Slope Stability Analysis for Landfills and Embankments

Geotechnical Design

CGN 4801

By

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FOR ANALYSIS AND DESIGN SLOPE FAILURE ARE DIVIDED INTO:

I. Planar Failures

(Determinate Problems)

- ***I- Infinite Slopes*** → Small Depth, Long Failure Surface
- ***II- Finite Slopes*** → Simple Wedge

(Determinate & Indeterminate Problems)

II. Circular Failures

→
- 1- Above the Toe
- 2- Through the Toe
- 3- Deep Seated

III. Wedge Failures

→ Multiple Planar Failure Surface

IV. Complex Failures

→ Combination of Planar & Circular

FACTORS AFFECTING SLOPE STABILITY

1- Soil Type
2- Geometry of the cross section (Height, slope angle, etc.)
3- Moisture Content
4- Pore water pressure
5- Additional loads
6- Shear Strength reduction
7- Vibrations and Earthquake

METHODS OF ANALYSIS

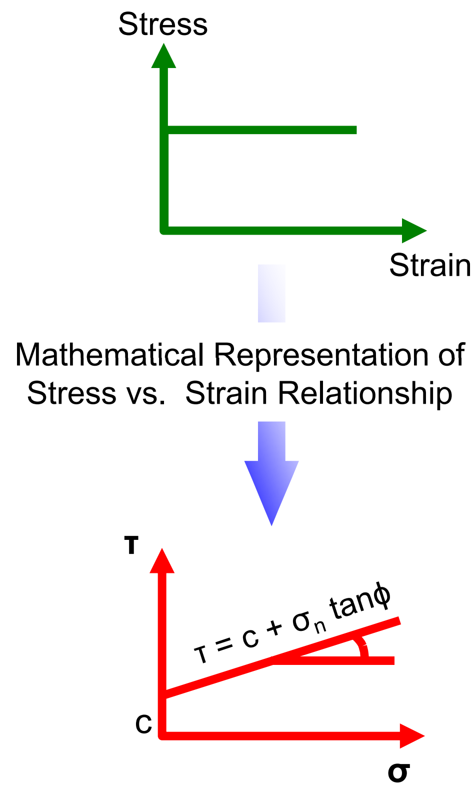
A state of equilibrium is said to exist when the shear stress along the failure surface is expressed as:

Shear Strength

Shear Stress $S = \tau / Fs$

Safety Factor

$$\tau = c + \sigma_n \tan\phi$$



FACTOR OF SAFETY

1- For Shear Strength

$\tau_{developed} = \tau / FS$

$\tau_{developed} = (c + \sigma \tan\phi) / FS$

2- For Shear Parameters

$c_d = c / FS$

$\tan\phi_d = \tan\phi / FS$

3- For Height of the Slope

$H_{design} = H_c / FS$

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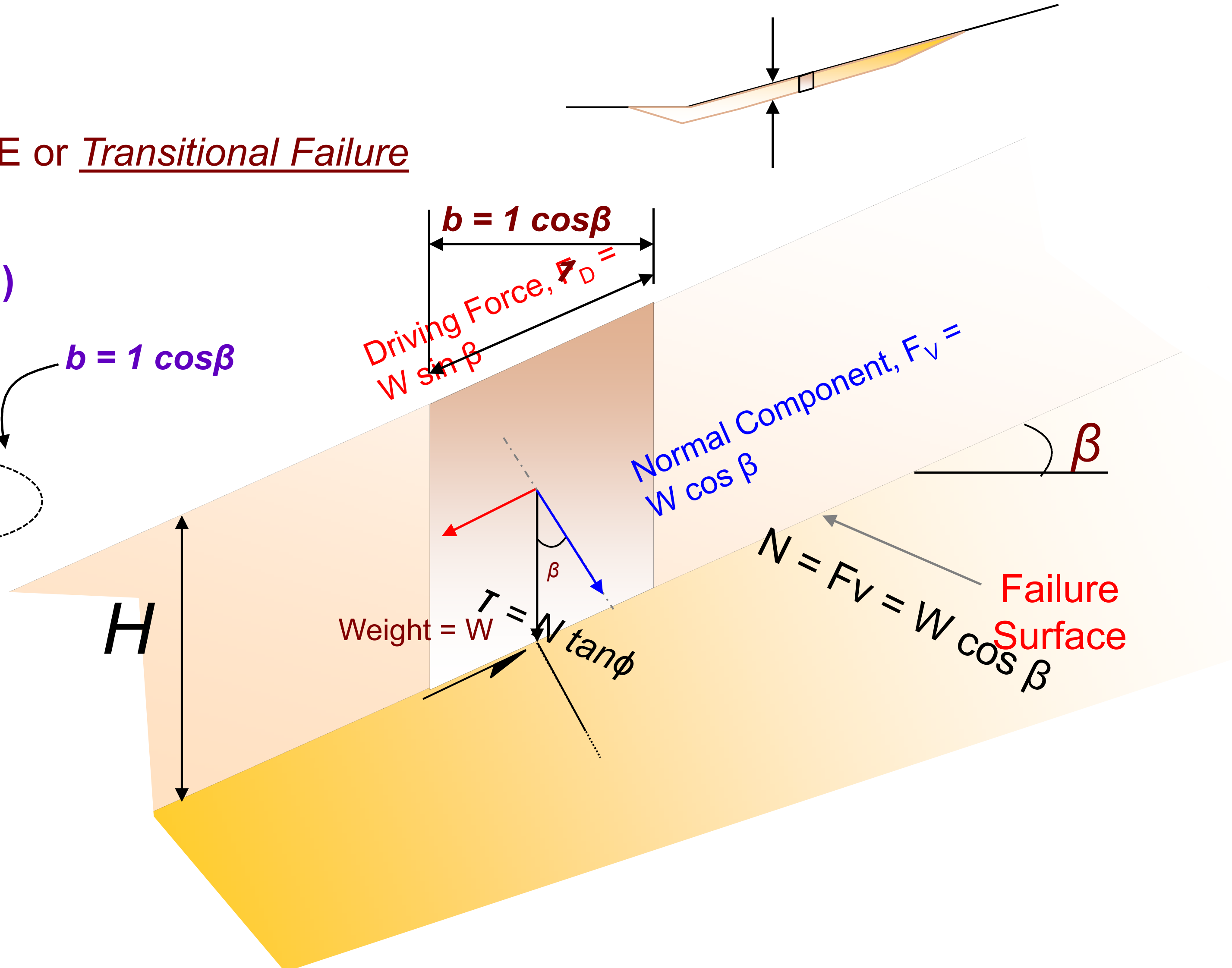
INFINITE SLOPE

I. PLANAR FAILURE or *Transitional Failure*

A- Dry Soil (ϕ soil)

b = 1 cosβ

$W = \gamma H \cos\beta$



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<u>A- Dry Soil</u> (ϕ soil)

$$W = \gamma\ H\ \cos\beta \quad \leftarrow b = 1\ \cos\beta$$

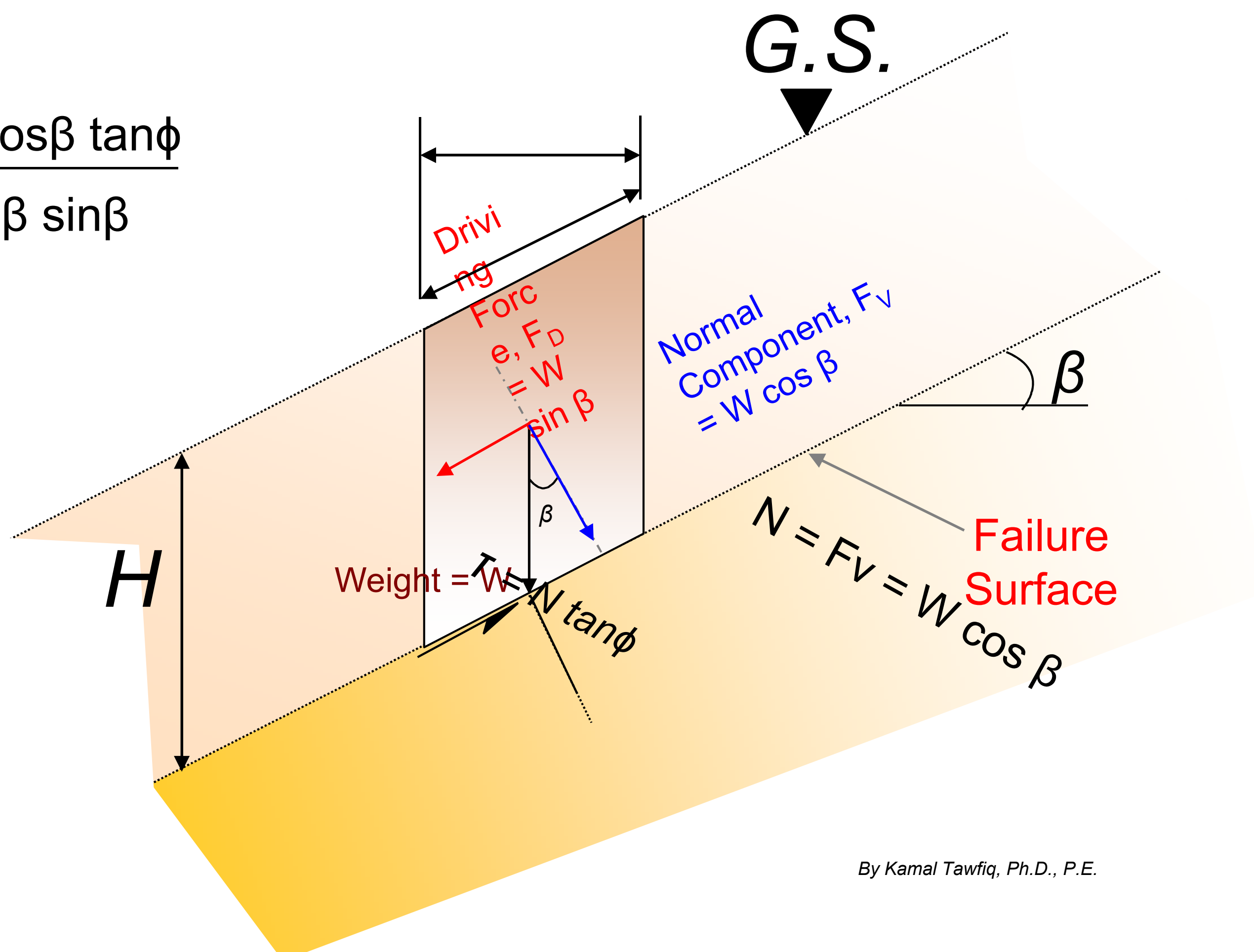
Driving Force = $F_D = \gamma\ H \cos\beta \sin\beta$

Resisting Force = $F_R = \gamma\ H \cos\beta \cos\beta \tan\phi$

$FS = F_R/F_D$

$$FS = \frac{\gamma\ H \cos\beta \cos\beta \tan\phi}{\gamma\ H \cos\beta \sin\beta}$$

$$FS = \frac{\tan\phi}{\tan\beta}$$



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<u>B- Submarged Soil</u> (ϕ soil)

