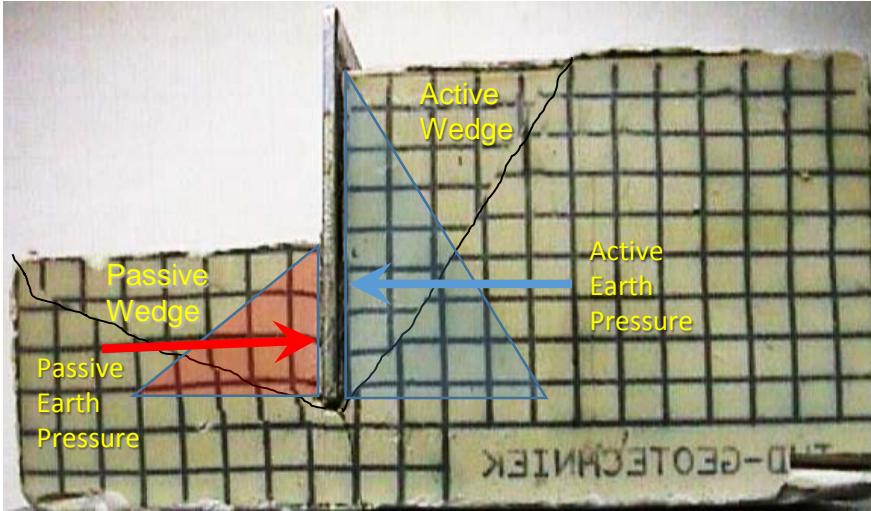
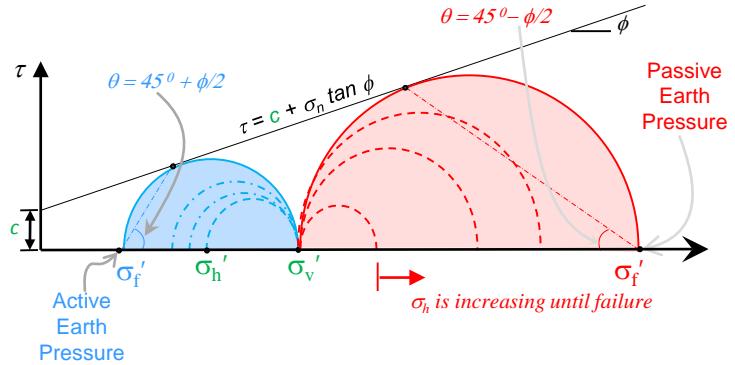


Rankine's Earth Pressure Method for (c- ϕ) Soil

Rankine's Active and Passive Earth Pressure in (c- ϕ) Soil



Rankine's Active Earth Pressure in (f) Soil with inclined backfill

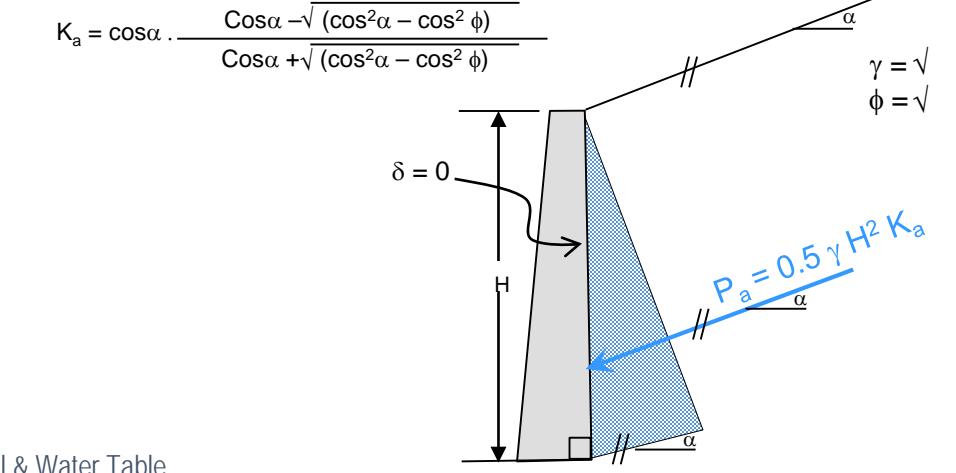


Fig.1

Active Earth Pressure

$$\sigma'_f = \sigma'_V \tan^2 \left(45^\circ - \frac{\phi}{2} \right) + 2 c \tan \left(45^\circ - \frac{\phi}{2} \right)$$

Or

$$\sigma'_f = \sigma'_V K_a - 2 c \sqrt{K_a}$$

$$K_a = \tan^2 \left(45^\circ - \frac{\phi}{2} \right) = \frac{1 - \sin \phi}{1 + \sin \phi}$$

Coefficient of active earth pressure

Passive Earth Pressure

$$\sigma'_f = \sigma'_V \tan^2 \left(45^\circ + \frac{\phi}{2} \right) + 2 c \tan \left(45^\circ + \frac{\phi}{2} \right)$$

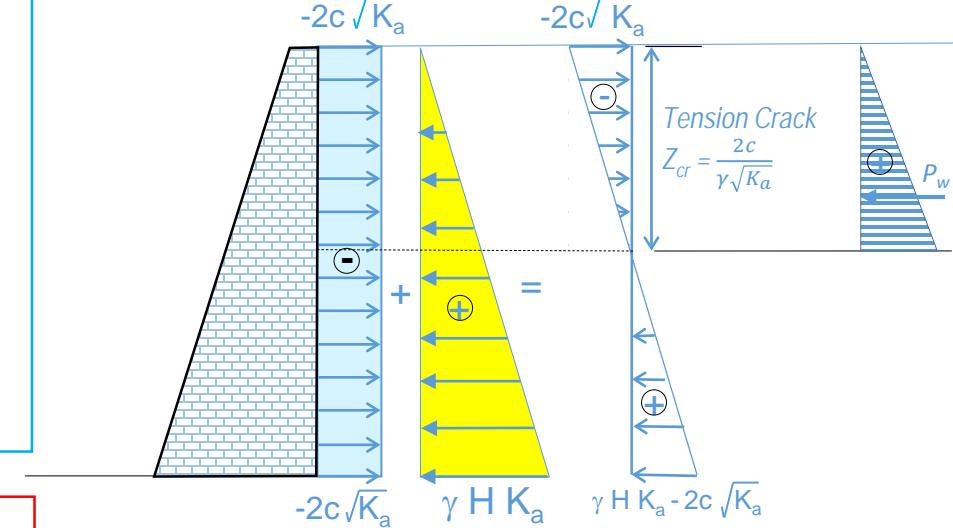
Or

$$\sigma'_f = \sigma'_V K_p + 2 c \sqrt{K_p}$$

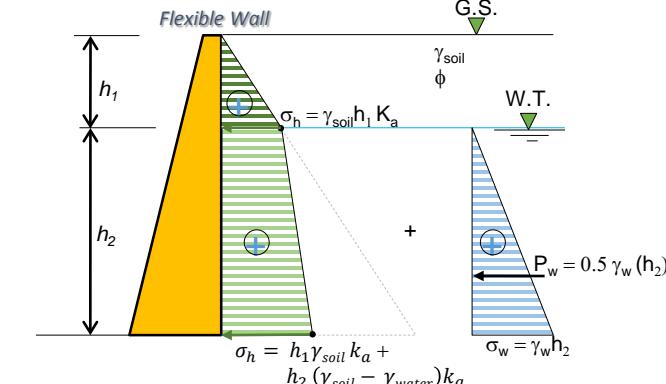
$$K_p = \tan^2 \left(45^\circ + \frac{\phi}{2} \right) = \frac{1 + \sin \phi}{1 - \sin \phi}$$

