9.9 Vertical Stress Due to Embankment Loading

Figure 9.17 shows the cross section of an embankment of height H. For this two-dimensional loading condition the vertical stress increase may be expressed as

$$\Delta\sigma_z = \frac{q_o}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right] \quad (9.25)$$

where $q_o = \gamma H$
γ = unit weight of the embankment soil
H = height of the embankment

$$\alpha_1\,(\text{radians}) = \tan^{-1}\left(\frac{B_1 + B_2}{z}\right) - \tan^{-1}\left(\frac{B_1}{z}\right) \quad (9.26)$$

$$\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right) \quad (9.27)$$

For a detailed derivation of the equation, see Das (1997). A simplified form of Eq. (9.25) is

$$\Delta\sigma_z = q_o I_3 \quad (9.28)$$

where I_3 = a function of B_1/z and B_2/z.

The variation of I_3 with B_1/z and B_2/z is shown in Figure 9.18 (Osterberg, 1957).

Figure 9.17 Embankment loading

Example 9.8

An embankment is shown in Figure 9.19a. Determine the stress increase under the embankment at points A_1 and A_2.

Solution

$$\gamma H = (17.5)(7) = 122.5 \text{ kN/m}^2$$

Stress Increase at A_1

The left side of Figure 9.19b indicates that $B_1 = 2.5$ m and $B_2 = 14$ m. So,

$$\frac{B_1}{z} = \frac{2.5}{5} = 0.5; \frac{B_2}{z} = \frac{14}{5} = 2.8$$

According to Figure 9.18, in this case, $I_3 = 0.445$. Because the two sides in Figure 9.19b are symmetrical, the value of I_3 for the right side will also be 0.445. So,

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = q_o[I_{3(\text{Left})} + I_{3(\text{Right})}]$$
$$= 122.5[0.445 + 0.445] = \mathbf{109.03\ kN/m^2}$$

Figure 9.18 Osterberg's ch[art] for determinat[ion] of vertical stre[ss] due to embank-ment loading