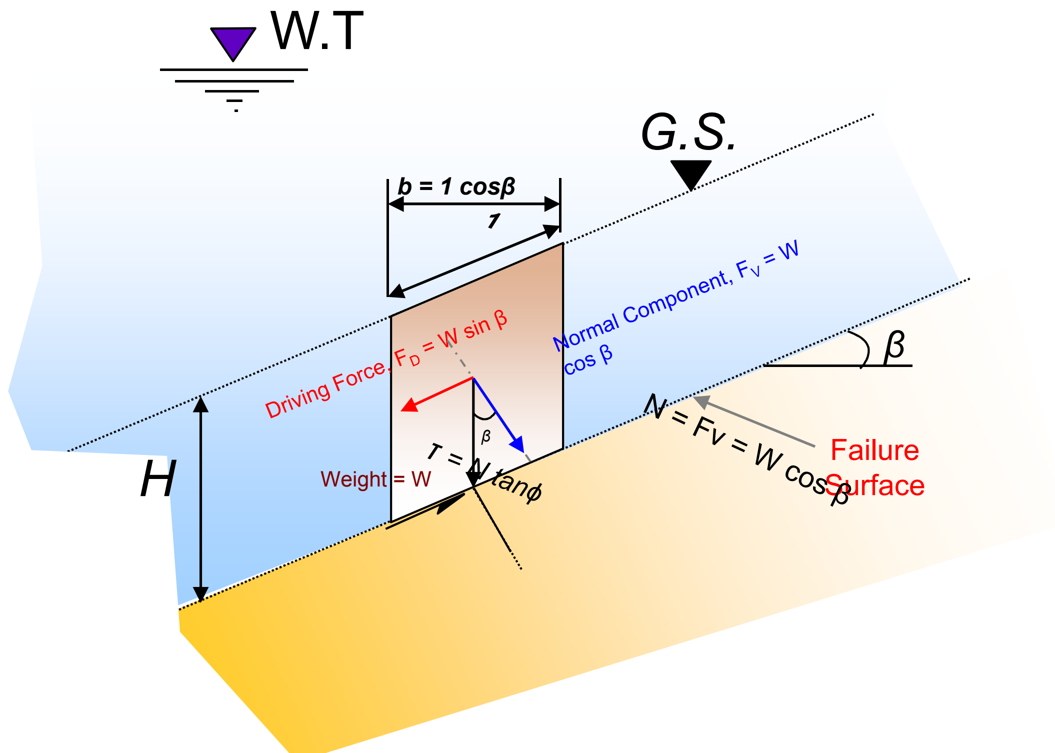
$$W = \bar{\gamma} H \cos\beta$$

Driving Force = $F_D = \bar{\gamma}\ H \cos\beta \sin\beta$
Resisting Force = $F_R = \bar{\gamma}\ H \cos\beta \cos\beta \tan\phi$

FS = F_R/F_D

$$FS = \frac{\bar{\gamma}\ H \cos\beta \cos\beta \tan\phi}{\bar{\gamma}\ H \cos\beta \sin\beta}$$

$$FS = \frac{\tan\phi}{\tan\beta}$$



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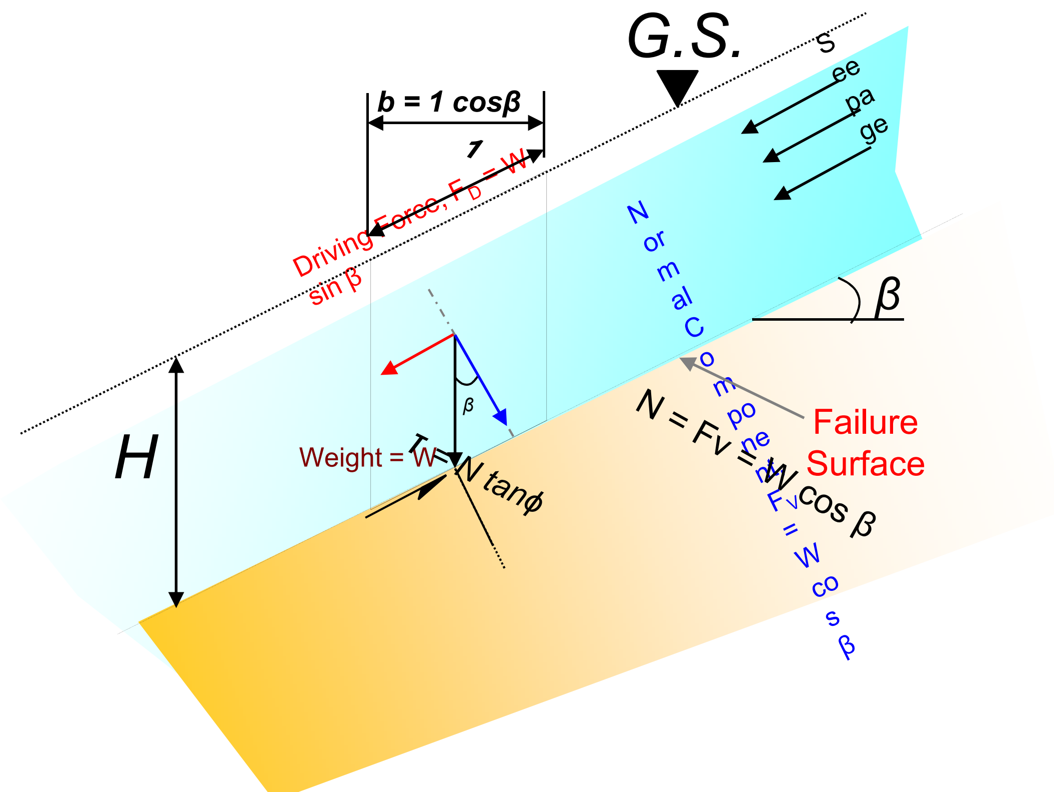
C- Seepage Parallel to Slope (ϕ soil)

$$FS = \frac{\tan\phi}{\tan\beta}\left(1 - \frac{\gamma_w Z}{\gamma_{soil} H \cos^2\beta}\right)$$



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D- Infinite Slope in c - ϕ soil *(with seepage)*

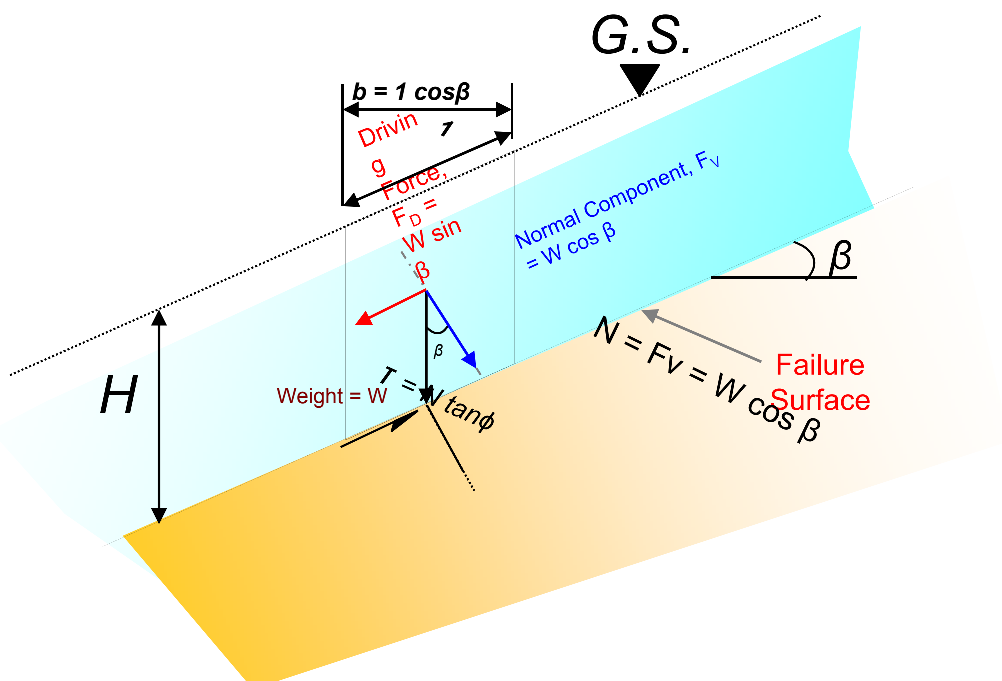
$$FS = \frac{c}{\gamma_{soil}\, H \cos\beta \sin\beta} \left(1 - \frac{u}{\gamma_{soil}\, H \cos^2\beta}\right) \frac{\tan\phi}{\tan\beta}$$

If no seepage:

$u = 0$

If Submarged Slope:

$u = 0 \quad \gamma = \gamma'$



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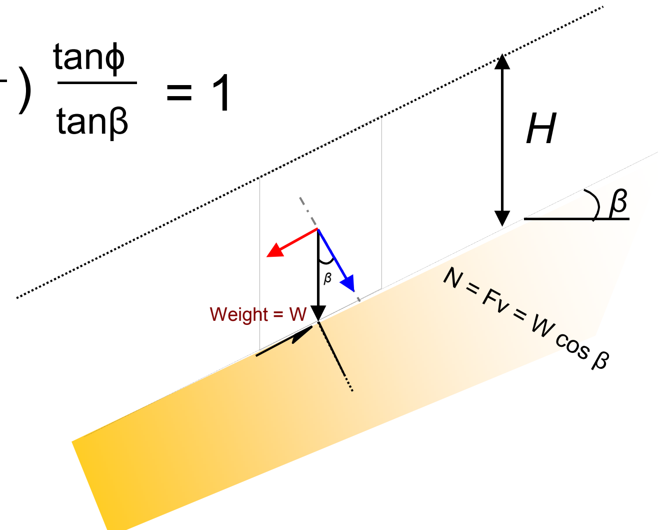
Critical Height Hc at FS = 1

$$FS = \frac{c}{\gamma_{soil} H \cos\beta \sin\beta} \left(1 - \frac{u}{\gamma_{soil} H \cos^2\beta}\right) \frac{\tan\phi}{\tan\beta} = 1$$

$$H_c = \frac{c - u \tan\phi}{\gamma_{soil} \cos^2\beta (\tan\beta - \tan\phi)}$$



Stability Number Ns

$$Ns = \frac{c}{\gamma Hc} \qquad r_u = \frac{u}{\gamma H} = \text{pore water pressure ratio}$$

General Equation:

$$H_c = \frac{c}{\gamma_{soil} [\sin\beta \cos\beta - \tan\phi (\cos^2\beta - r_u)]}$$