Coefficient of passive earth pressure

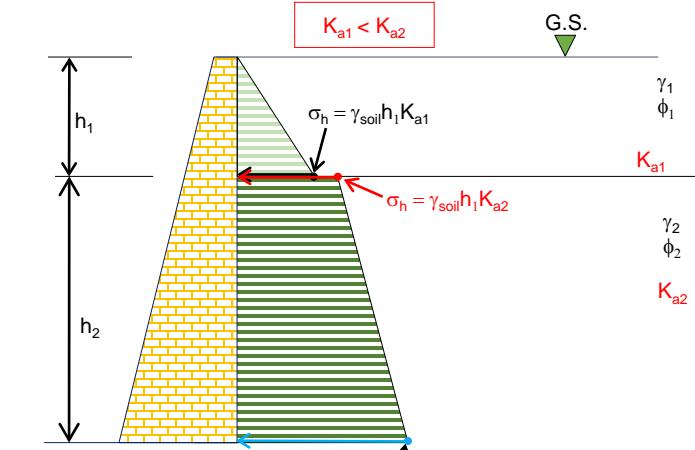
Effect of Cohesion of the Rankine's Active and Passive Earth Pressure



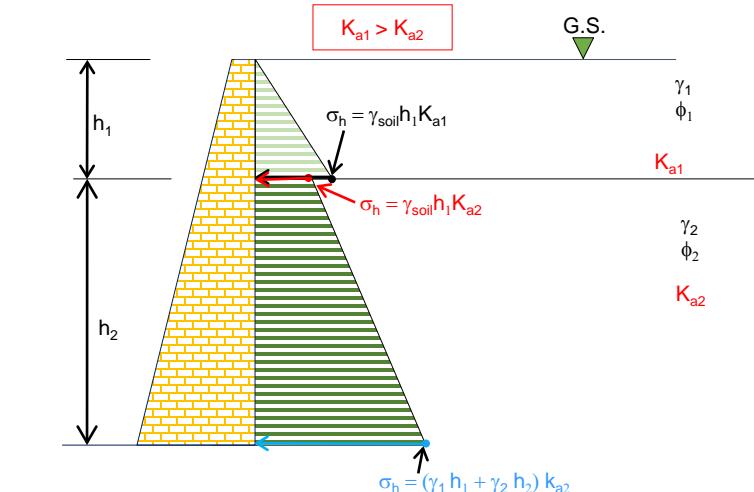
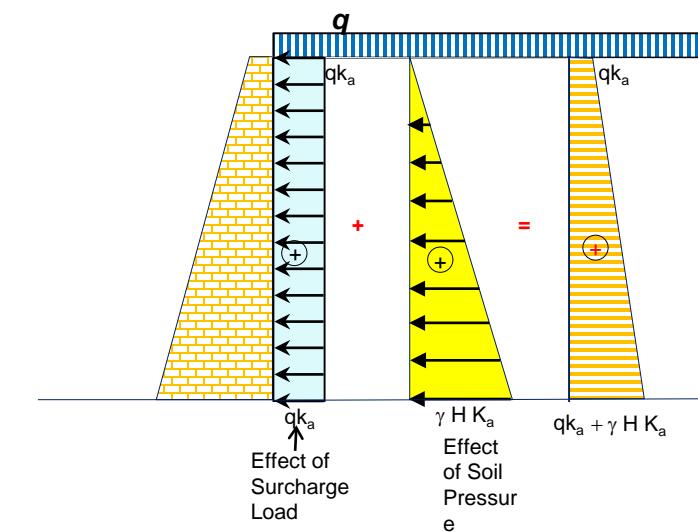
Rankine's Active Earth Pressure in f - Soil & Water Table



