressure in c- ϕ Soil




*Coefficient of passive earth pressure*

$$\sigma_f' = \sigma_v' \tan^2 (45 + \phi/2) + 2\, c \tan (45 + \phi/2)$$

$$\textit{where } K_p = \tan^2 (45 + \phi/2) = \frac{1 + \sin\phi}{1 - \sin\phi}$$

$$\sigma_f' = \sigma_v' K_p + 2\, c \sqrt{K_p}$$

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## Rankine's Active Earth Pressure in $\phi$ – Soil

Case 1: For Inclined Surface

$$K_a = \cos\alpha \frac{\cos\alpha - (\cos^2\alpha - \cos^2\phi)^{1/2}}{\cos\alpha + (\cos^2\alpha - \cos^2\phi)^{1/2}}$$

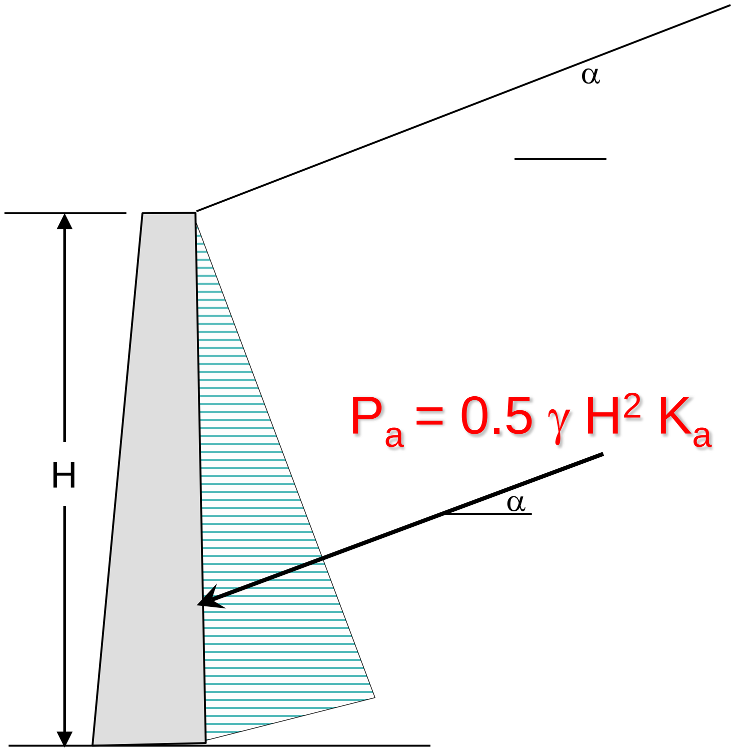
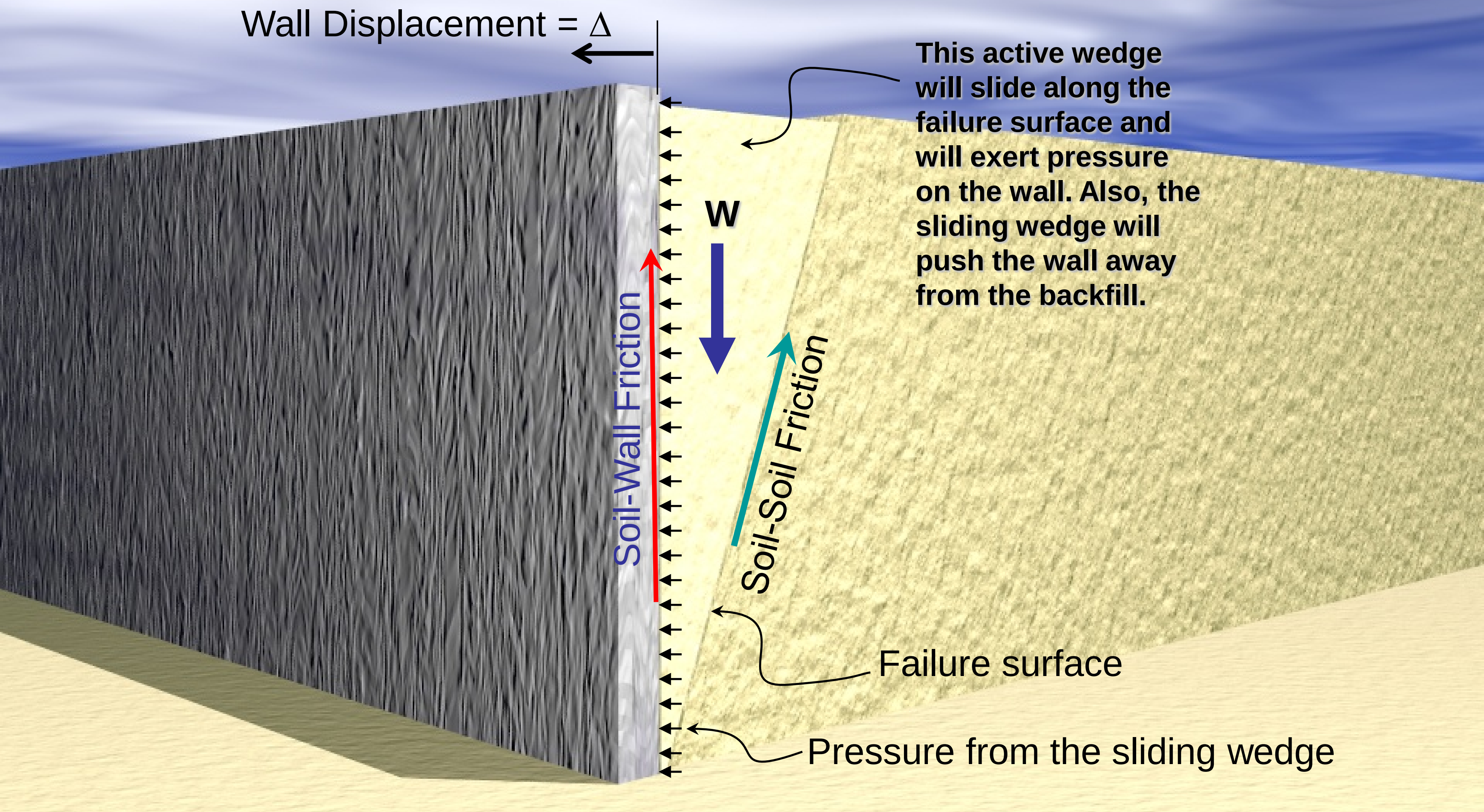



Fig.1

# Coulomb Earth Pressure Method (1776)

# Coulomb Earth Pressure Method (1776)

Forces acting on the wall.