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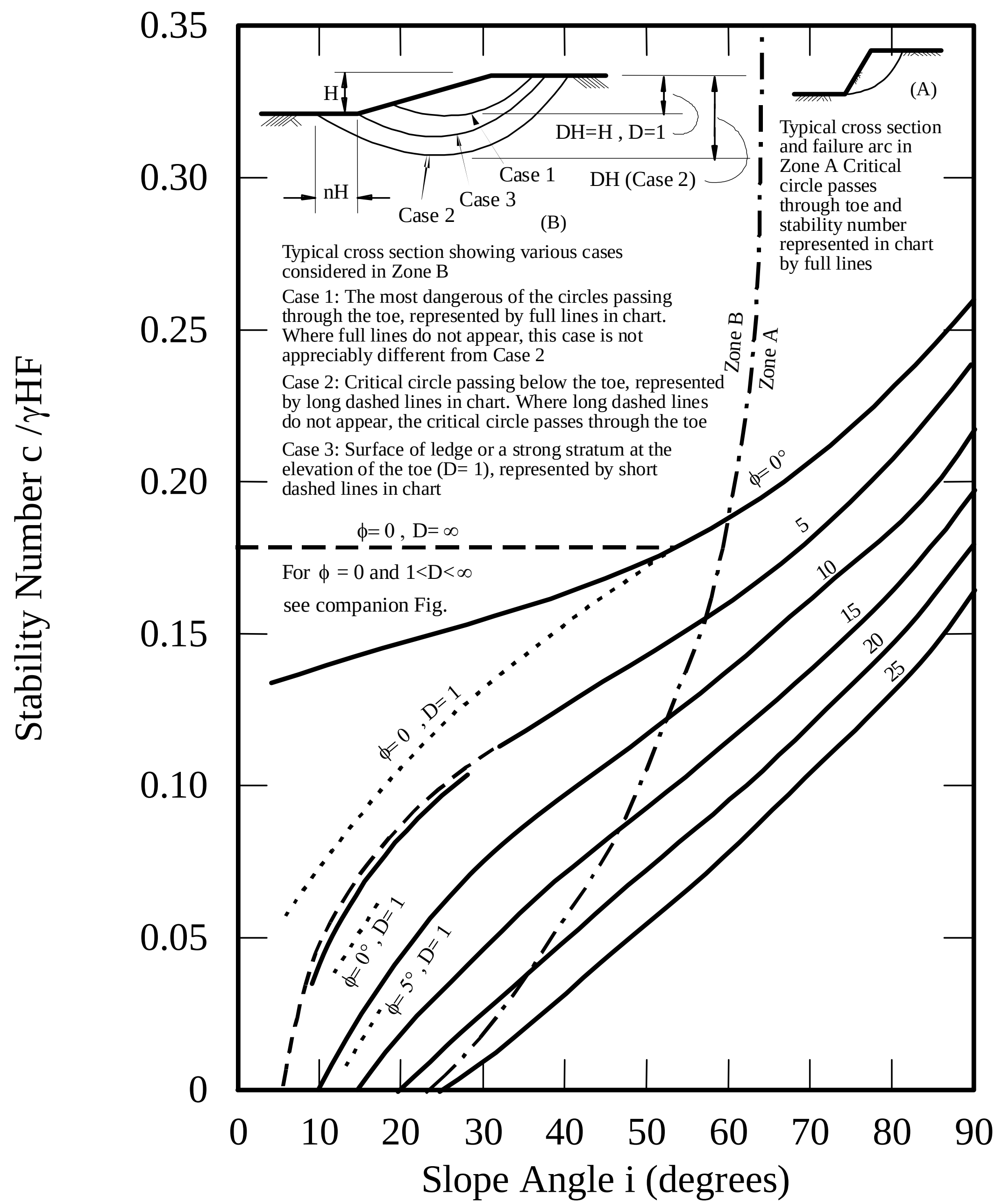
Stability Number

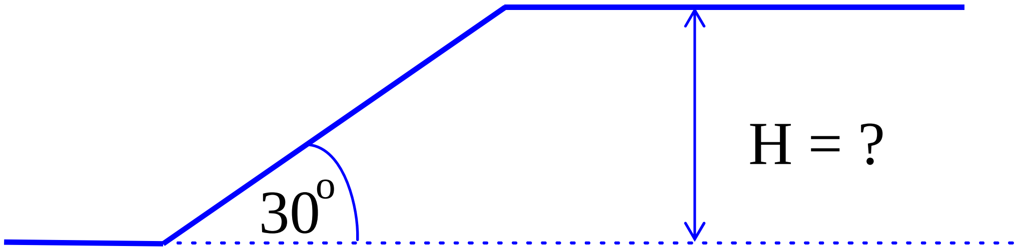
- A variety of charted solutions exist for the simple geometry considered above.
- For the undrained (total stress) analysis of slopes charts produced by Taylor are often used.
- The charts are based on the analysis of circular failure surfaces, and assume that soil strength is given by a Mohr-Coulomb analysis
- Tension cracks are not considered

Taylor's Chart



Taylor's Chart

Example - 1



c_u = 20 kN/m²
ϕ_u = 10°
γ_{bulk} = 15 kN/m³
F.S. = 1.5

Since the factor of safety is given, then the problem indicates a design of a new slope

Solution:

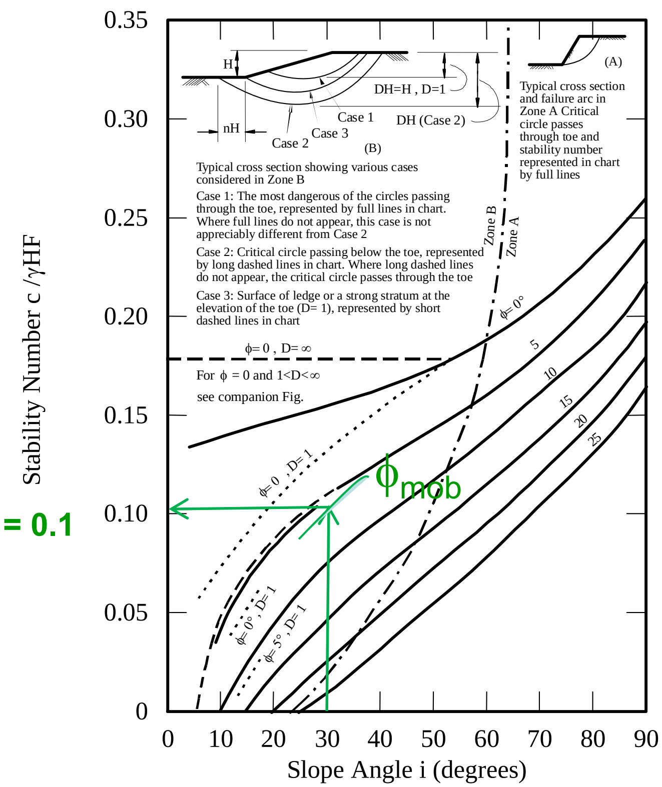
$\phi_{mobilized} = \phi_{developed}$

Use the chart with $i = 30°$, and $\phi_{mob} = \tan^{-1}\left(\frac{\tan 10°}{1.5}\right) = 6.7°$

$$SN = 0.1 = \frac{20/1.5}{15 \times H_{design}}$$

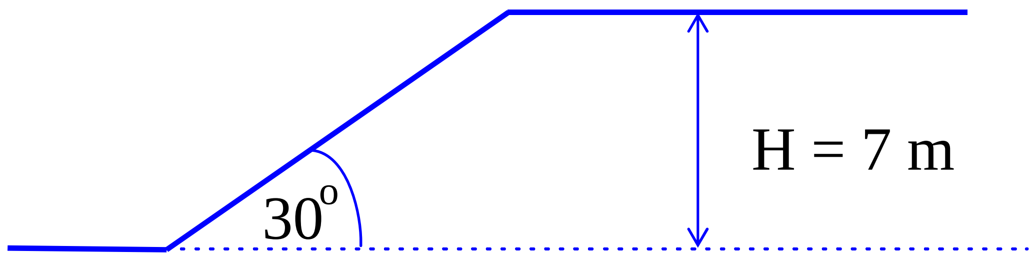
$$H_{design} = \frac{20/1.5}{15 \times 0.1} = 8.88 \text{ m}$$

SN = 0.1



Taylor's Chart

Example - 2



c_u = 20 kN/m²
ϕ_u = 10°
γ_{bulk} = 15 kN/m³
F.S. = ????

Since the factor of safety is not given, then the problem indicates an existing slope that we need to analyze its safety.

$\phi_{mobilized} = \phi_{developed}$

Solution:

Trial # 1

1- Assume $FS_\phi = 1$

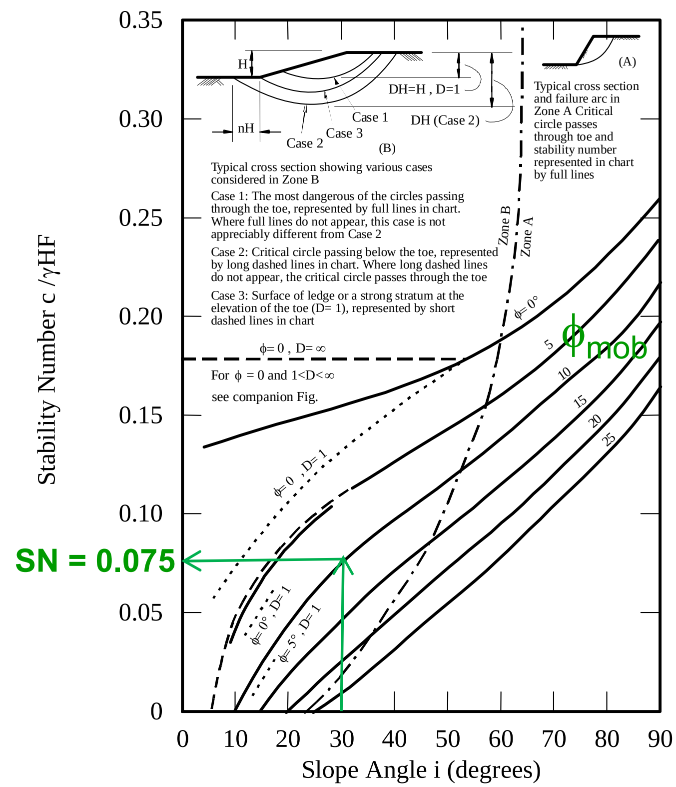
2- Use the chart with *i* = 30°, and $\phi_{mob} = \tan^{-1}\left(\frac{\tan 10^o}{1.0}\right) = 10^o$

3- Go to the chart and find SN for $\phi_{mob} = 10^o$

$$SN = 0.075 = \frac{c_{mob}}{15 \times 7}$$

4- $c_{mob} = 15 \times 7 \times 0.075 = 7.87\ kN/m^2$

5- $FS_c = c / c_{mob} = 20/7.87 = 2.5$



Therefore the Assumed $FS_\phi \neq$ the calculated FS_c

This means the assumed factor of safety was not the right one. So we need to assume another FSϕ and solve the problem again for FS_c.

Trial # 2

1- Assume FS_ϕ = 1.5

2- Use the chart with i = 30°, and $\phi_{mob} = \tan^{-1}\left(\frac{\tan 10^o}{1.5}\right) = 6.7^o$

3- Go to the chart and find SN for ϕ_{mob} = 6.7°

$$SN = 0.1 = \frac{c_{mob}}{15 \times 7}$$

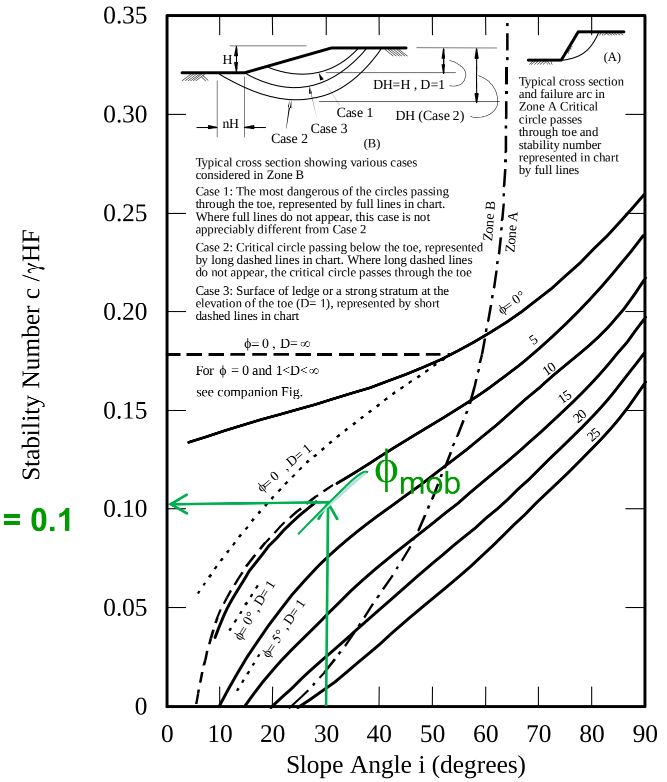
4- c_{mob} = 15 x 7 x 0.10 = 10.5 kN/m²

5- $FSc = c / c_{mob}$ = 20/10.5 = 1.9

Therefore the Assumed FSϕ ≠ the calculated FS_c

This means the assumed factor of safety was not the right one. So we need to assume another FSϕ and solve the problem again for FS_c.