Effect of Two Soil Layers on Active Earth Pressure



Effect of Surcharge (q) Load on Active Earth Pressure



Internal Stability

Structural Design

Steel Reinforcement
and Thicknesses

Structural Design

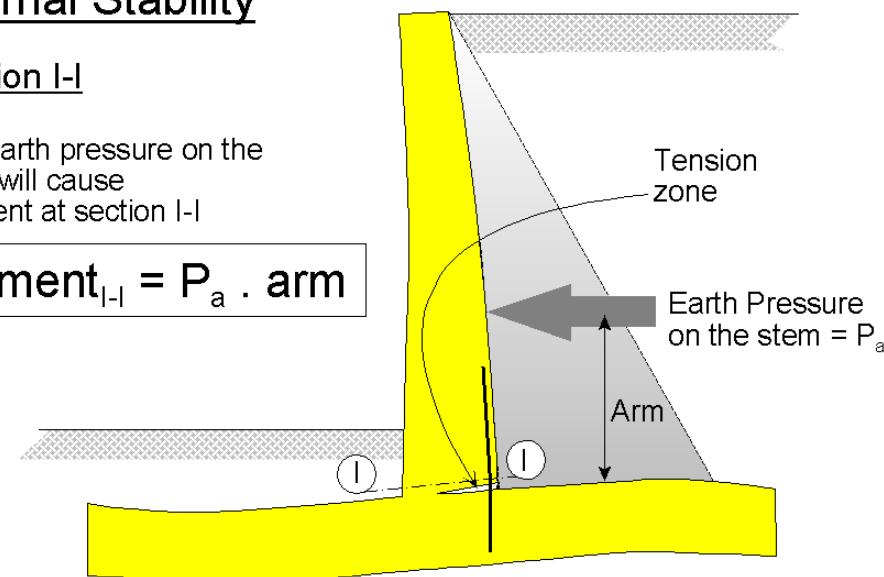
2-

Internal Stability

Section I-I

The earth pressure on the stem will cause moment at section I-I

$$\text{Moment}_{I-I} = P_a \cdot \text{arm}$$

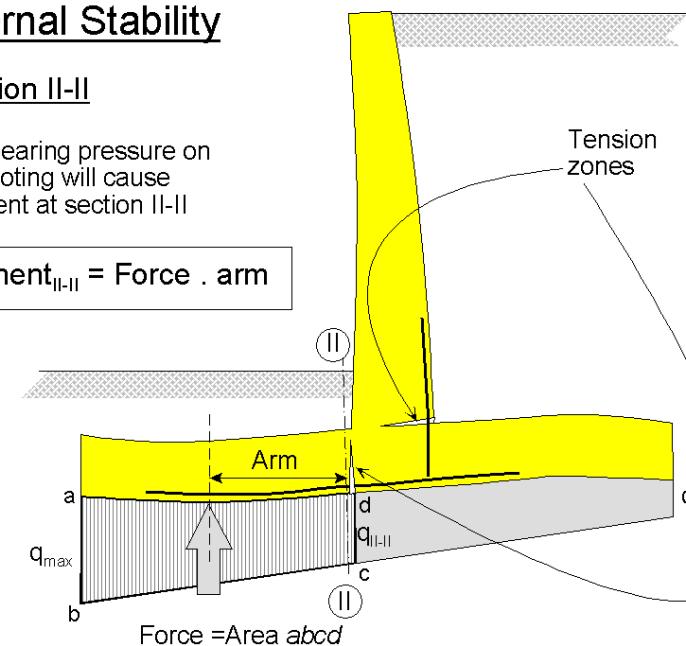


Internal Stability

Section II-II

The bearing pressure on the footing will cause moment at section II-II

$$\text{Moment}_{II-II} = \text{Force} \cdot \text{arm}$$

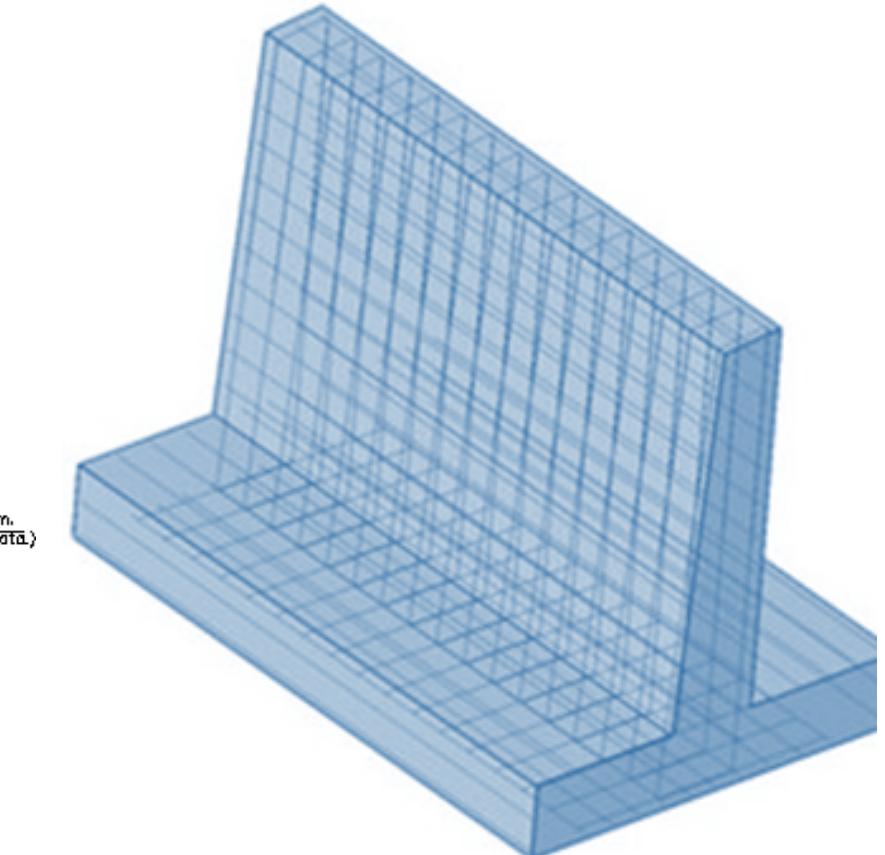
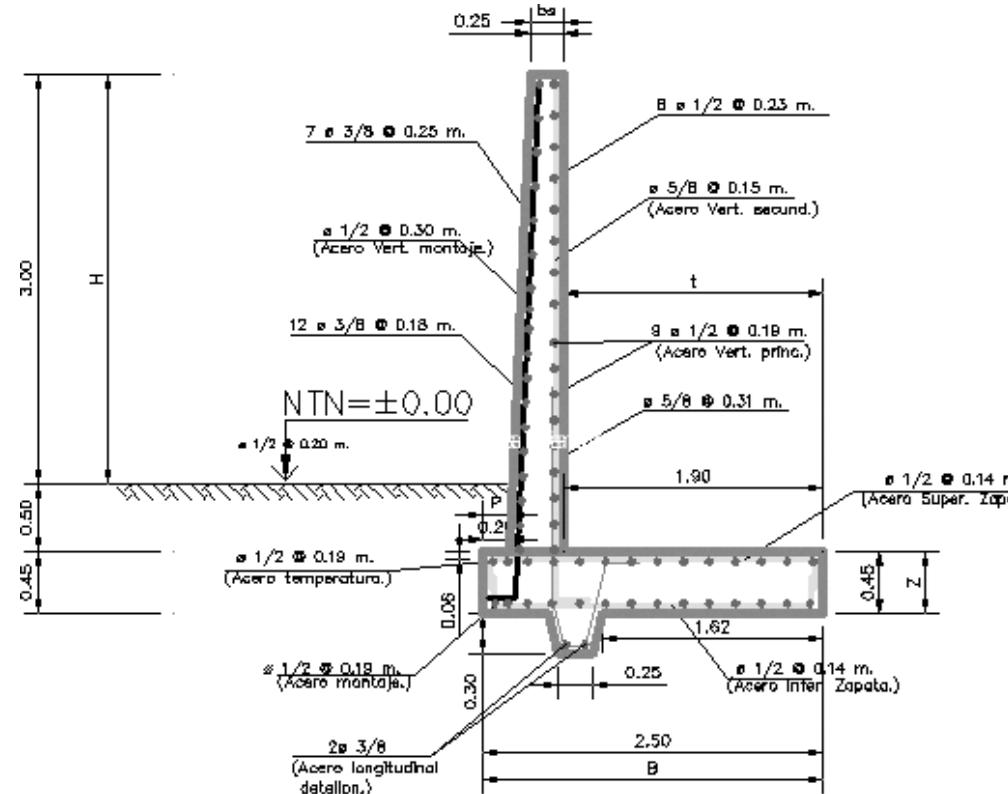
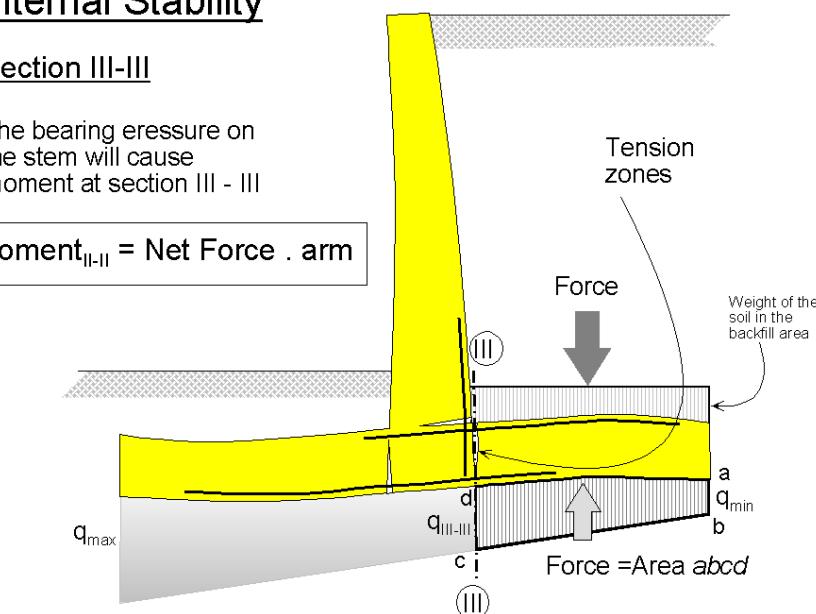