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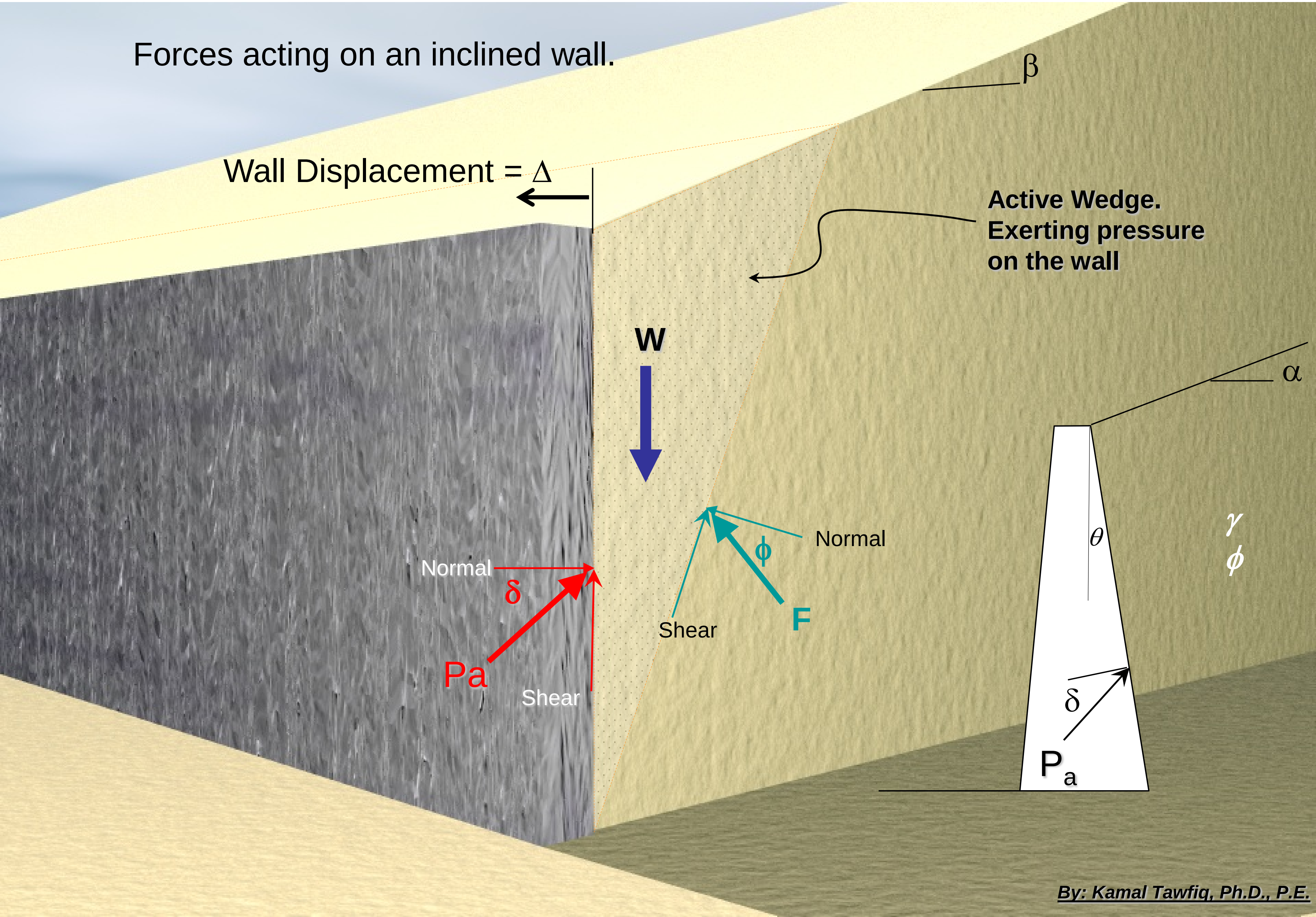
# Coulomb Earth Pressure Method (1776)

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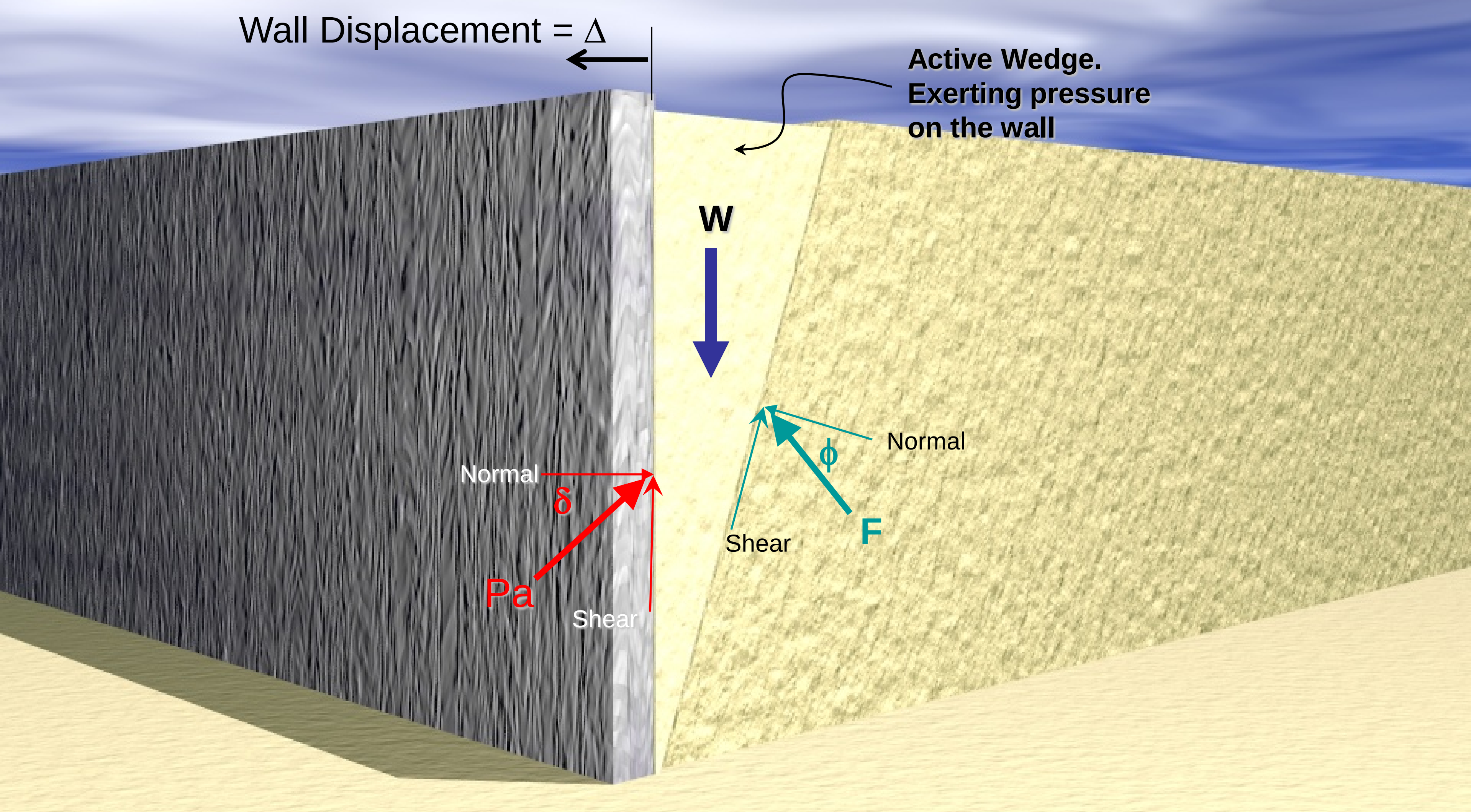
# Coulomb Earth Pressure Method (1776)

Forces acting on the wall.

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# Coulomb Earth Pressure Method (1776)

**W** = weight of the soil wedge
**R** = resultant of the shear and normal forces on the failure surface BC
**$P_a$** = the active force per unit length of the wall. The direction of Pa is inclined at an angle d to the normal drawn and the face of the wall that supports the soil

Δ = the angle of friction between the soil and the wall

**The force triangle for the wedge is shown in the Figure 2. From the sine law, the forces can be set as follows:**

$$\frac{W}{\sin(90+\theta+\delta-\beta+\phi)} = \frac{P_a}{\sin(\beta-\phi)} \quad (1)$$

$$P_a = \frac{\sin(\beta-\phi)\;W}{\sin(90+\theta+\delta-\beta+\phi)} \quad (2)$$

Substitute for W, Eq. 2 can be written as

$$P_a = 0.5\,\gamma H^2\left[\frac{\cos(\beta-\phi)\cos(\phi-\alpha)\sin(\beta-\phi)}{\cos^2\theta\,\sin(\beta-\alpha)\,\sin(90+\theta-\delta-\beta+\phi)}\right] \quad (2)$$

To determine the critical value for β for maximum $P_a$

$$(dP_a/d\beta) = 0$$

$$P_a = ½\,K_a\,\gamma\,H^2$$

Where $K_a$ = Coulomb's active earth pressure coefficient

$$K_a = \frac{\cos^2(\phi-\theta)}{\cos^2\theta\,\cos(\delta-\theta)\left[1+\sqrt{\dfrac{\sin(\delta+\phi)\,\sin(\phi-\alpha)}{\cos(\delta+\theta)\,\cos(\theta-\alpha)}}\right]^2}$$