SN = 0.1



Trial # 3

1- Assume $FS_\phi = 1.8$

2- Use the chart with $i = 30^o$, and $\phi_{mob} = \tan^{-1}\left(\frac{\tan 10^o}{1.8}\right) = 5.5^o$

3- Go to the chart and find SN for $\phi_{mob} = 5.5^o$

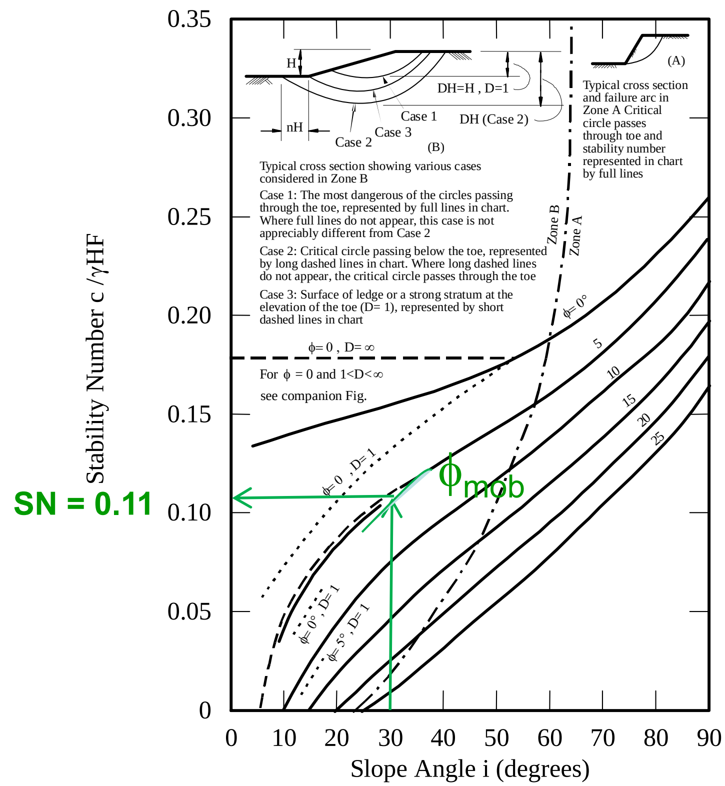
$$SN = 0.11 = \frac{c_{mob}}{15 \times 7}$$

4- $c_{mob} = 15 \times 7 \times 0.11 = 11.55 \text{ kN/m}^2$

5- $FSc = c / c_{mob} = 20/11.55 = 1.73$

Therefore the Assumed $FS\phi \neq$ the calculated FS_c

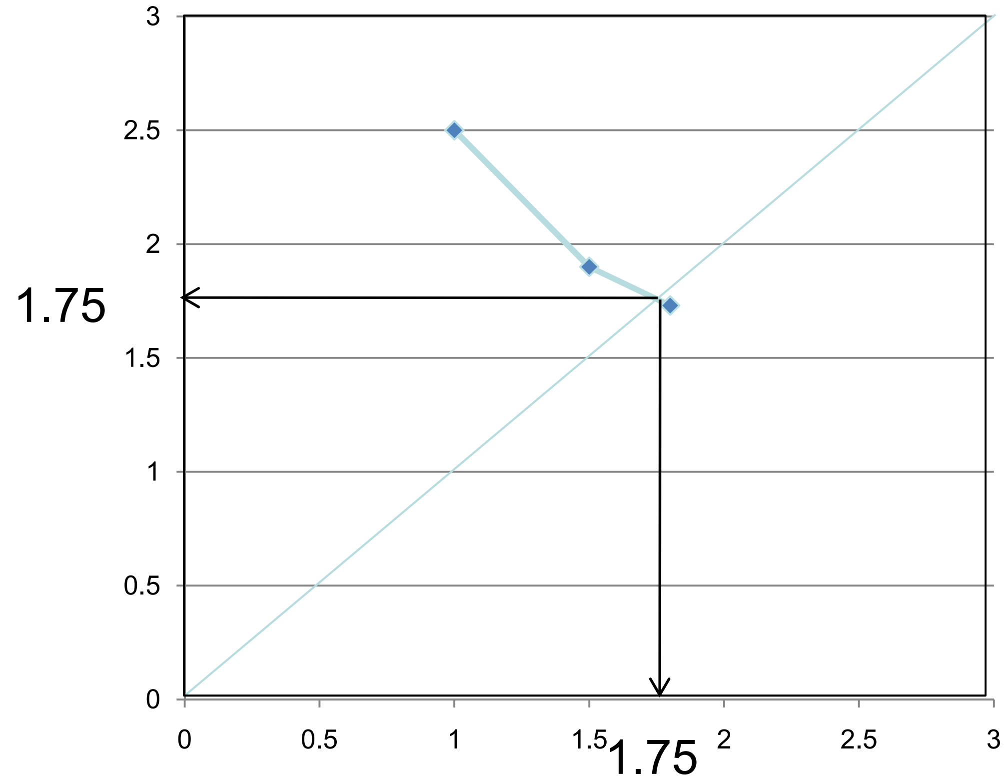
This means the assumed factor of safety was not the right one. So we need to assume another $FS\phi$ and solve the problem again for FS_c.



Since we complied three different trials, we are ready to find the right factor of safety by using the 45^{o} line method

Assumed FSϕ	Calculated FS$_c$
1.0	2.5
1.5	1.9
1.8	1.73

So the correct FS is 1.75



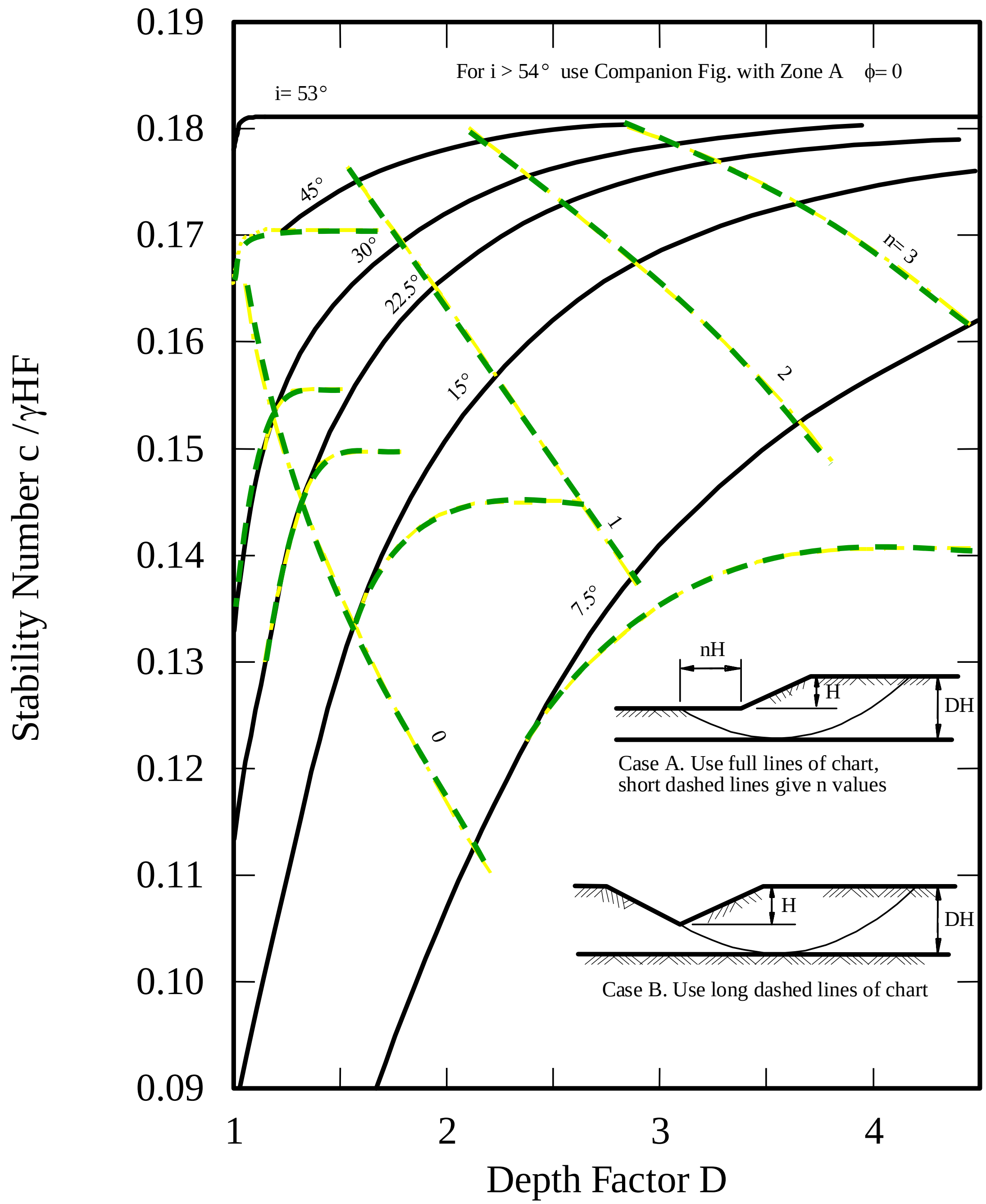
Taylor’s Chart - example

- Zones are marked on the chart indicating whether the failure mode will be shallow or deep-seated.

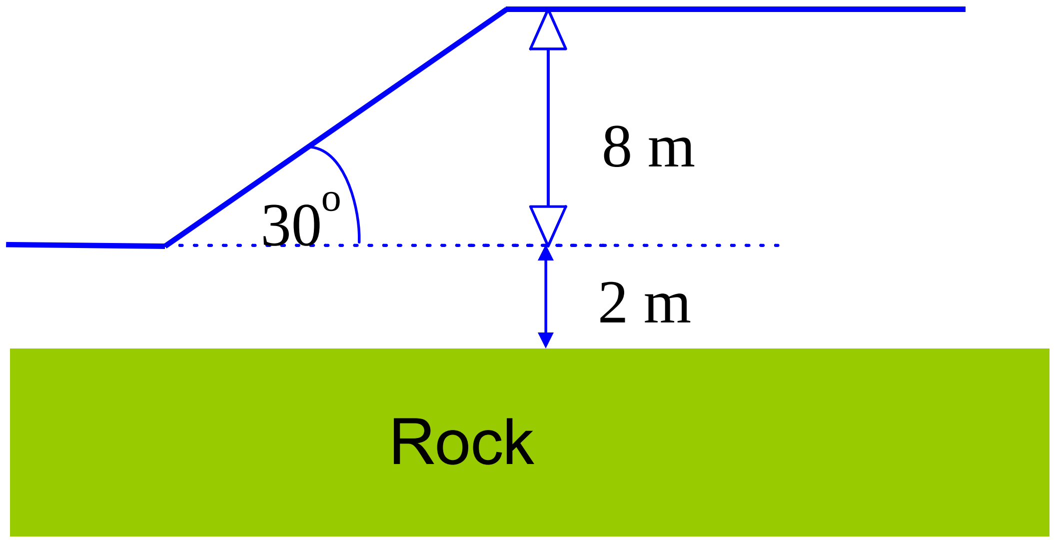
- If a deep-seated failure is indicated the soil layer must be sufficiently deep to enable this mechanism to occur.