Internal Stability

Section III-III

The bearing pressure on the stem will cause moment at section III - III

$$\text{Moment}_{III-III} = \text{Net Force} \cdot \text{arm}$$

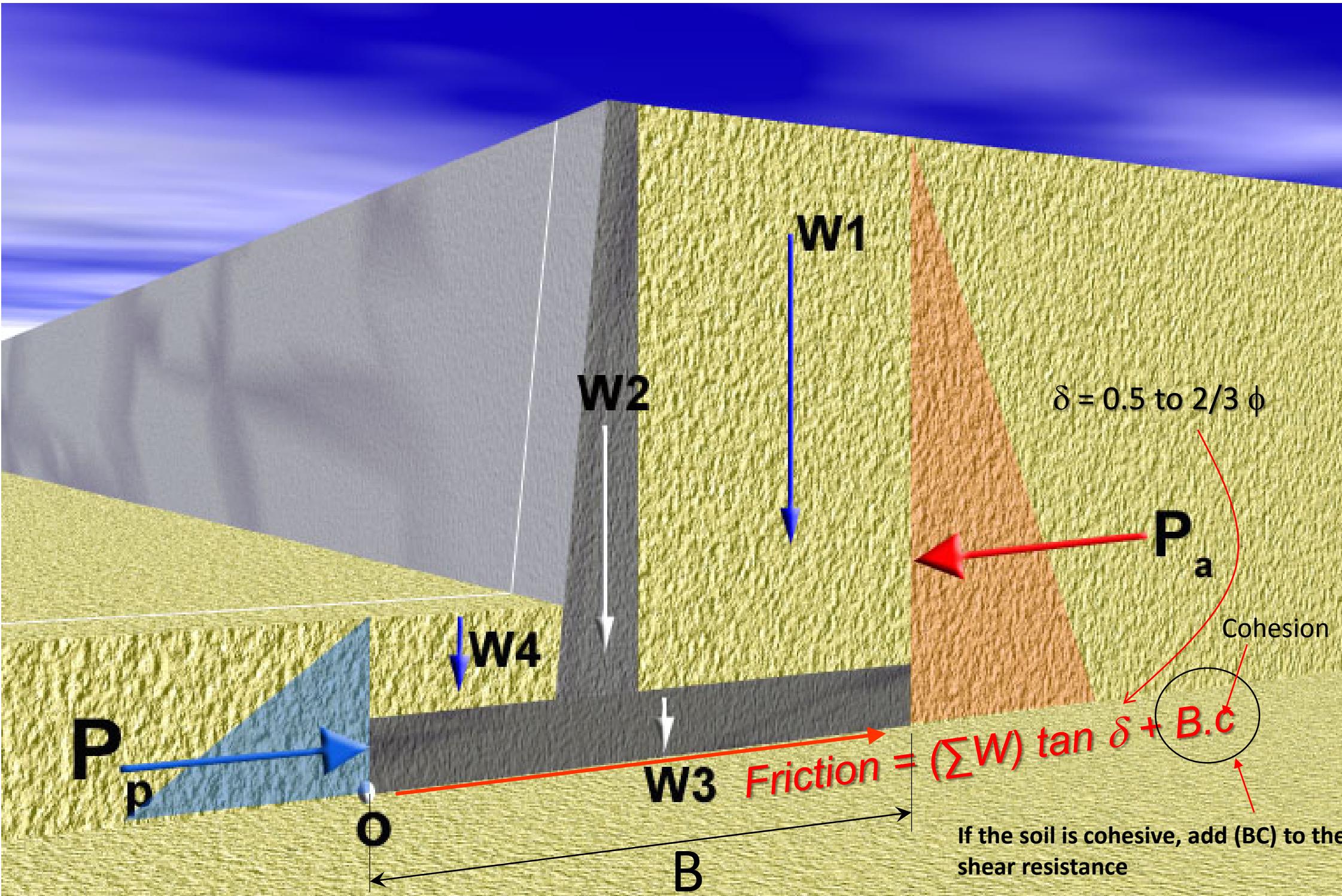


DETALLES DE MURO DE CONTENCION

I. External Stability

1- Sliding

Factor of Safety Against Sliding = $\frac{\text{Resisting Force}}{\text{Driving Force}} = \frac{F_R}{F_D}$



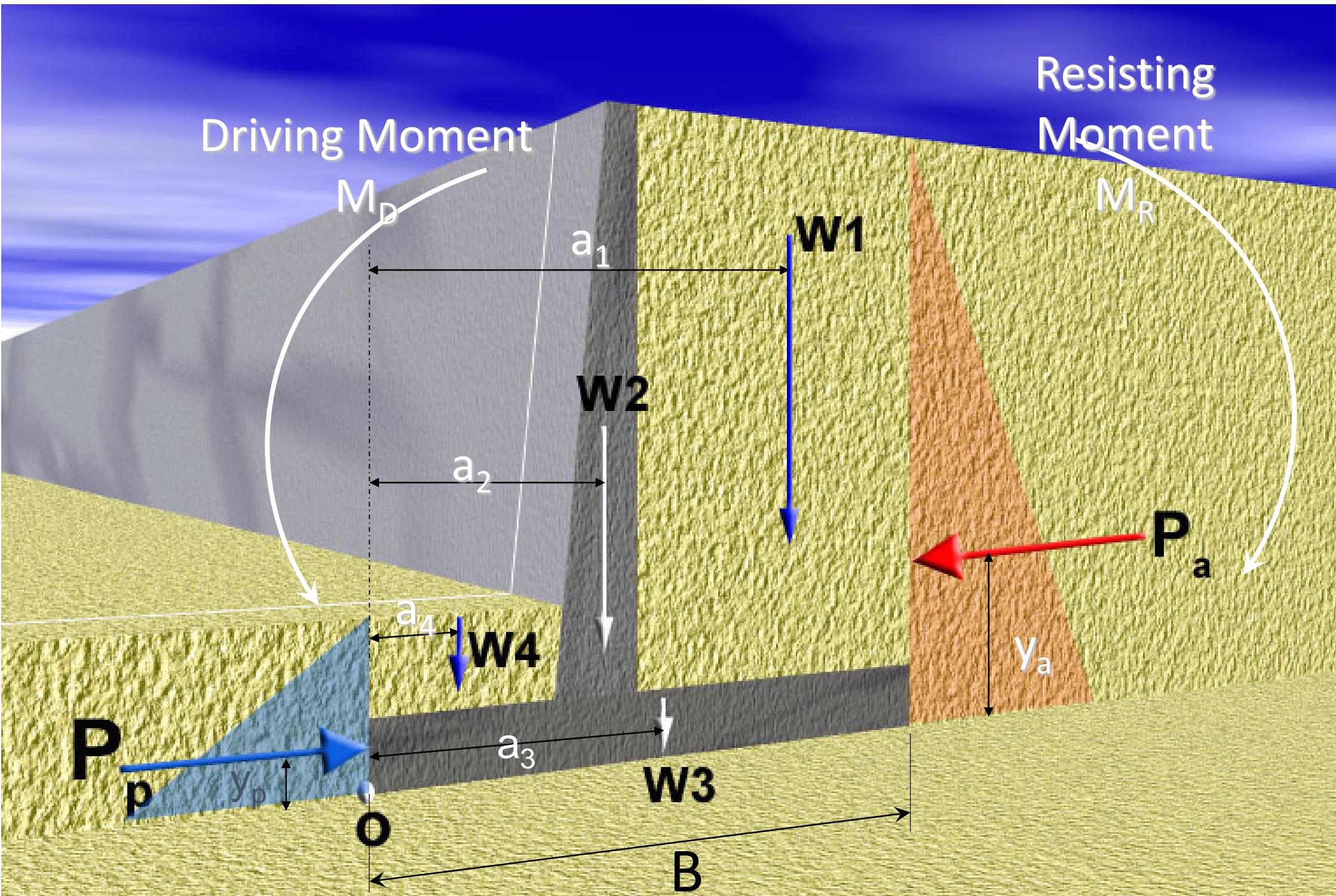
$$F_D = P_a$$

$$F_R = P_p + \text{Friction}$$

I. External Stability

2- Overturning

$$\text{Factor of Safety Against Sliding} = \frac{\text{Resisting Moment}}{\text{Driving Moment}} = \frac{M_R}{M_D}$$