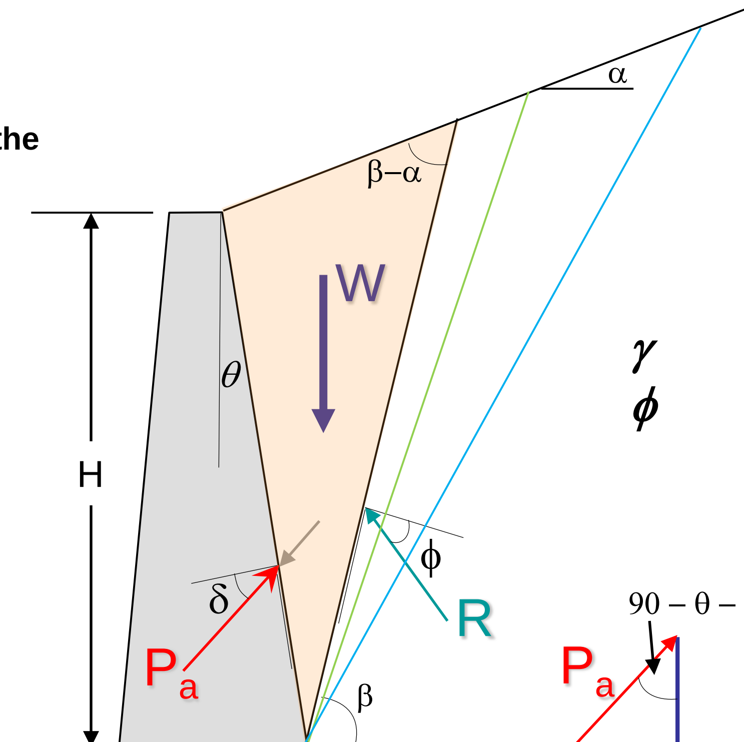

Fig.1

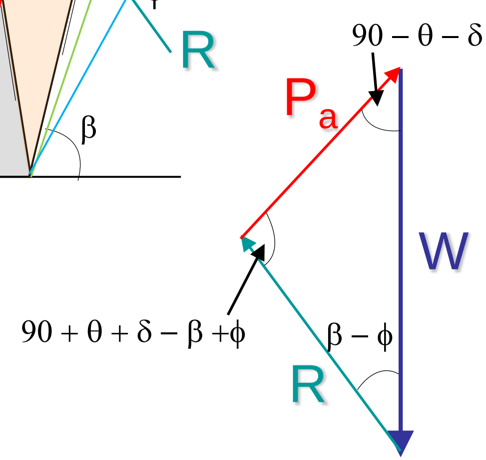

Fig.2

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# Coulomb's Earth Pressure

$\phi = \surd$

$\theta = 0$

$\delta = 0$

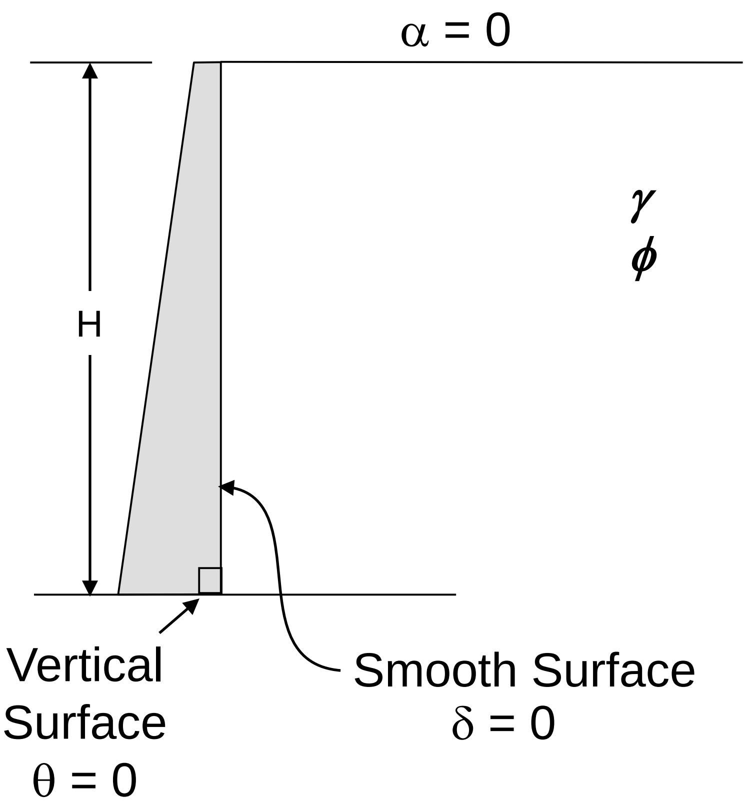
$\alpha = 0$

$$K_a = \frac{\cos^2(\phi - \cancel{\theta})}{\cos^2\cancel{\theta}\,\cos(\cancel{\delta} - \cancel{\theta})\left[1 + \sqrt{\dfrac{\sin(\cancel{\delta} + \phi)\,\sin(\phi - \cancel{\alpha})}{\cos(\cancel{\delta} + \theta)\,\cos(\theta - \cancel{\alpha})}}\right]^2}$$

Under the given wall and backfill conditions, $K_a$ of Coulomb's active earth pressure becomes equivalent to $K_a$ of Rankine's

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$P_a = \tfrac{1}{2} K_a \gamma H^2$$

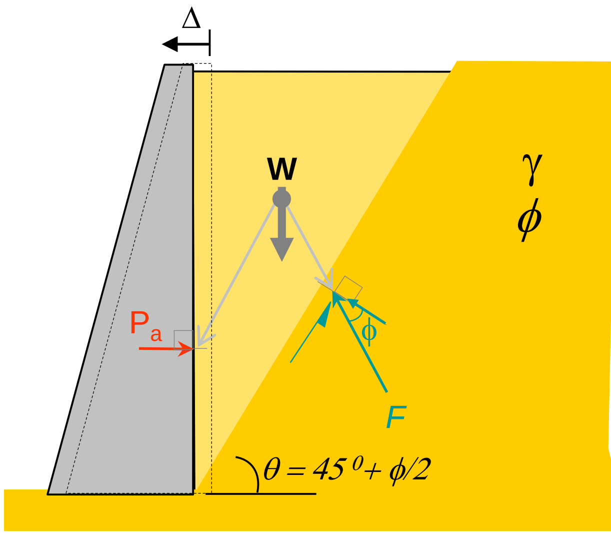


# Active Earth Pressure in ϕ – Soil

## Example -1

**Given**:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ($\gamma$) = 115 pcf
- Angle of soil friction ($\phi$) = 30$^o$
- Assume wall to be smooth




**Find**:

- Lateral force $P_a$ acting on the wall

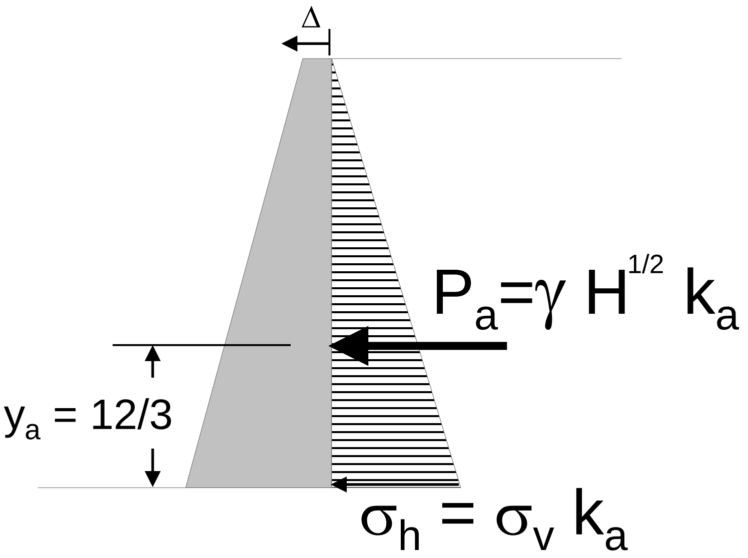
**Solution**:

$\sigma_h = \sigma_v\, k_a$

$P_a = \gamma\ H^{0.5}\, k_a$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$P_a = 115 \times 12^2 \times 0.5$