- There is a second chart due to Taylor which can be used when the depth of soil below the base of the slope is limited

- This chart is only valid for $\phi = 0$


For i > 54° use Companion Fig. with Zone A ϕ= 0
i= 53°
45°
30°
22.5°
15°
7.5°
n= 3
2
1
0
nH
H
DH
Case A. Use full lines of chart,
short dashed lines give n values
Case B. Use long dashed lines of chart
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0.09
1
2
3
4
Stability Number c /γHF
Depth Factor D

Taylor's Chart - example with finite depth



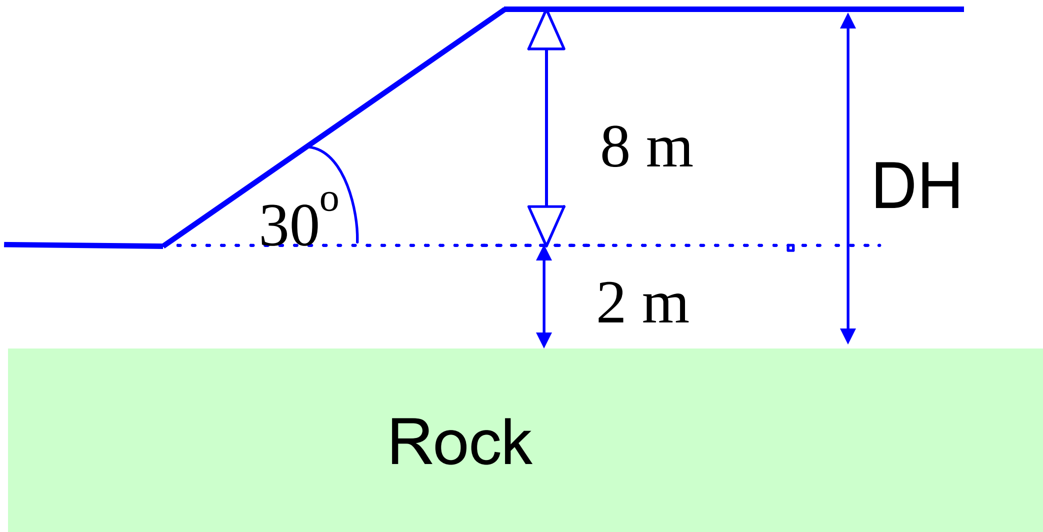
c_u = 20 kN/m^2

ϕ_u = 0

γ_{bulk} = 15 kN/m^3

Calculate the Depth Factor D

Taylor's Chart - example with finite depth



c_u = 20 kN/m^2

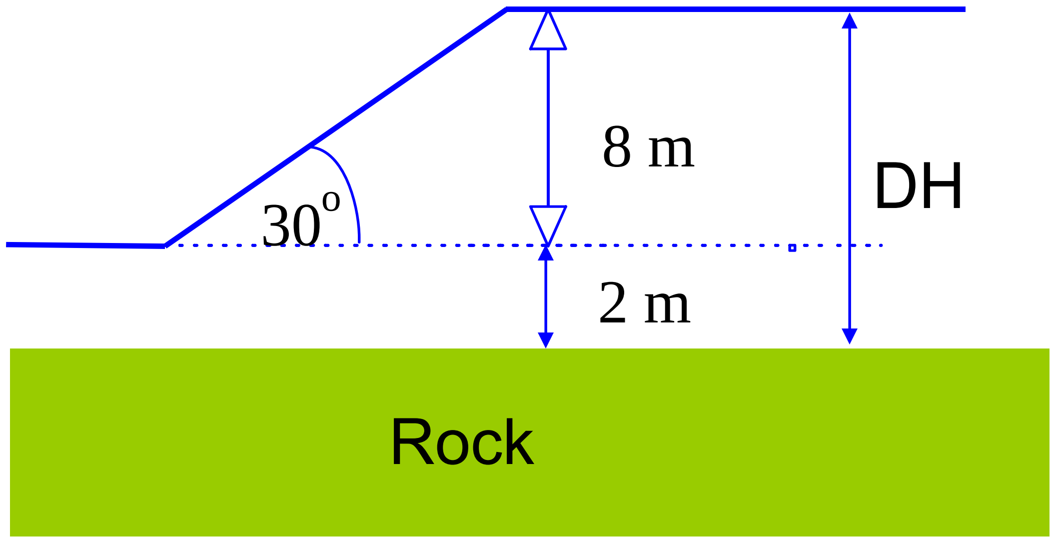
ϕ_u = 0

γ_{bulk} = 15 kN/m^3

Calculate the Depth Factor D

DH = 10 m

Taylor's Chart - example with finite depth



$c_u = 20 \text{ kN/m}^2$

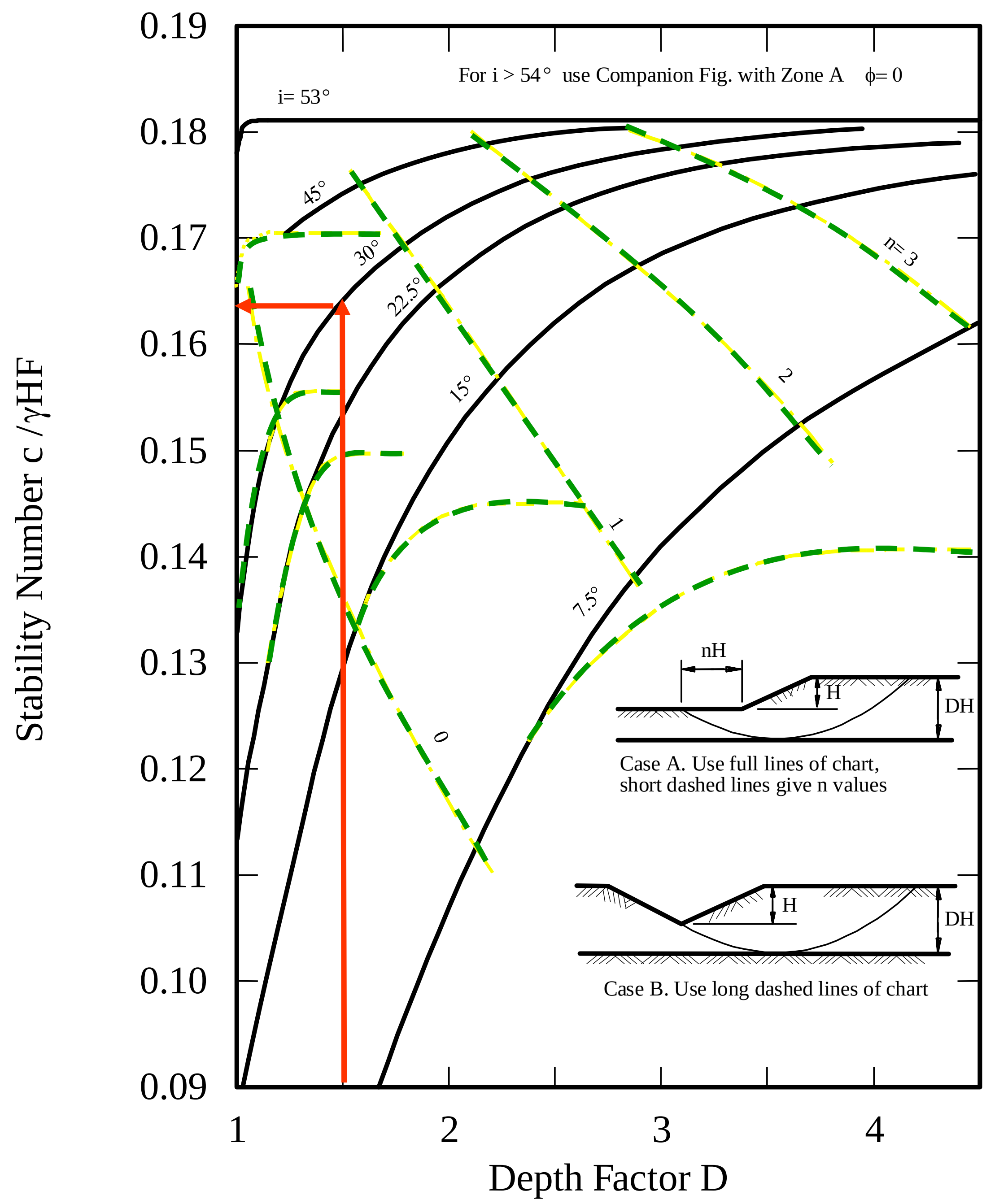
$\phi_u = 0$

$\gamma_{bulk} = 15 \text{ kN/m}^3$

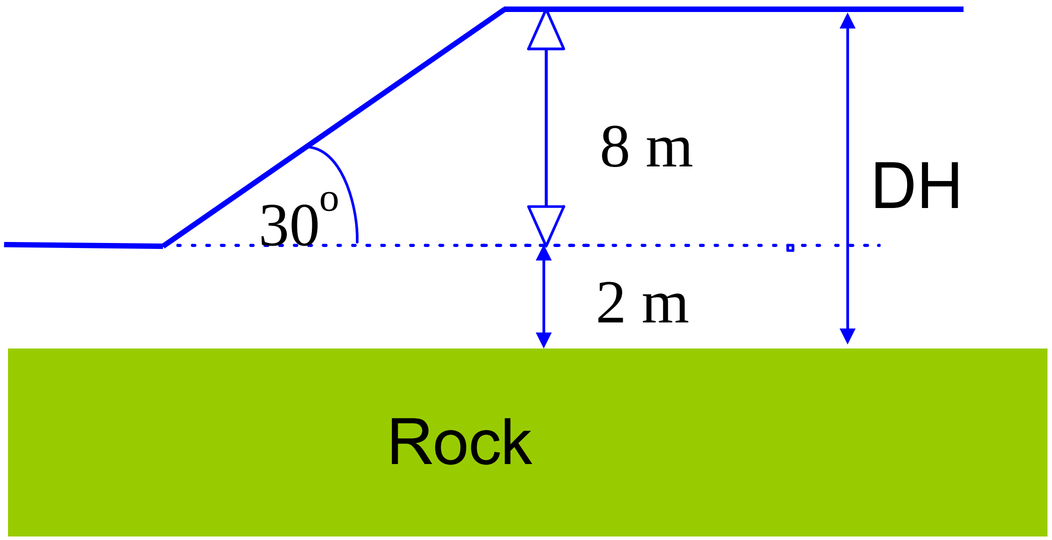
Calculate the Depth Factor D

DH = 10 m, H = 8m

D = 1.25


0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0.09
1
2
3
4
Stability Number c /γHF
Depth Factor D
For i > 54° use Companion Fig. with Zone A ϕ= 0
i= 53°
45°
30°
22.5°
15°
7.5°
n= 3
2
1
0
nH
H
DH
Case A. Use full lines of chart,
short dashed lines give n values
H
DH
Case B. Use long dashed lines of chart