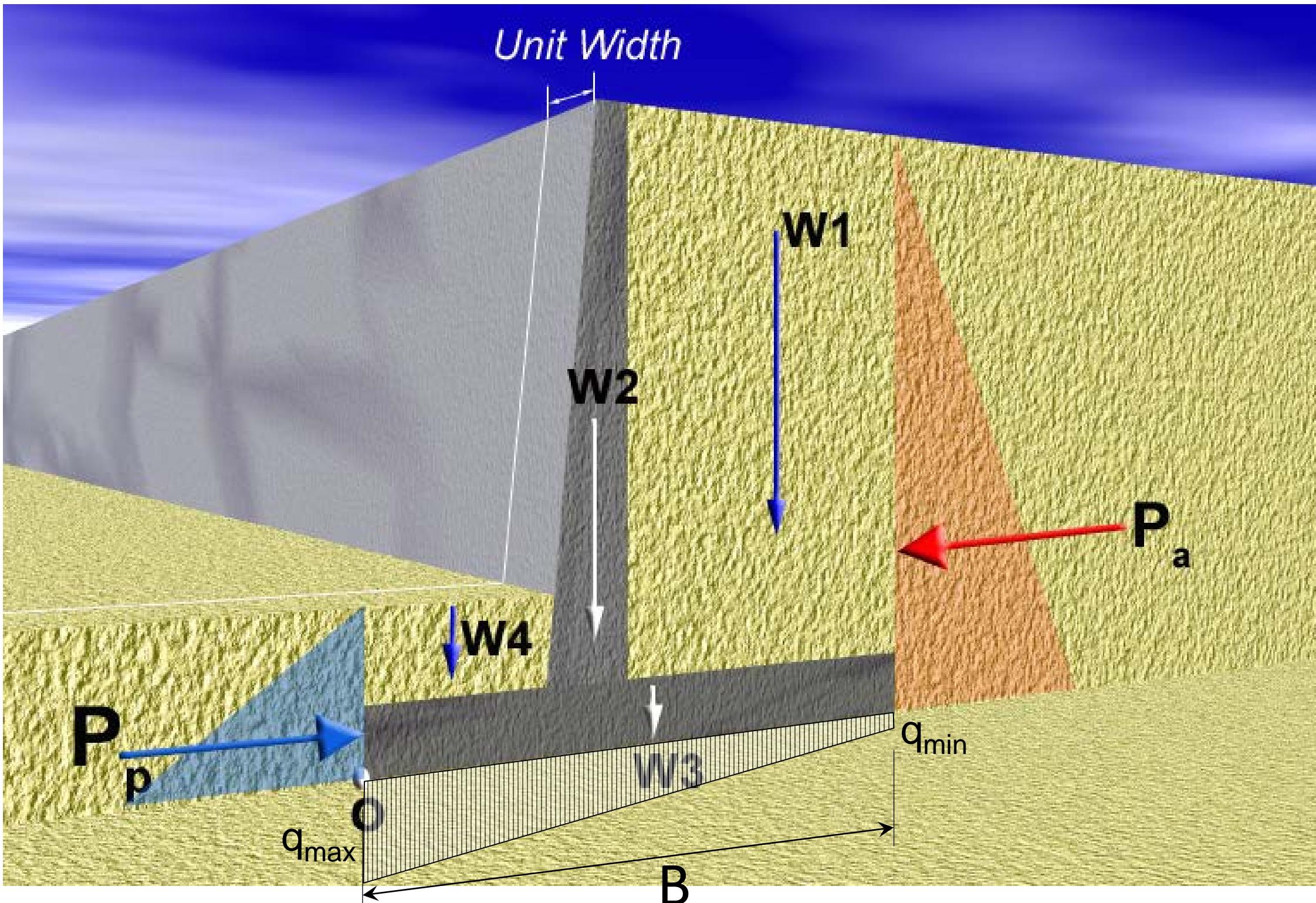
Moment About o

$$M_D = P_a \cdot y_a$$

$$M_R = P_p \cdot y_p + W_1 \cdot a_1 + W_2 \cdot a_2 + W_3 \cdot a_3 + W_4 \cdot a_4$$

I. External Stability

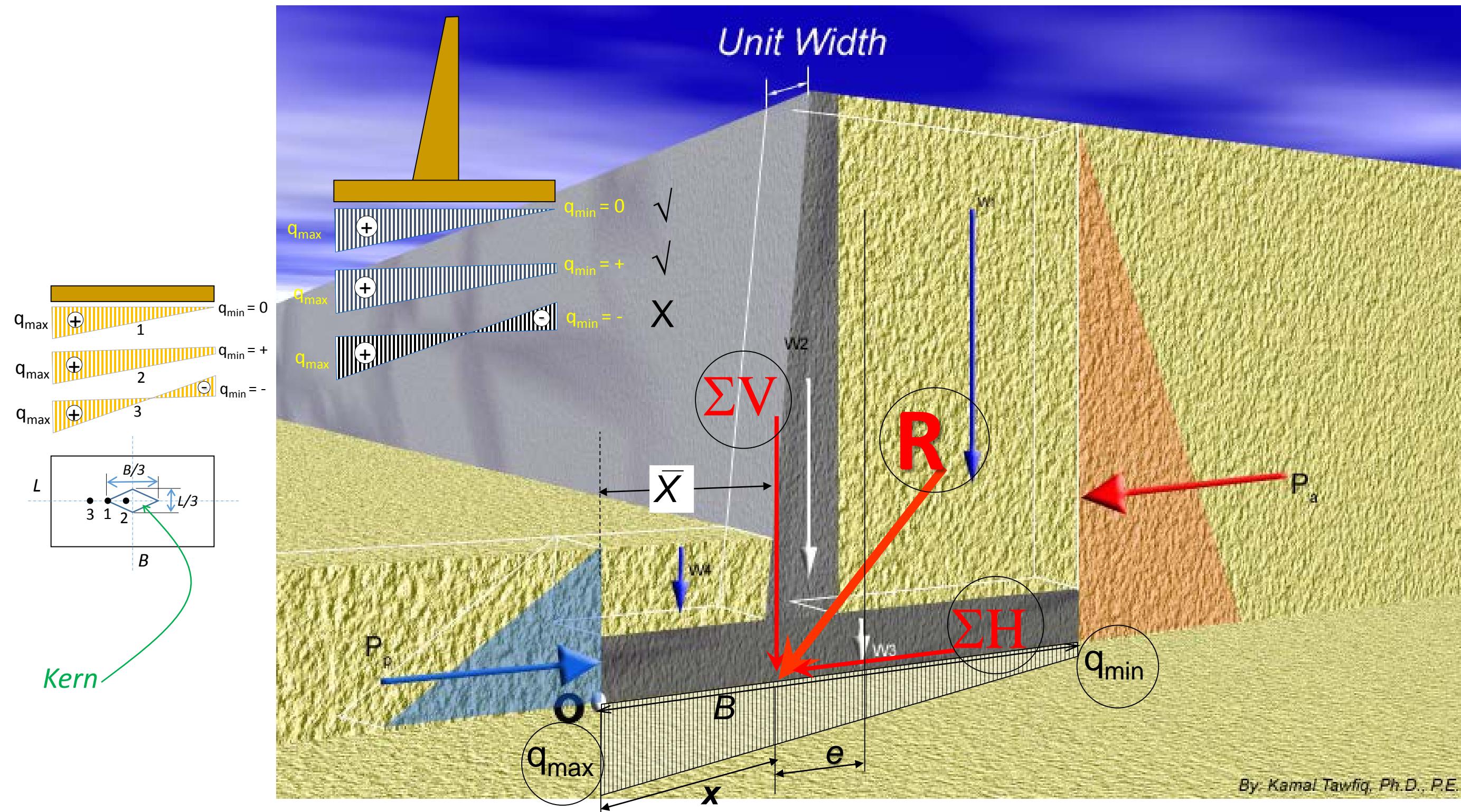
3- Bearing Capacity Failure



3- Check for Bearing Capacity Failure

Factor of Safety Against Bearing Capacity Failure =

$$\frac{q_{\text{all}}}{q_{\text{max}}}$$



By: Kamal Tawfiq, Ph.D., P.E.