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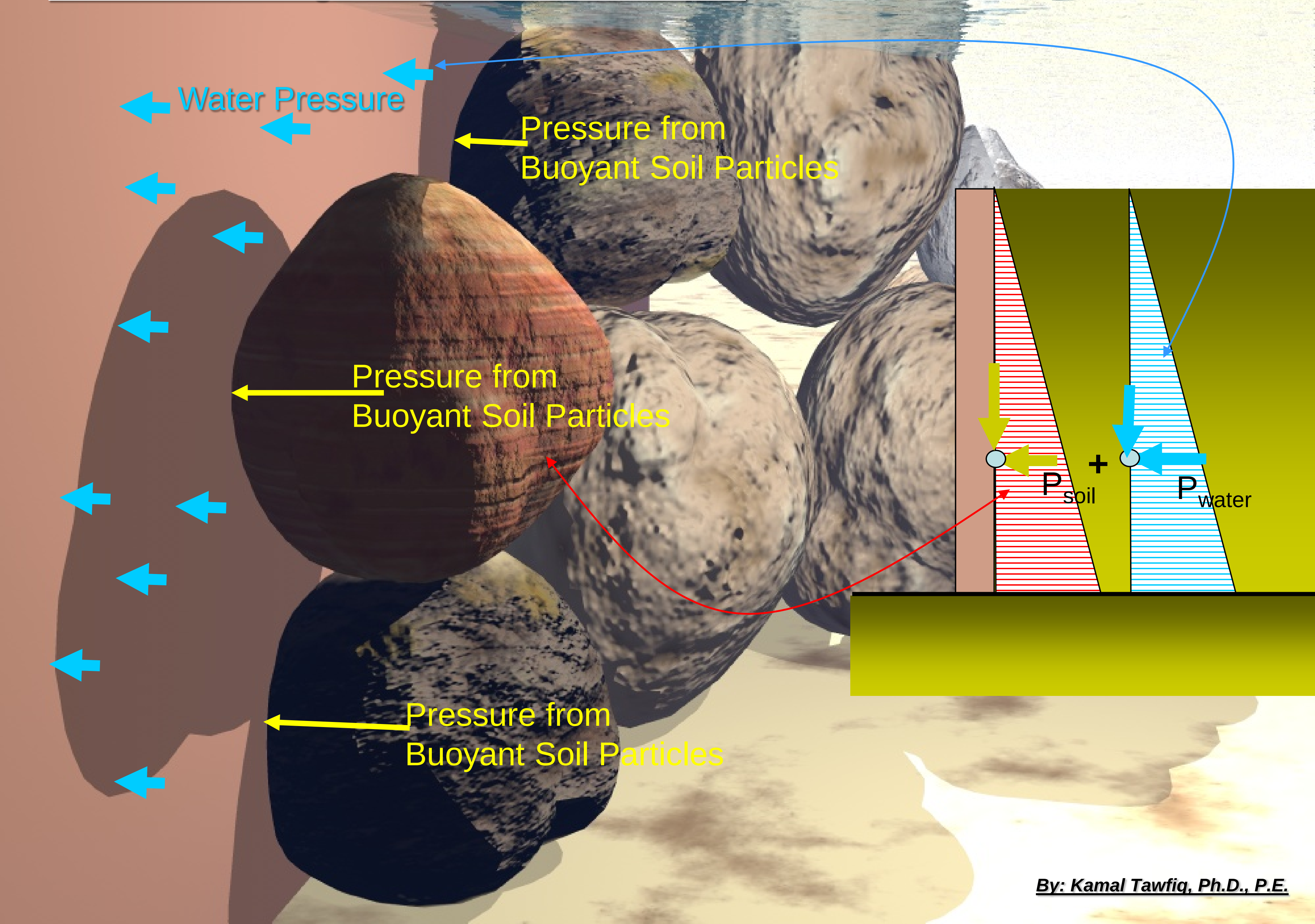
# Effect of Submerged Soil of Lateral Pressure

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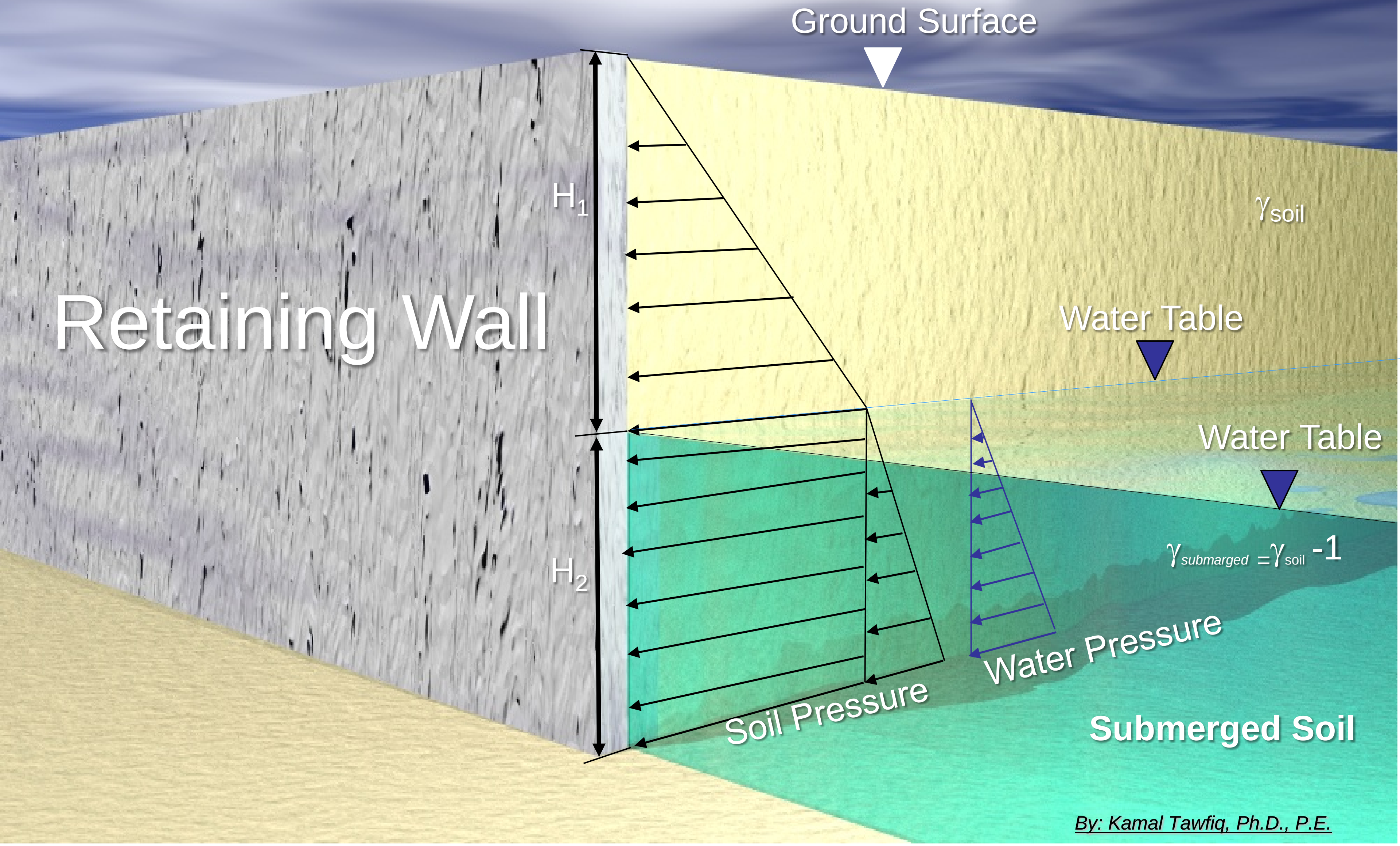
# Effect of Submerged Soil of Lateral Pressure