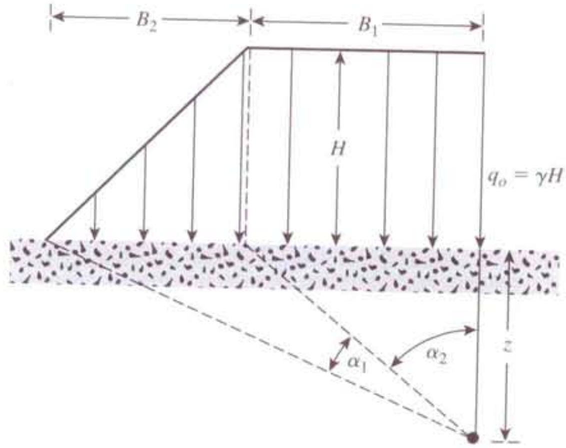

Figure 9.17 Embankment loading

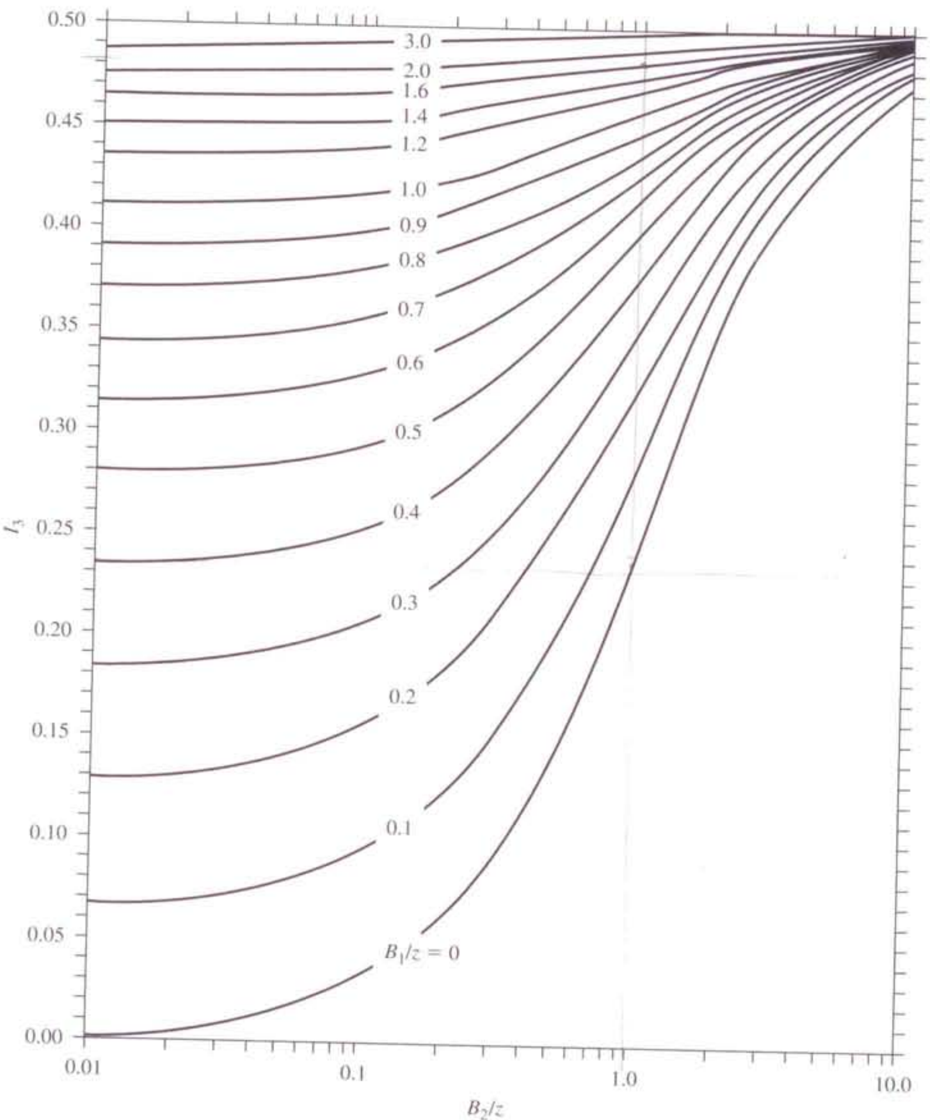

Figure 9.18 Osterberg's chart for determination of vertical stress due to embankment loading