ΣV = sum of all vertical loads

ΣH = sum of all horizontal loads

$$R = \sqrt{(\Sigma V)^2 + (\Sigma H)^2}$$

$$M_{\text{net}} = \Sigma M_R - \Sigma M_D$$

$$\Sigma V \bar{X} = M_{\text{net}}$$

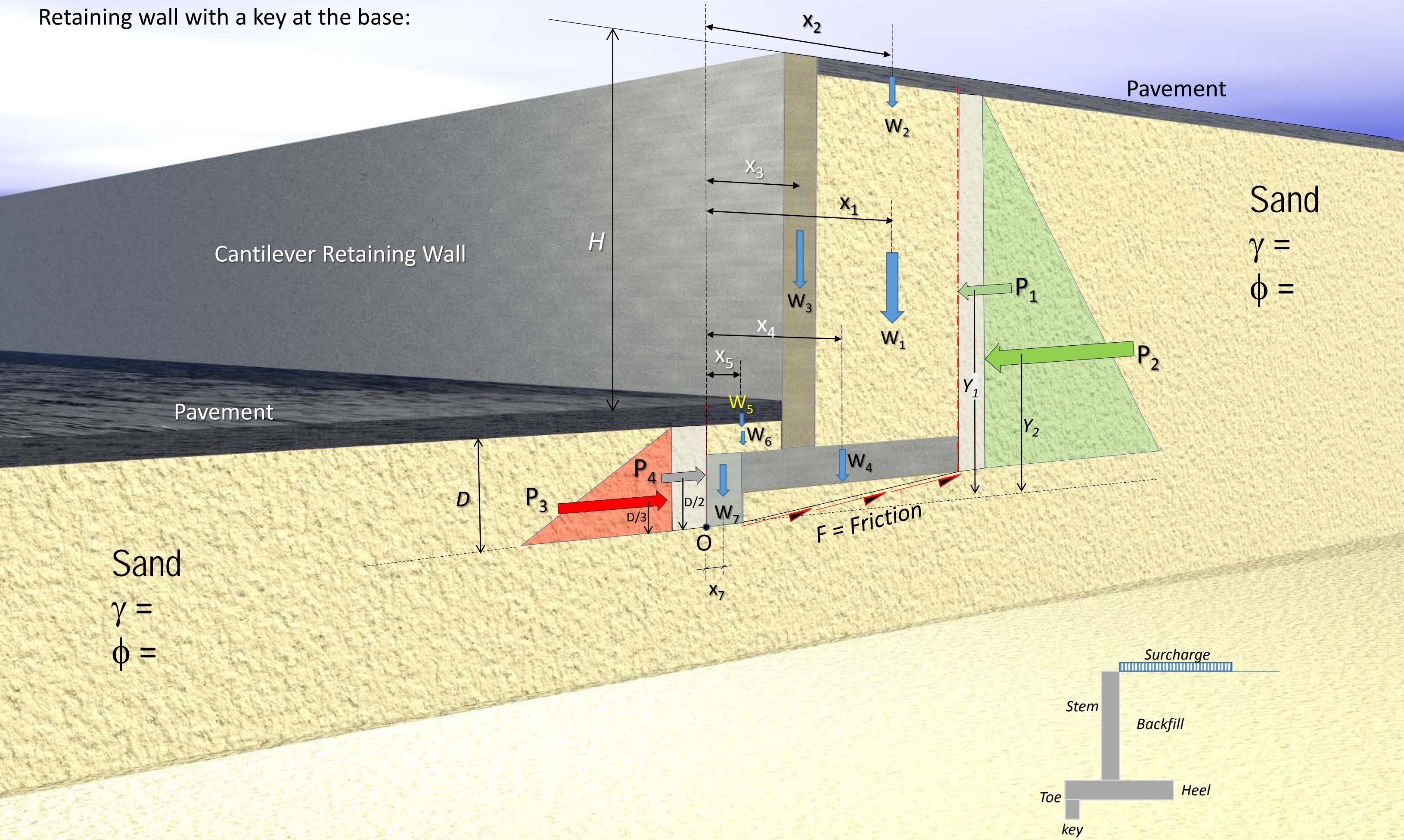
$$e = \frac{B}{2} - \bar{X}$$

$$q = \frac{\Sigma V}{A} \pm \frac{M_{\text{net}} y}{I}$$

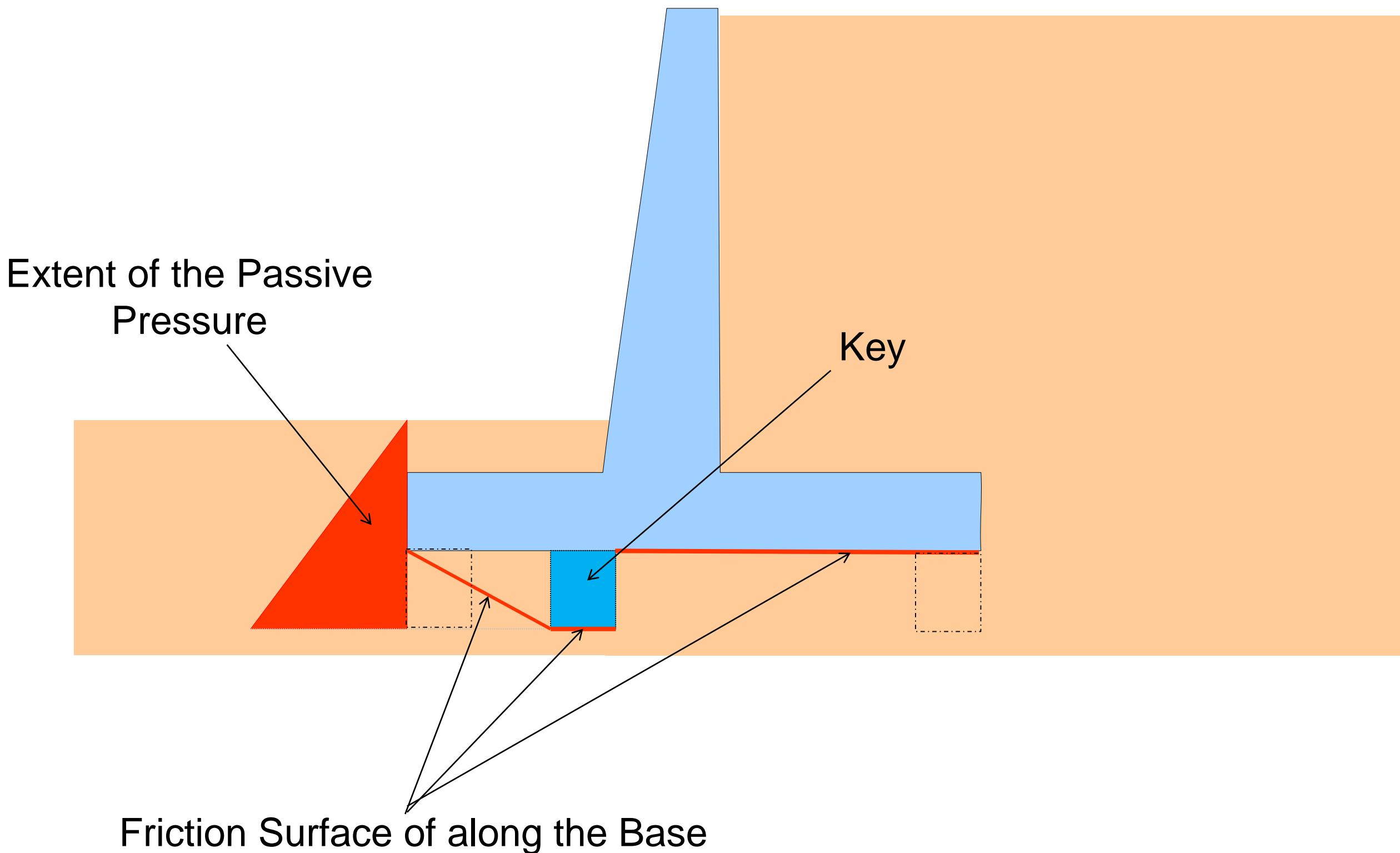
$$q_{\text{max}} = \frac{\Sigma V}{B} \left(1 \pm \frac{6e}{B} \right)$$