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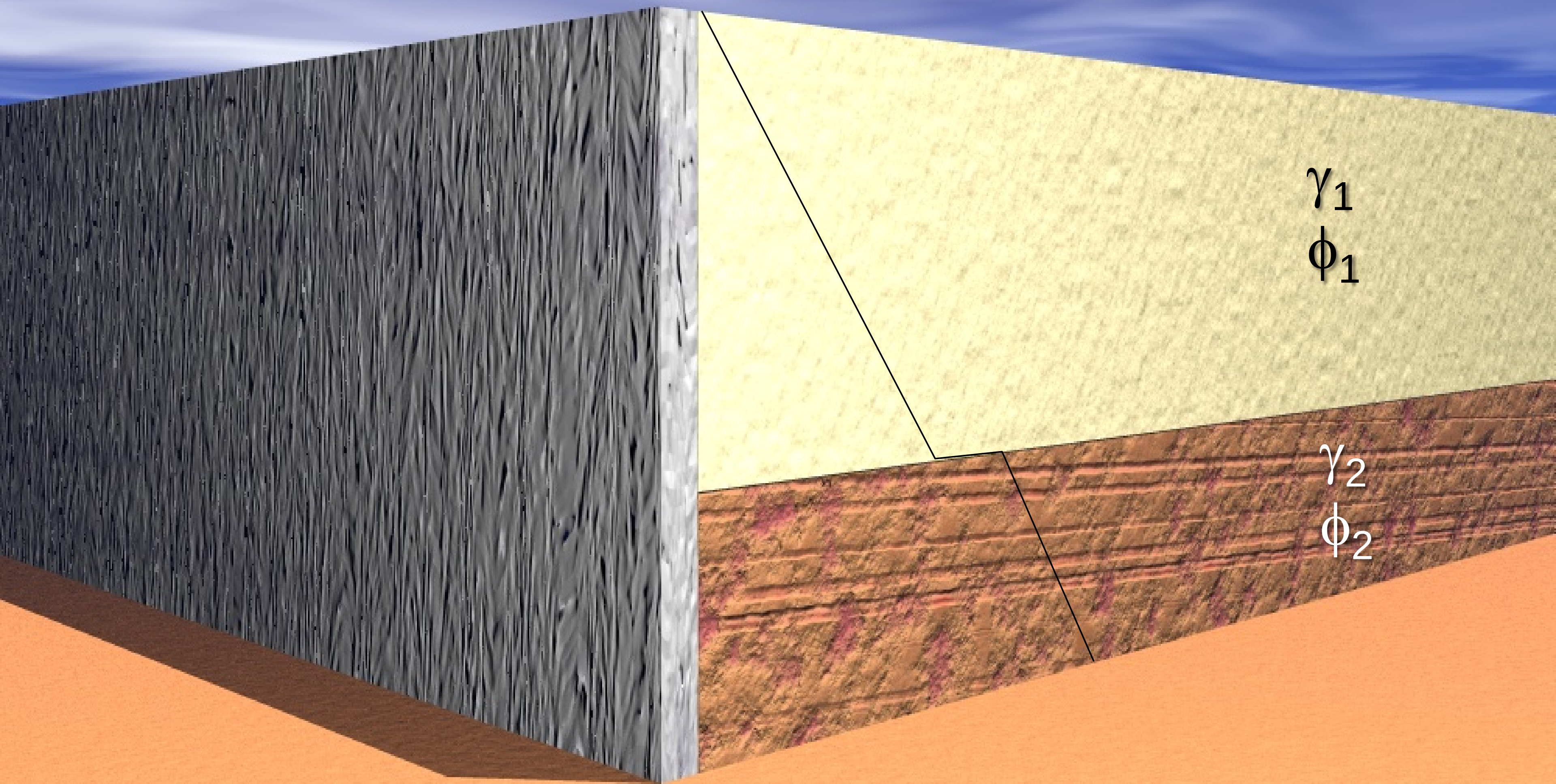
Effect of Soil Layers on Lateral Pressure

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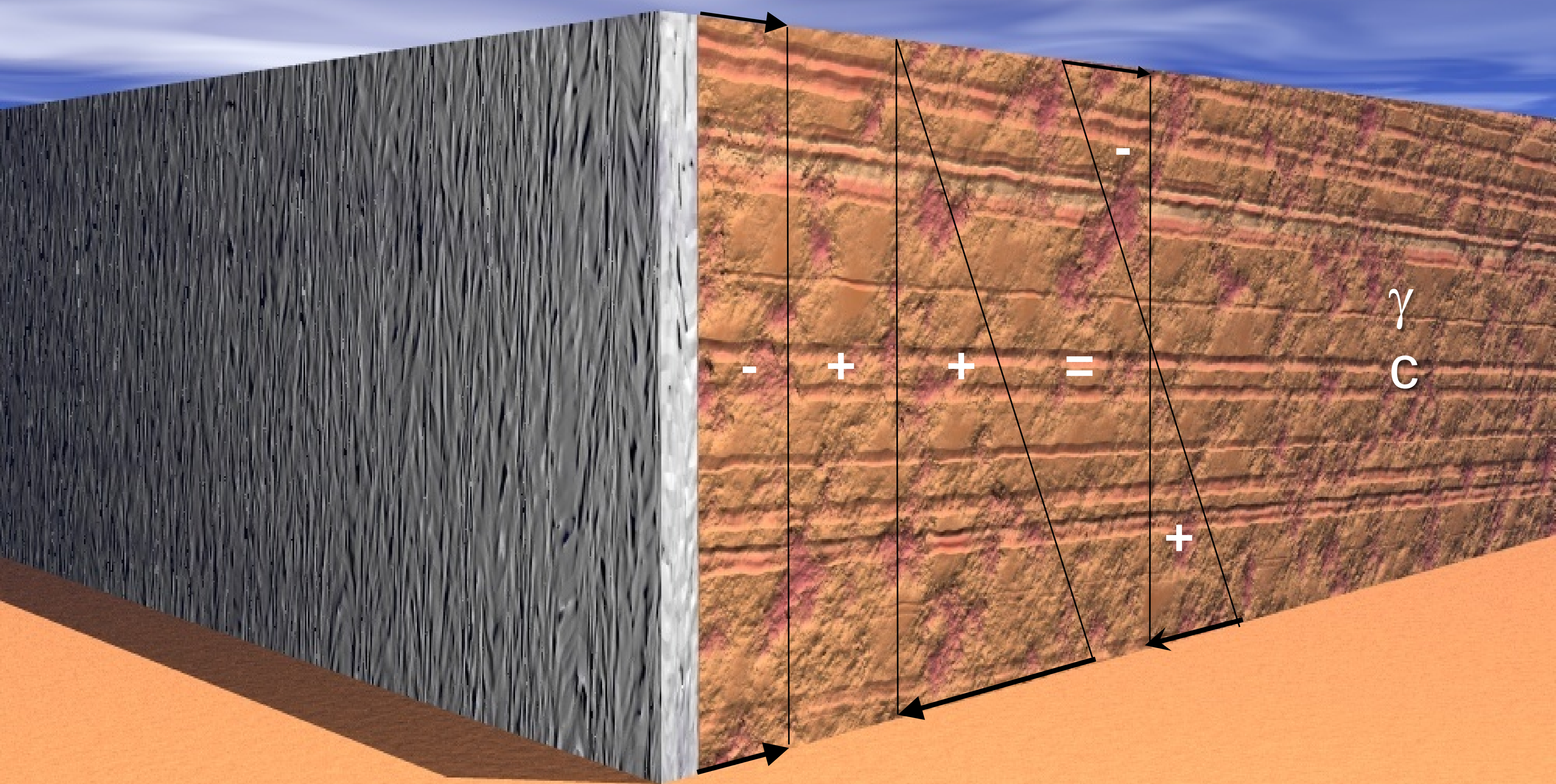
# Effect of Clay on Lateral Pressure