Taylor's Chart - example with finite depth



$c_u = 20 \text{ kN/m}^2$

$\phi_u = 0$

$\gamma_{bulk} = 15 \text{ kN/m}^3$

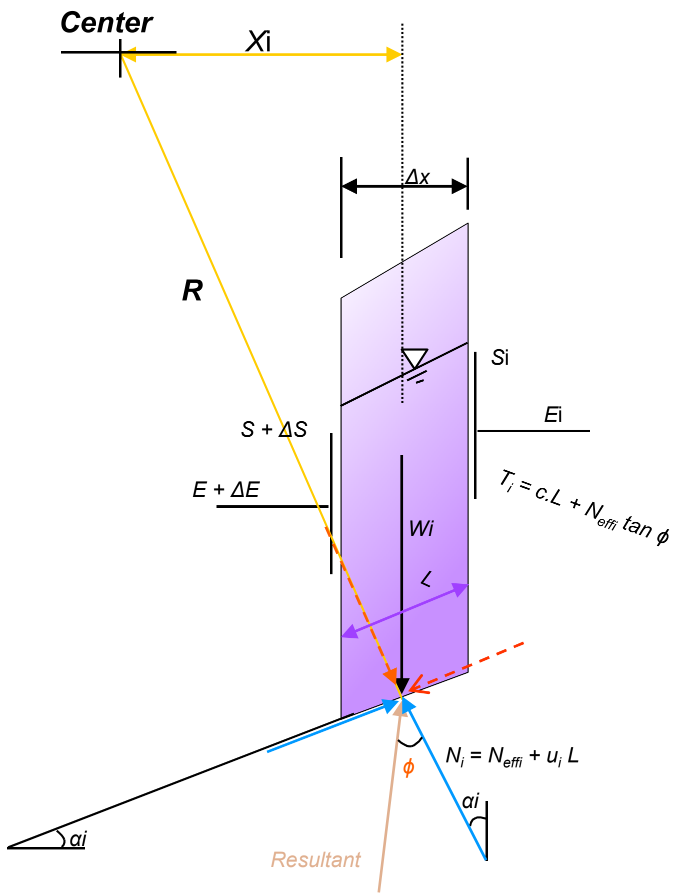
D = 1.25

$$\frac{C_{dev}}{\gamma H} = \frac{C_{dev}}{15 \times 8} = 0.165$$

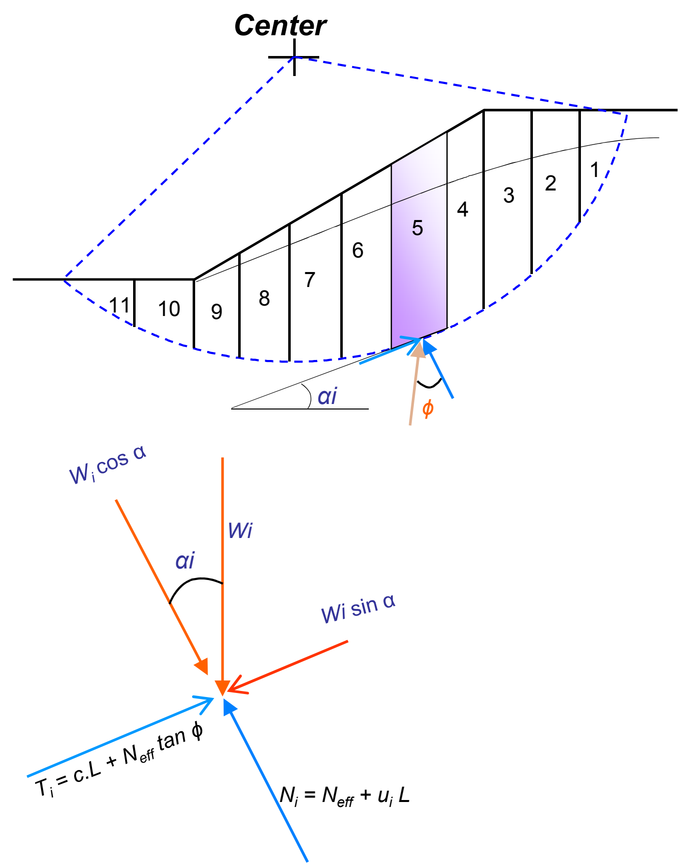
$C_{dev} = 0.165 \times 15 \times 8 = 19.8 \text{ kN/m}^2$

FS = 1.01

STATICALLY INDETERMINATE PROBLEMS



METHOD OF SLICES



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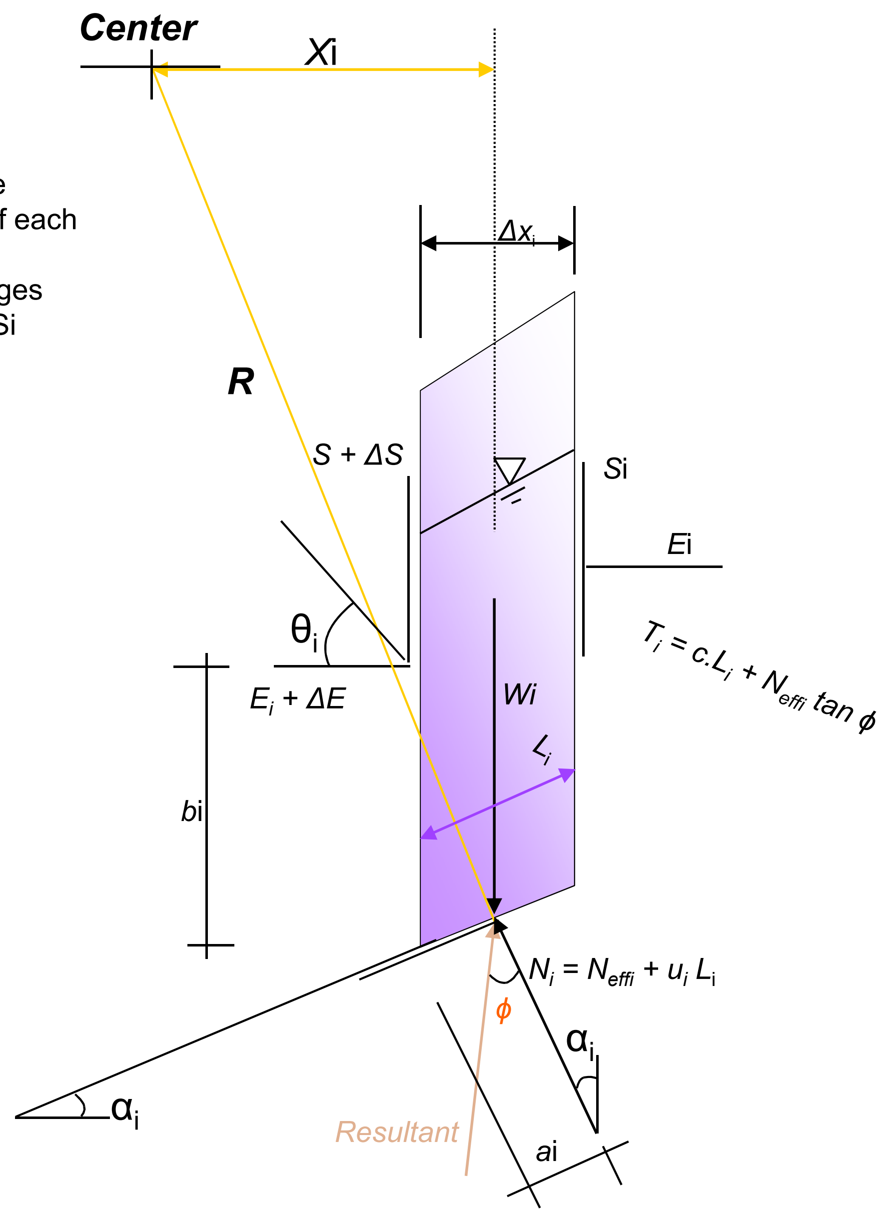
Unknowns Associated with Force Equilibrium

n = Resultant normal forces N_i on the base of each slice or wedge
1 = Safety factor, which permits the shear forces Ti on the base of each slice to be expressed in terms of Ni
n-1 = Resultant normal forces Ei on each interface between slices or wedges
n-1 = Angles αi which express the relationships between the shear force Si and the normal force Ei on each interface



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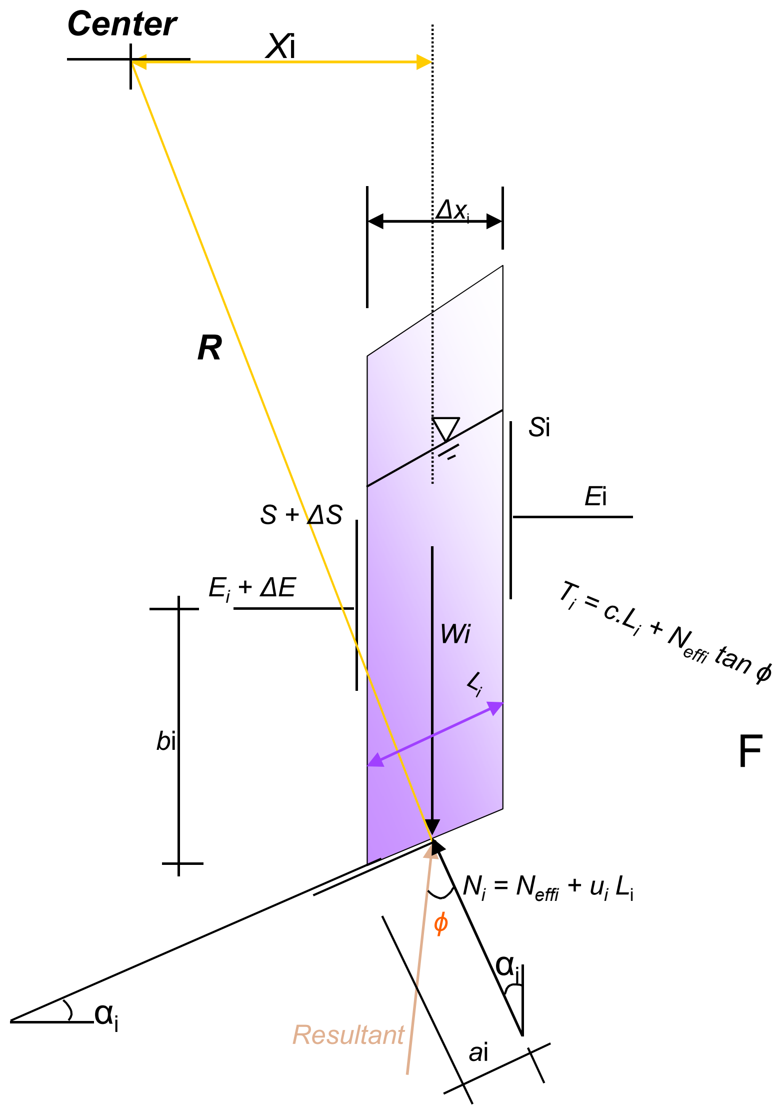
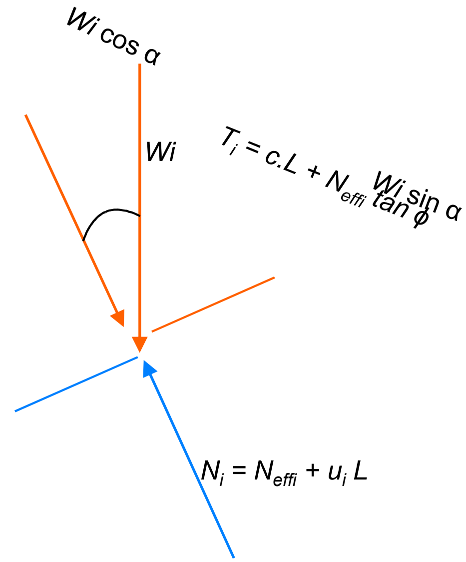
FELLENIUS METHOD (ORDINARY METHOD, SWEDISH METHOD)





$$F = \frac{M_R}{M_D} = \frac{\sum CL_i + (w_i \cos\theta_i - u_i L_i)\tan\phi}{\sum w_i \sin\theta_i}$$