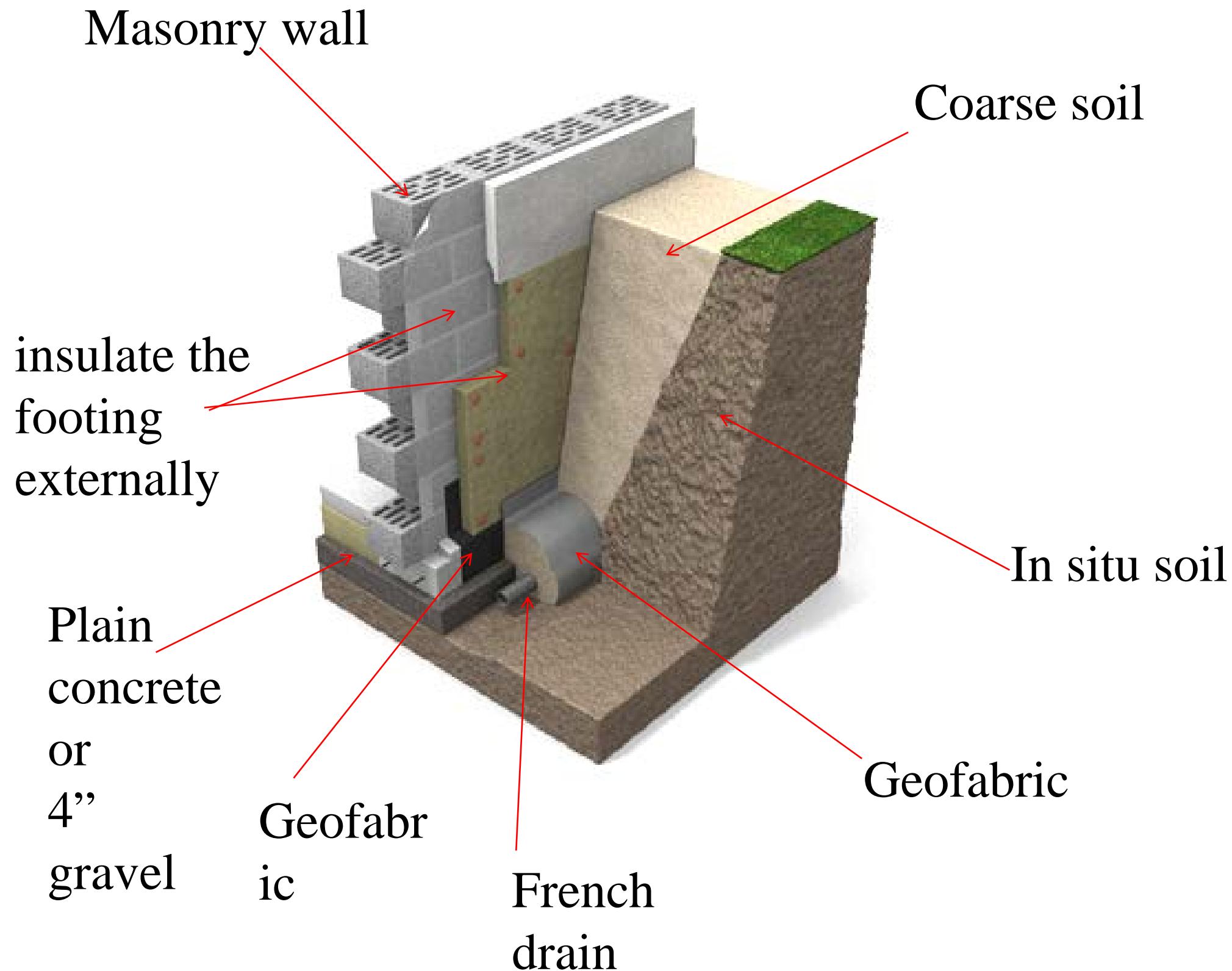
Kamal Tawfiq

Retaining wall with a key at the base:



Using Key at the Base to Improve Sliding Resistance





Active Earth Pressure in ϕ – Soil

Example -1

Given:

- Vertical retaining wall (Rigid)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30°
- Assume wall to be smooth
- Angle of friction between the base and the soil $\delta = 20^\circ$

Determine:

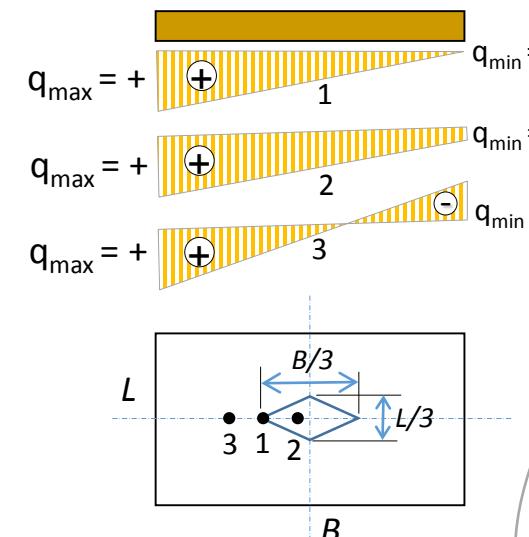
The stability of the wall

Solution:

$$\sigma_h = \sigma_v k_o \quad \text{For Rigid Wall use } k_o \\ P_o = 0.5 \gamma H^2 k_o$$

$$K_o = 1 - \sin\phi$$

$$P_o = 0.5 \times 115 \times 12^2 \times 0.5 = 4,140 \text{ lb/ft}$$

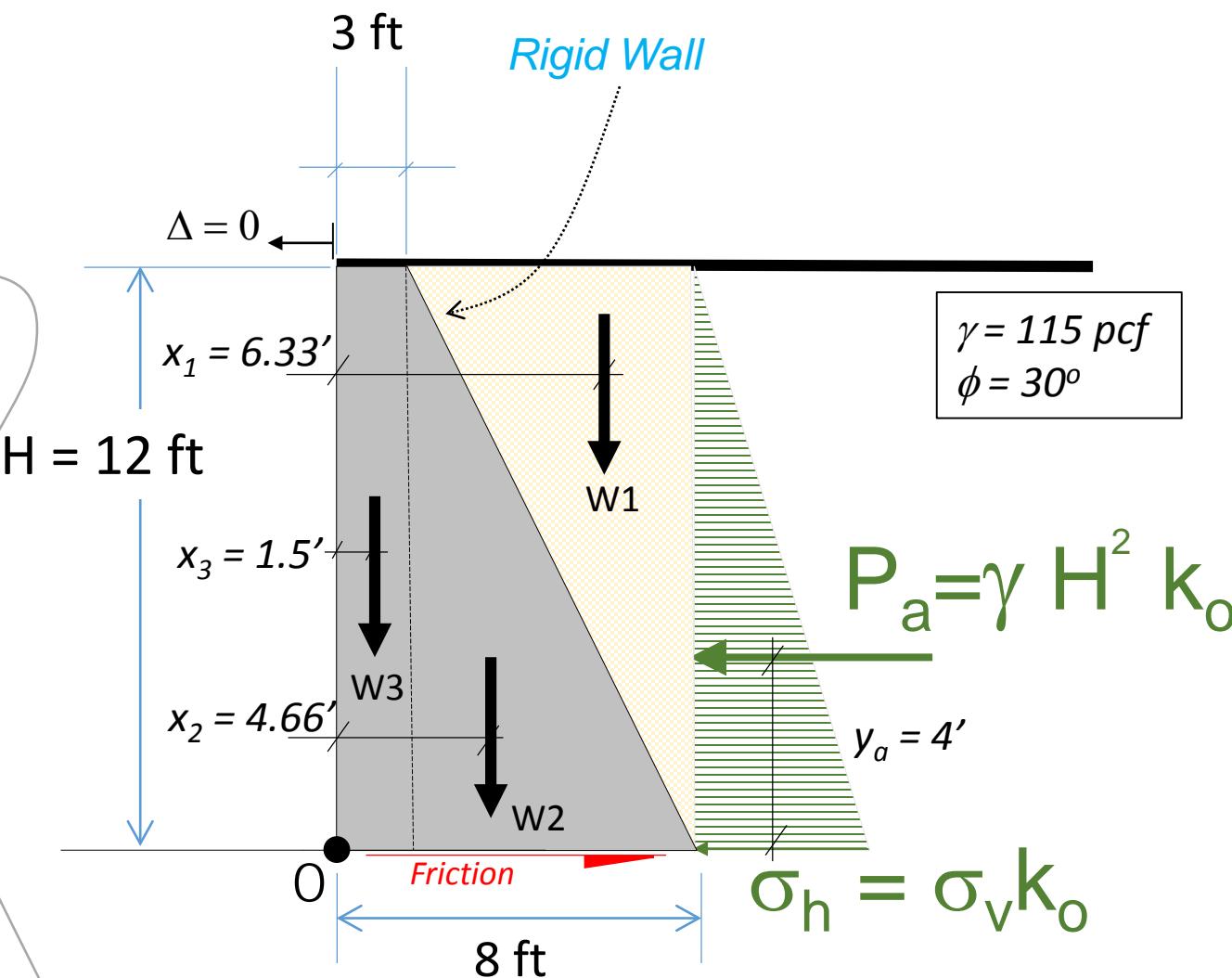


1- Factor of Safety Against Sliding

$$FS_{(\text{sliding})} = \frac{\sum V \tan(20^\circ) + P_p}{P_a} \quad P_p = 0 \\ = \frac{[(13350) \tan 20^\circ]}{4140} = 1.2 > 1.5 \text{ Not OK}$$