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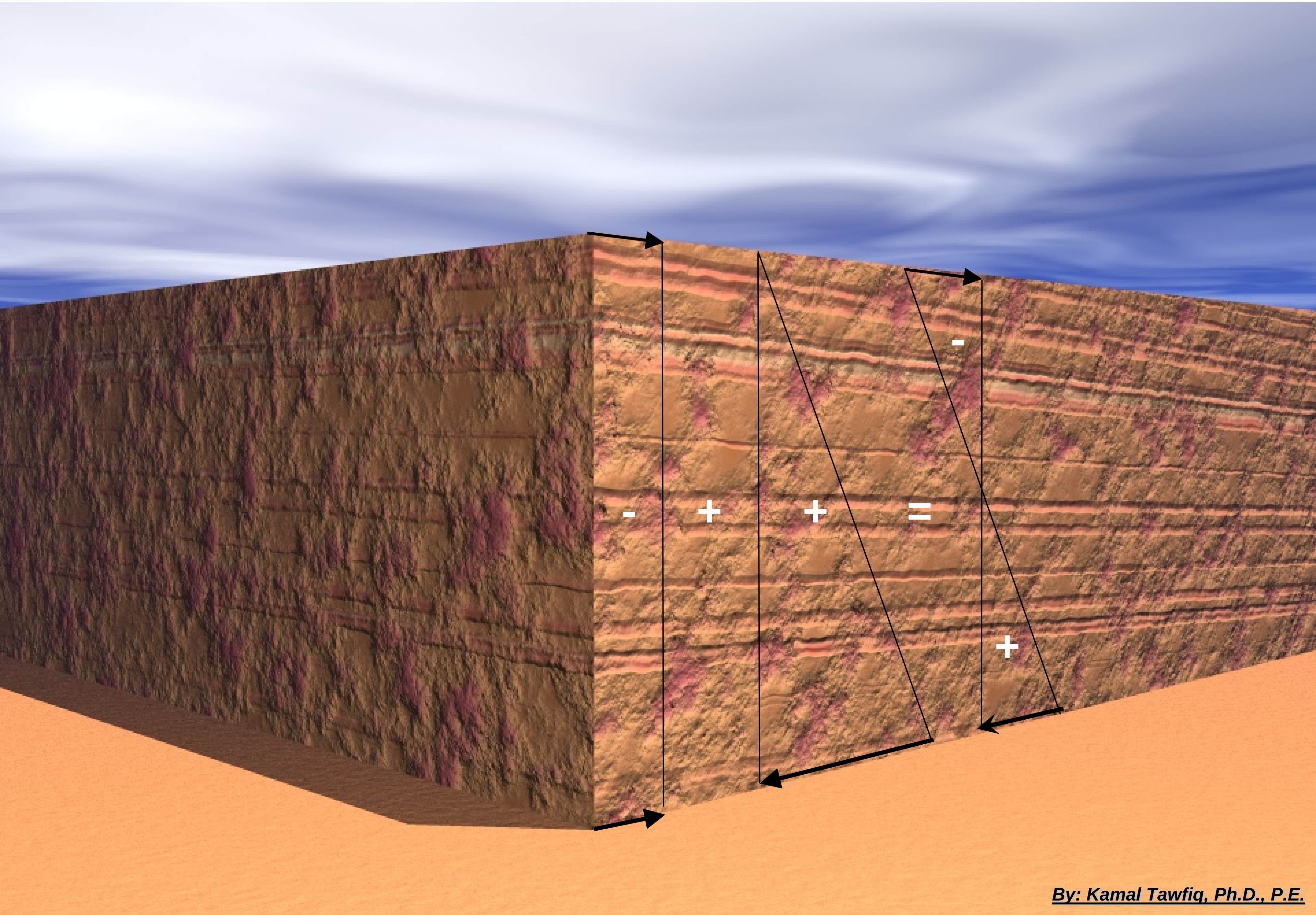


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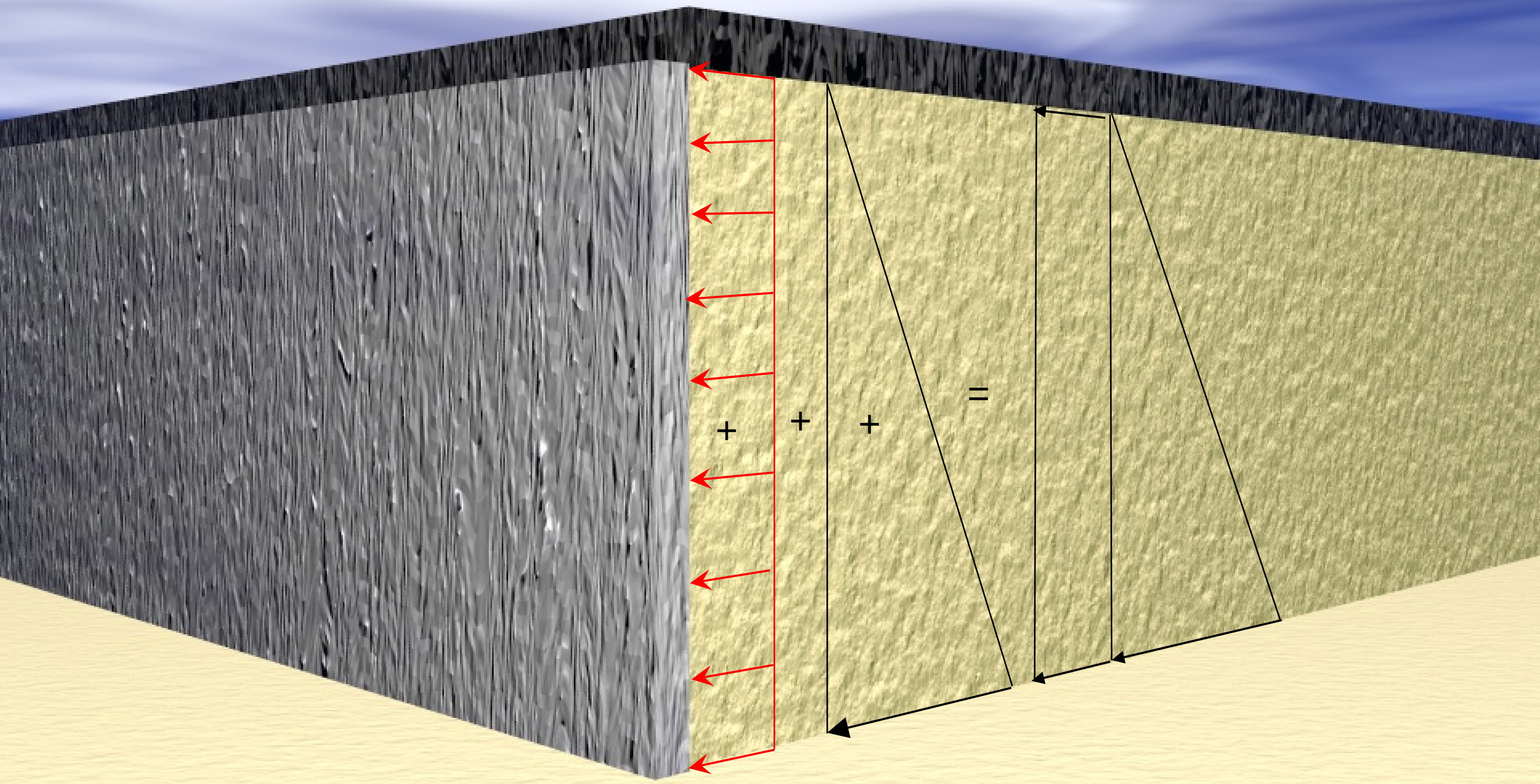
# Effect of Surcharge Load on Earth Pressure