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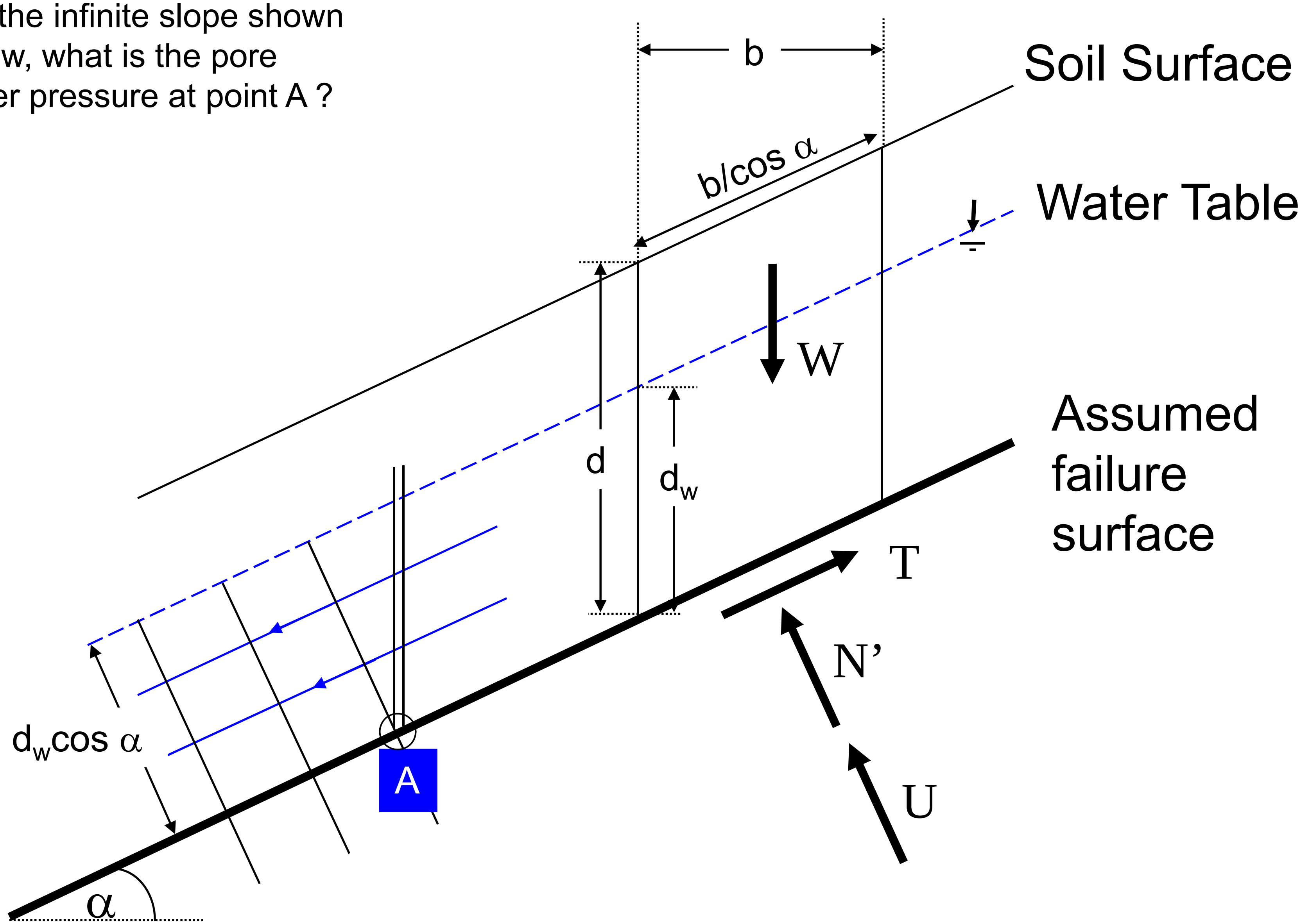
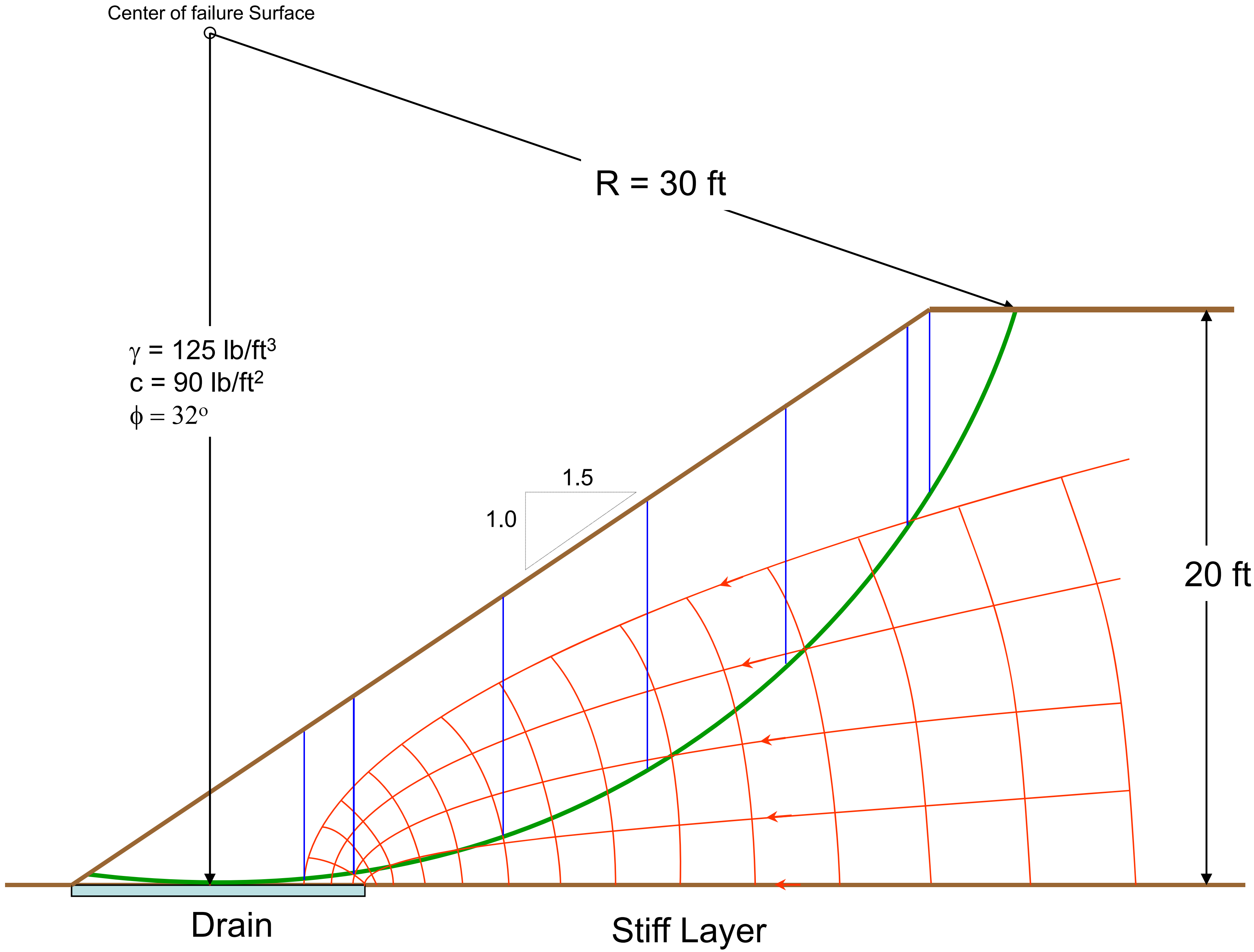
Quiz # 4
Fall 2009

For the infinite slope shown below, what is the pore water pressure at point A ?





Center of failure Surface
R = 30 ft
γ = 125 lb/ft³
c = 90 lb/ft²
φ = 32°
1.5
1.0
20 ft
Drain
Stiff Layer

$$F = \frac{\bar{c}L + \tan \bar{\phi} \sum_{i=1}^{i=n} (W_i \cos \theta_i - u_i \Delta l_i)}{\sum_{i=1}^{i=n} W_i \sin \theta_i}$$

Given. Slope in Example 24.3.
Find. Safety factor by ordinary method of slices.
Solution. See Table E24.4.

Table E24.4

Slice	W_i (kips)	$\sin \theta_i$	$W_i \sin \theta_i$ (kips)	$\cos \theta_i$	$W_i \cos \theta_i$ (kips)	u_i (kips/ft)	Δl_i (ft)	U_i (kips)	$\bar{N}_i$ (kips)
1	0.9	−0.03	0	1.00	0.9	0	4.4	0	0.9
2	1.7	0.05	0.1	1.00	1.7	0	3.2	0	1.7
2A	1.3	0.14	0.2	0.99	1.3	0.03	1.9	0.05	1.25
3	4.6	0.25	1.2	0.97	4.5	0.21	5.3	1.1	3.4
4	5.6	0.42	2.3	0.91	5.1	0.29	5.6	1.6	3.5
5	5.8	0.58	3.4	0.81	4.7	0.25	6.2	1.55	3.15
6	4.6	0.74	3.4	0.67	3.1	0.11	6.7	0.7	2.4
6A	0.5	0.82	0.4	0.57	0.3	0	1.2	0	0.3
7	1.5	0.87	1.3	0.49	0.7	0	7.3	0	0.7
			12.3				41.8		17.3

$$F = \frac{0.09(41.8) + 17.3 \tan 32^\circ}{12.3} = \frac{3.76 + 10.82}{12.3} = \frac{14.58}{12.3} = 1.19$$

Note. That $r \Sigma W_i \sin \theta_i = 30(12.3) = 369$ kip-ft should equal the moment in the last column of Table E24.3. The slight difference results from rounding errors. ◀

$$F = \frac{\sum_{i=1}^{i=n} [\bar{c}\,\Delta x_i + (W_i - u_i\,\Delta x_i)\tan\bar{\phi}][1/M_i(\theta)]}{\sum_{i=1}^{i=n} W_i \sin\theta_i}$$

$$M_i(\theta) = \cos\theta_i\left(1 + \frac{\tan\theta_i \tan\bar{\phi}}{F}\right)$$

►Example 24.5

Given. Slope in Example 24.3.
Find. Safety factor by simplified Bishop method of slices.
Solution. See Table E24.5.