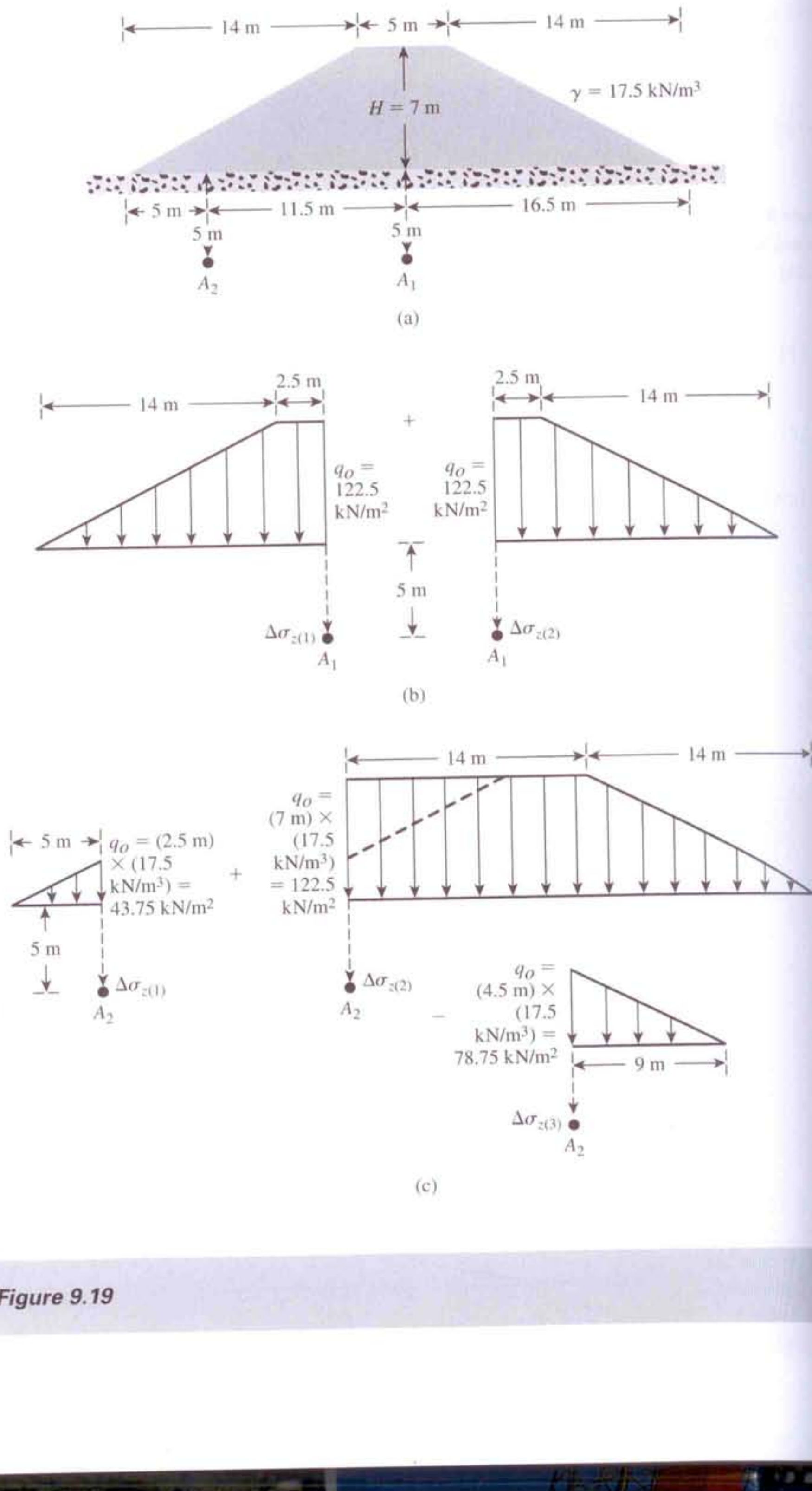

Figure 9.19

9.10

Stress Increase at A_2

Refer to Figure 9.19c. For the left side, $B_2 = 5$ m and $B_1 = 0$. So,

$$\frac{B_2}{z} = \frac{5}{5} = 1; \frac{B_1}{z} = \frac{0}{5} = 0$$

According to Figure 9.18, for these values of B_2/z and B_1/z, $I_3 = 0.24$. So,

$$\Delta\sigma_{z(1)} = 43.75(0.24) = 10.5 \text{ kN/m}^2$$

For the middle section,

$$\frac{B_2}{z} = \frac{14}{5} = 2.8; \frac{B_1}{z} = \frac{14}{5} = 2.8$$

Thus, $I_3 = 0.495$. So,

$$\Delta\sigma_{z(2)} = 0.495(122.5) = 60.64 \text{ kN/m}^2$$

For the right side,

$$\frac{B_2}{z} = \frac{9}{5} = 1.8; \frac{B_1}{z} = \frac{0}{5} = 0$$

and $I_3 = 0.335$. So,

$$\Delta\sigma_{z(3)} = (78.75)(0.335) = 26.38 \text{ kN/m}^2$$

Total stress increase at point A_2 is

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} - \Delta\sigma_{z(3)} = 10.5 + 60.64 - 26.38 = \mathbf{44.76 \text{ kN/m}^2}$$ ■

9.10 Vertical Stress Below the Center of a Uniformly Loaded Circular Area

Using Boussinesq's solution for vertical stress $\Delta\sigma_z$ caused by a point load [Eq. (9.12)], one also can develop an expression for the vertical stress below the center of a uniformly loaded flexible circular area.

From Figure 9.20, let the intensity of pressure on the circular area of radius R be equal to q. The total load on the elemental area (shaded in the figure) is equal to $qr\,dr\,d\alpha$. The vertical stress, $d\sigma_z$, at point A caused by the load on the elemental area (which may be assumed to be a concentrated load) can be obtained from Eq. (9.12):

$$d\sigma_z = \frac{3(qr\,dr\,d\alpha)}{2\pi} \frac{z^3}{(r^2 + z^2)^{5/2}} \tag{9.29}$$

The increase in the stress at point A caused by the entire loaded area can be found by integrating Eq. (9.29):

$$\Delta\sigma_z = \int d\sigma_z = \int_{\alpha=0}^{\alpha=2\pi} \int_{r=0}^{r=R} \frac{3q}{2\pi} \frac{z^3 r}{(r^2 + z^2)^{5/2}}\,dr\,d\alpha$$