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Geotechnical Design
CEG 4801
Fall 2004
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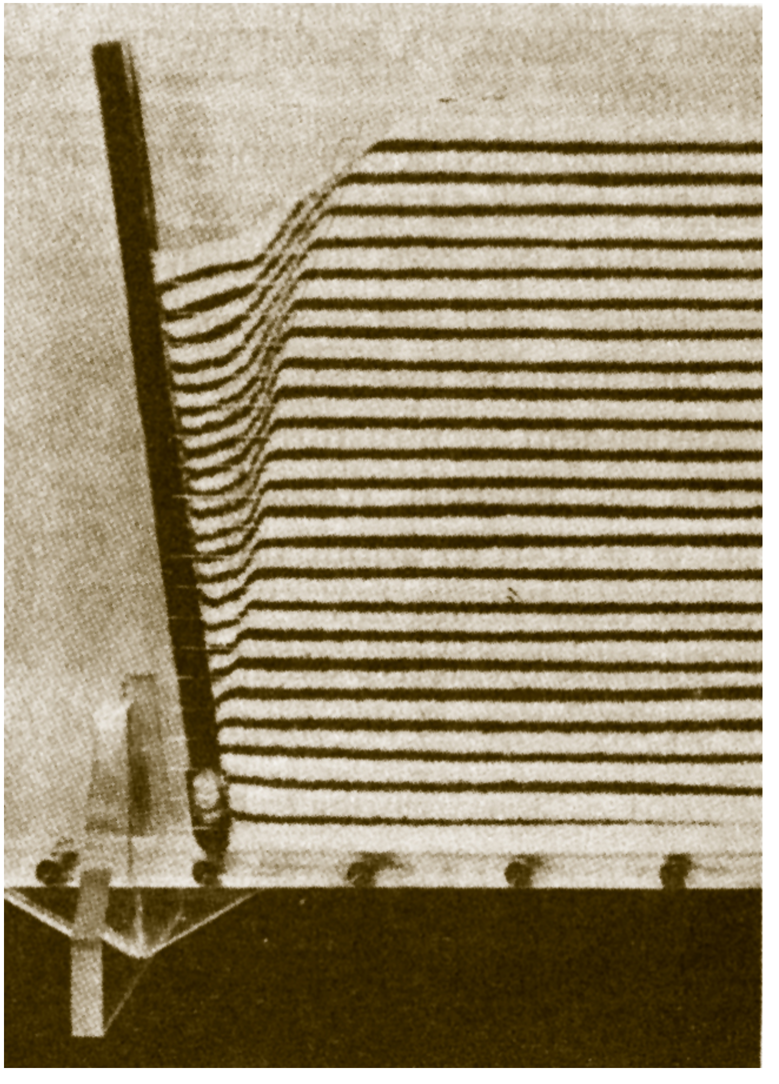
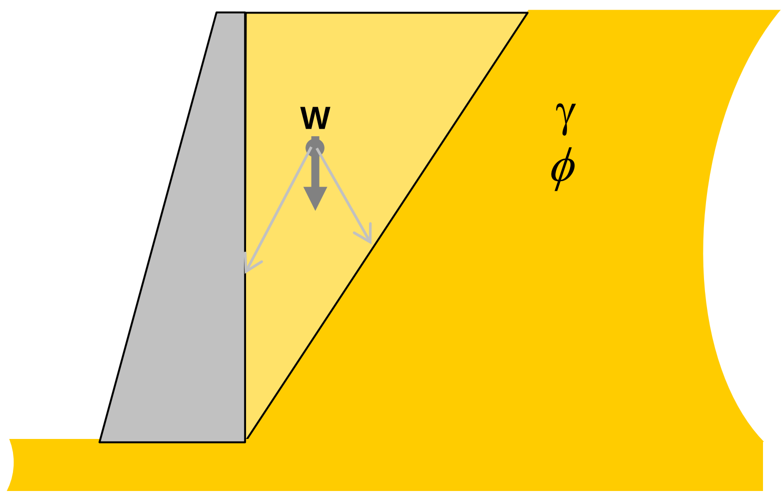
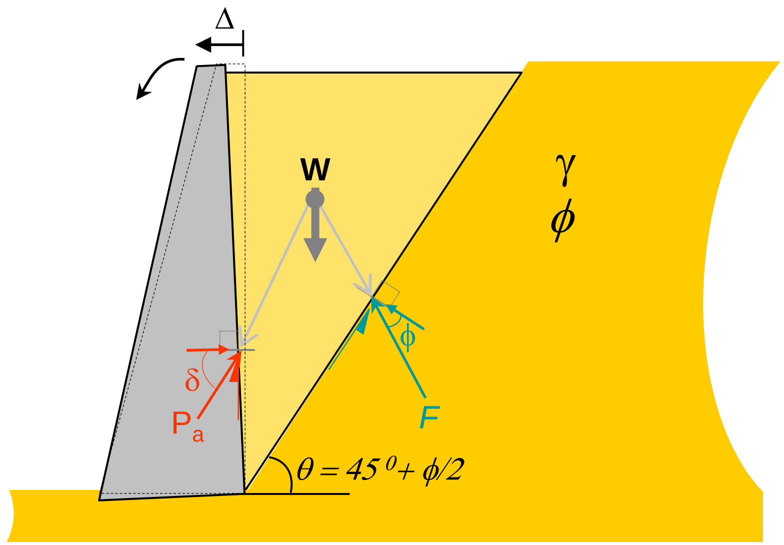
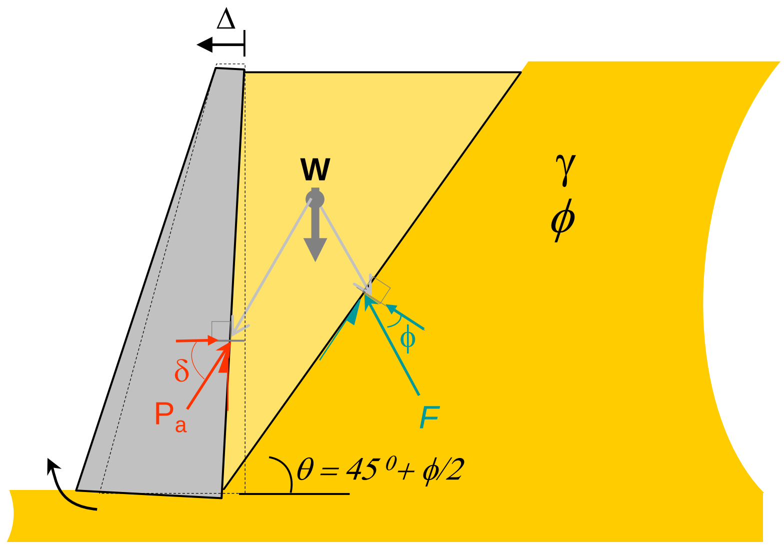