Table E24.5

(1) Slice	(2) Δx_i (ft)	(3) $\bar{c}\Delta x_i$ (kips)	(4) $u_i\Delta x_i$ (kips)	(5) $W_i - u_i\Delta x_i$ (kips)	(6) (5) tan $\bar{\phi}$ (kips)	(7) (3) + (6) (kips)	(8) M_i $F = 1.25$	(8) M_i $F = 1.35$	(9) (7) ÷ (8) $F = 1.25$	(9) (7) ÷ (8) $F = 1.35$
1	4.5	0.40	0	0.9	0.55	0.95	0.97	0.97	1.0	1.0
2	3.2	0.29	0	1.7	1.05	1.35	1.02	1.02	1.3	1.3
2A	1.8	0.16	0.05	1.25	0.80	0.95	1.06	1.05	0.9	0.9
3	5.0	0.45	1.05	3.55	2.25	2.70	1.09	1.08	2.5	2.5
4	5.0	0.45	1.45	4.15	2.55	3.00	1.12	1.10	2.7	2.75
5	5.0	0.45	1.25	4.55	2.7	3.15	1.10	1.08	2.85	2.9
6	4.4	0.40	0.50	4.1	2.65	3.05	1.05	1.02	2.9	2.95
6A	0.6	0.05	0	0.5	0.30	0.35	0.98	0.95	0.35	0.4
7	3.2	0.29	0	1.5	0.95	1.25	0.93	0.92	1.3	1.35
									15.8	16.05

For assumed $F = 1.25$ $F = \frac{15.8}{12.3} = 1.29$

$F = 1.35$ $F = \frac{16.05}{12.3} = 1.31$

A trial with assumed $F = 1.3$ would give $F = 1.3$. ◄

Ordinary Method of Slices

In this method,[5] it is assumed that the forces actin; upon the sides of any slice have *zero resultant in th direction normal to the failure arc for that slice.* Thi: situation is depicted in Fig. 24.12. With this assumption

$$\bar{N}_i + U_i = W_i \cos\theta_i$$

or

$$\bar{N}_i = W_i \cos\theta_i - U_i = W_i \cos\theta_i - u_i \Delta l_i \quad (24.9$$

Combining Eqs. 24.8 and 24.9,

$$F = \frac{\bar{c}L + \tan\bar{\phi} \sum_{i=1}^{i=n} (W_i \cos\theta_i - u_i\,\Delta l_i)}{\sum_{i=1}^{i=n} W_i \sin\theta_i} \quad (24.10$$

The use of Eq. 24.10 to compute F is illustrated i Example 24.4.

Here the assumption regarding side forces involve: $n - 1$ assumptions, while there are only $n - 2$ unknowns Hence the system of slices is overdetermined and i general it is not possible to satisfy statics. Thus th safety factor computed by this method will be in error Numerous examples have shown that the safety facto obtained in this way usually falls *below the lower boun of solutions that satisfy statics.* In some problems, / from this method may be only 10 to 15% below th range of equally correct answers, but in other problem:

[5] Also known as Swedish Circle Method or Fellenius Method Consideration of slices within the trial wedge was first proposed b Fellenius (1936).

the error may be as much as 60% (e.g., see Whitman and Bailey, 1967).

Despite the errors, this method is widely used in practice because of its early origins, because of its simplicity, and because it errs on the safe side. Hand calculations are feasible, and the method has been programmed for computers. It seems unfortunate that a method which may involve such large errors should be so widely used and it is to be expected that more accurate

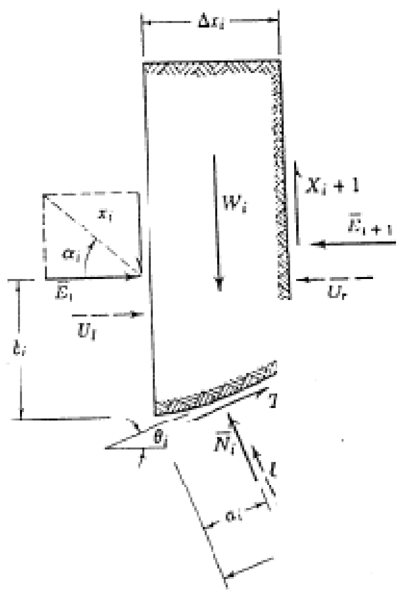
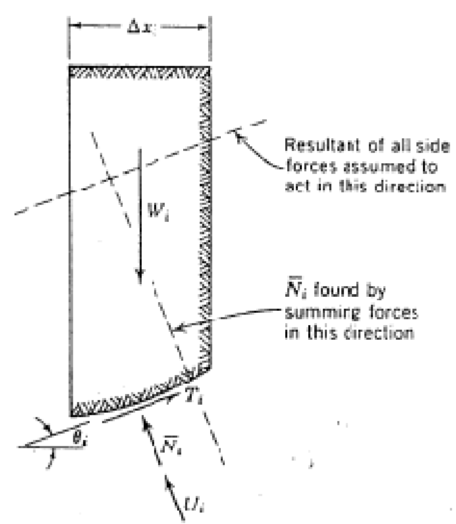


Fig. 24.11 Complete system of fo



ig. 24.12 Forces considered in ordinary method of slices.

Simplified Bishop Method of Slices

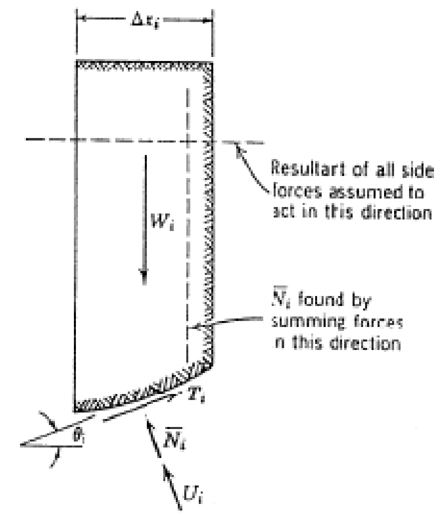
In this newer method[6] it is assumed that the forces acting on the sides of any slice have ***zero resultant in the vertical direction.*** The forces $\bar{N}_i$ are found by considering the equilibrium of the forces shown in Fig. 24.13. A value of safety factor must be used to express the shear forces T_i, and it is assumed that this safety factor equals the F defined by Eq. 24.8. Then:

$$\bar{N}_i = \frac{W_i - u_i \Delta x_i - (1/F)\bar{c}\,\Delta x_i \tan\theta_i}{\cos\theta_i[1 + (\tan\theta_i \tan\bar{\phi})/F]} \qquad (24.11)$$

Combining Eqs. 24.8 and 24.11 gives

$$F = \frac{\sum_{i=1}^{i=n} [\bar{c}\,\Delta x_i + (W_i - u_i \Delta x_i)\tan\bar{\phi}][1/M_i(\theta)]}{\sum_{i=1}^{i=n} W_i \sin\theta_i} \qquad (24.12)$$

[6] The method was first described by Bishop (1955); the simplified version of the method was developed further by Janbu et al. (1956).



13 Forces considered in simplified Bishop method of

where

$$M_i(\theta) = \cos\theta_i\left(1 + \frac{\tan\theta_i \tan\bar{\phi}}{F}\right) \qquad (24.13)$$

Equation 24.12 is more cumbersome than Eq. 24.10 from the ordinary method, and requires a trial and error solution since F appears on both sides of the equation. However, convergence of trials is very rapid. Example 24.5 illustrates the tabular procedure which may be used. The chart in Fig. 24.14 can be used to evaluate the function M_i.

The simplified Bishop method also makes $n - 1$ assumptions regarding unknown forces and hence overdetermines the problem so that in general the values of $\bar{N}_i$ and F are not exact. However, numerous examples have shown that this method gives values of F which fall within the range of equally correct solutions as determined by exact methods. There are cases where the Bishop method gives misleading results: e.g., with deep failure circles when F is less than unity (see Whitman and Bailey, 1967). Nonetheless, the Bishop method is recommended for general practice. Hand calculations are possible, and computer programs are available.

24.7 FINAL COMMENTS ON METHODS OF ANALYSIS

Sections 24.4 to 24.6 have presented in detail methods for computing the safety factor for a given cross section and given failure arc. There are additional considerations involved in applying these methods to practical problems.

It is necessary to make a trial and error search for the failure surface having the smallest factor of safety. When using circular failure surfaces, it is convenient to establish a grid for the centers of circles, to write at each grid point the smallest safety factor for circles centered on the grid point, and then to draw contours of equal safety factor. Figure 24.16 shows an example of contours of equal safety factor. I– [illegible] only circles passing tangent t[illegible] stratum were considered, but in [illegible] also be necessary to consider sh[illegible]

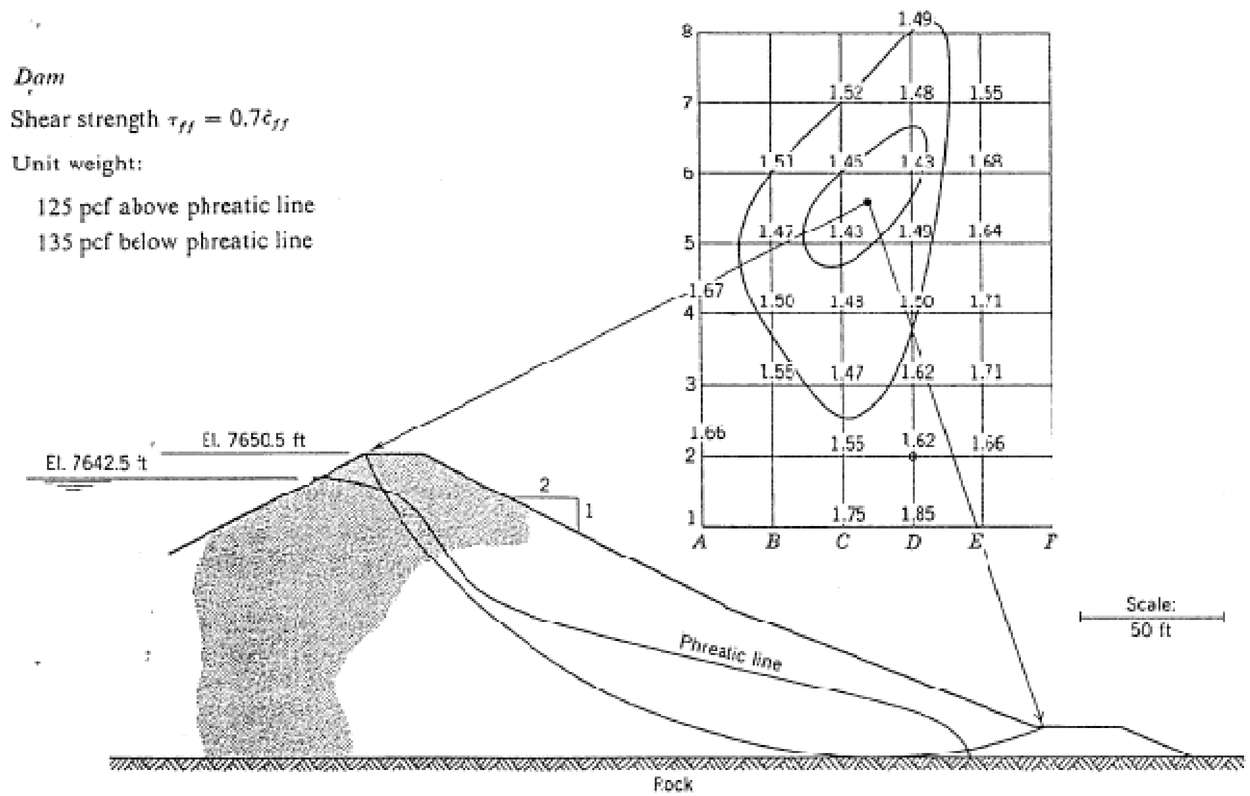


Fig. 24.16 Contours of safety factor.