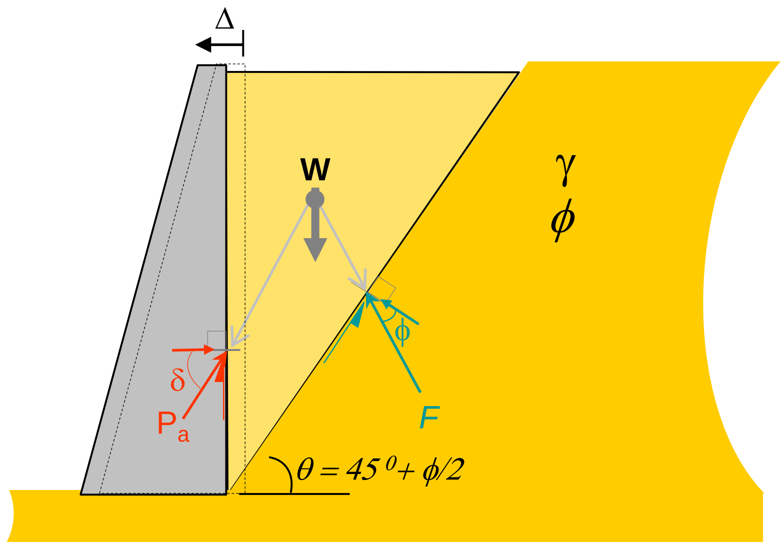
Earth Pressure Behind Retaining Wall

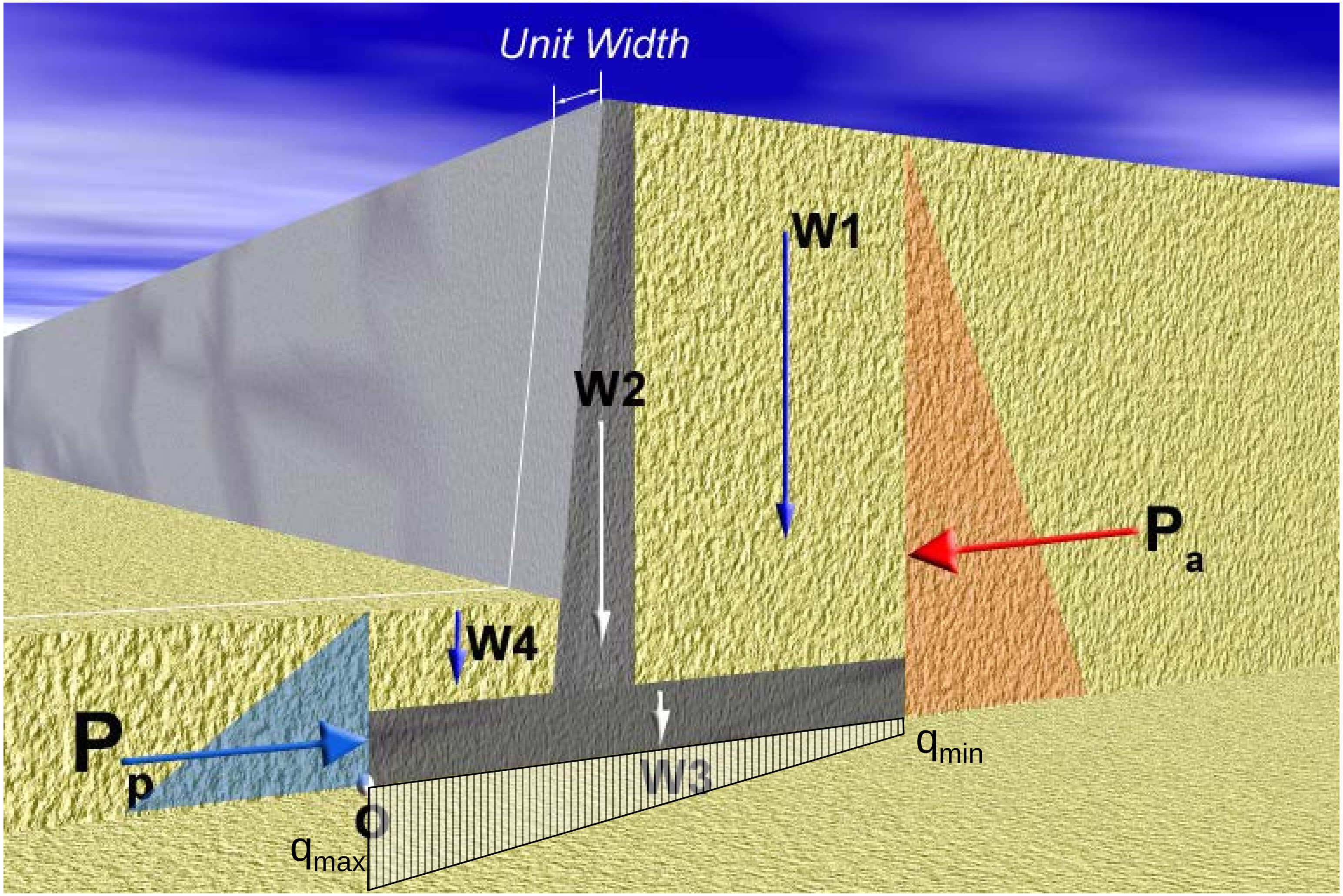


W
γ
ϕ

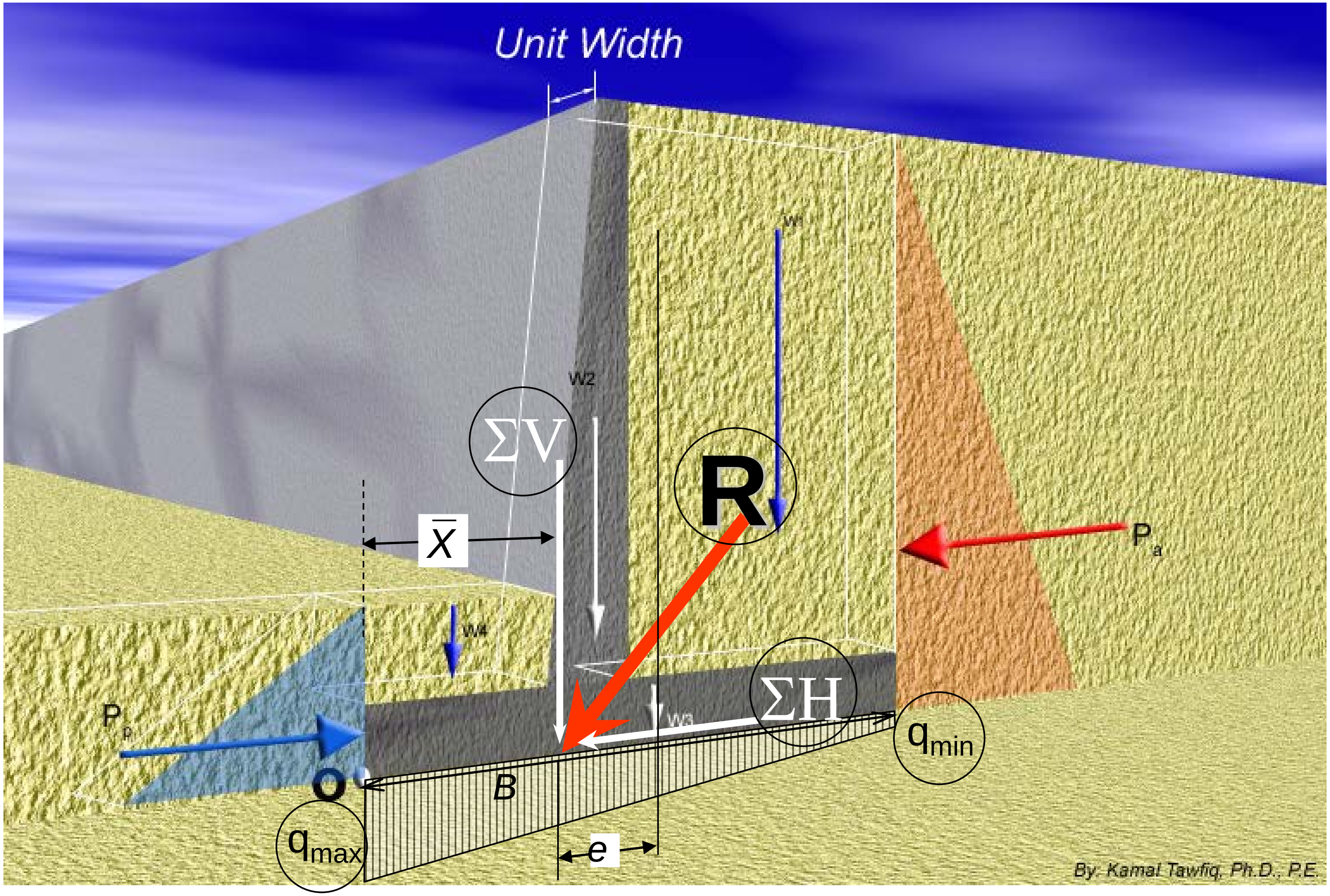

Δ
W
γ
ϕ
δ
Pa
ϕ
F
θ = 45° + ϕ/2


Δ
W
γ
ϕ
δ
P a
ϕ
F
θ = 45° + ϕ/2


Δ
W
γ
ϕ
δ
P a
ϕ
F
θ = 45 0 + ϕ/2


Unit Width
W1
W2
W4
W3
Pa
Pp
O
qmin
qmax

## 3- Check for Bearing Capacity Failure




ΣV = sum of all vertical loads
ΣH = sum of all horizontal loads

$$R = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$$

$$M_{net} = \Sigma M_R - \Sigma M_o$$

$$\bar{X} = \frac{M_{net}}{\Sigma V}$$

$$e = \frac{B}{2} - \bar{X}$$

$$q = \frac{\Sigma V}{A} \pm \frac{M_{net}\, y}{I}$$

$$q_{\substack{max \\ min}} = \frac{\Sigma V}{B}\left(1 \pm \frac{6e}{B}\right)$$