2- Factor of Safety Against Overturning

$$FS_{(\text{overturning})} = \frac{\sum M_R}{M_D} = \frac{50908.5}{16560} = 3.1 > 2.0 K$$



$$P_a = \gamma H^2 k_o$$

$$\sigma_h = \sigma_v k_o$$

3- Factor of Safety Bearing Capacity Failure = $FS_{(BC)}$

$$M_{\text{net}} = \sum M_R - \sum M_D = 50908.5 - 13350 = 37558.5 \text{ ft.lb/ft}$$

$$M_{\text{net}} = 37,558.5 = \sum F_y(X) = 13350 (X)$$

$$X = (M_{\text{net}} / \sum F_y) = 2.81 \text{ ft}$$

$$e = (8/2) - 2.81 = 1.18 \text{ ft} < B/6 \text{ or } 8/6 = 1.33 \text{ (Full contact)}$$

$$q_{\max} = \frac{\sum F_y}{B} \left(1 + \frac{6e}{B} \right) = \frac{13350}{8} \left(1 + \frac{(6)(1.18)}{8} \right) = 3145.6 \frac{\text{lb}}{\text{ft}^2} > q_{\text{all}} = 3,000 \frac{\text{lb}}{\text{ft}^2}$$

$$q_{\min} = \frac{\sum F_y}{B} \left(1 - \frac{6e}{B} \right) = \frac{13350}{8} \left(1 - \frac{(6)(1.18)}{8} \right) = 192 \frac{\text{lb}}{\text{ft}^2}$$

X - Force (lb)/ft	Vertical Distance (ft)	F_y (lb)/ft	Moment Arm X (ft)	Driving Moment (ft.lb)/ft	Resisting Moment (ft.lb)/ft
$P_a = 4,140$	4			$4,140 \times 4 = 16,560$	
		$W_1 = 0.5 \times 5 \times 12 \times 115 = 3,450$	6.33		$3,450 \times 6.33 = 21,838.5$
		$W_2 = 0.5 \times 5 \times 12 \times 150 = 4,500$	4.66		$4,500 \times 4.66 = 20,970$
		$W_3 = 3 \times 12 \times 150 = 5400$	1.5		$5,400 \times 1.5 = 8,100$
$P_p = 0$	0				0
		13,350		16,560	50,908.5

Active & Passive Earth Pressure in ϕ – Soil

Example -2

Given:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight (γ) = 115 pcf
- Angle of soil friction (ϕ) = 30°
- Assume wall to be smooth
- $\gamma_{\text{concrete}} = 150 \text{ lb/ft}^3$
- $D = 4 \text{ ft}$

Find:

- Resultant Force of the Wall

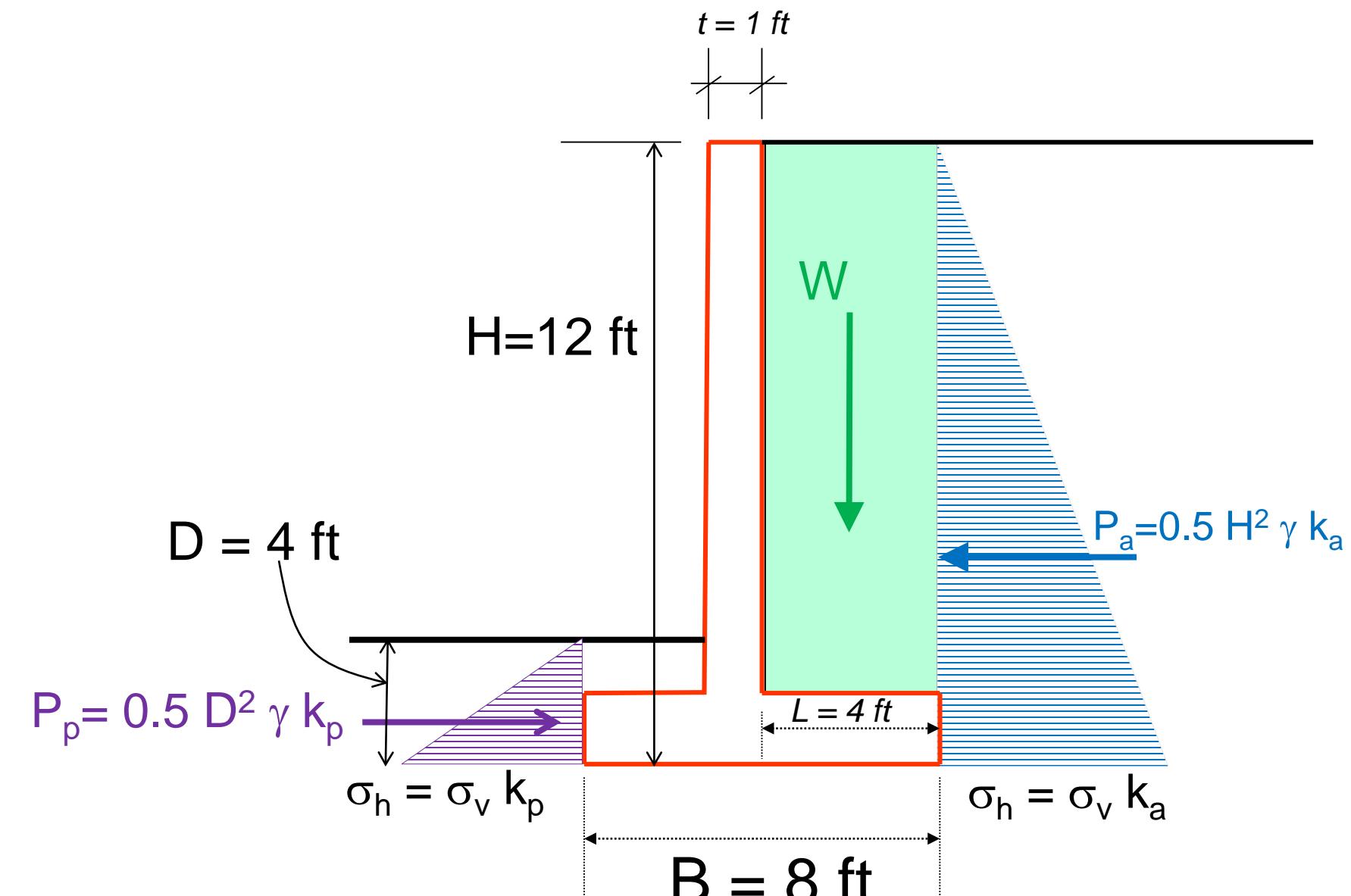
Solution:

$$\sigma_h = \sigma_v k_a$$

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$P_a = 0.5 \gamma H^2 k_a$$

$$P_a = 0.5 \times 12^2 \times 115 \times 0.33 = 2,732.4 \text{ lb/ft}^2$$



$$P_p = 0.5 D^2 \gamma k_p$$

$$P_p = 0.5 \times 4^2 \times 115 \times 3 = 2,760 \text{ lb/ft}^2$$

F_x (lb)/ft	Y (ft)	F_y (lb)/ft	Moment Arm X (ft)	Driving Moment (ft.lb)/ft	Resisting M (ft.lb)/ft
$P_a = 2,732.4$	4			$2,732.4 \times 4 = 10,929.6$	
		$W_1 = 4 \times 10 \times 115 = 4,600$	6		$4600 \times 6 = 27,600$
		$W_2 = 1 \times 10 \times 150 = 1,500$	3.5		$1,500 \times 3.5 = 5,250$
		$W_3 = 8 \times 1 \times 150 = 1,200$	4		$1,200 \times 4 = 4,800$
		$W_4 = 2 \times 3 \times 115 = 690$	1.5		$690 \times 1.5 = 1,035$
$P_p = 2,760$	1.33				$2,760 \times 1.33 = 3,680$
		7,990		10,929.6	42,365

$$1 - \text{Factor of Safety Against Sliding} = FS_{(\text{sliding})} = \frac{\sum F_y \tan(20^\circ) + Pp}{P_a} = \frac{[(7,990) \tan 20^\circ] + 2,760}{2,732.4} = 2.1 > 1.5 OK$$

$$2\text{-Factor of Safety Against Overturning} = FS_{(overturning)} = \frac{\sum M_{Resisting}}{M_{Driving}} = \frac{42,365}{10,929.6} = 3.8 > 2 \text{ OK}$$

3- Factor of Safety Bearing Capacity Failure = FS_(BC)

$$M_{net} = \sum M_r - \sum M_o = 42,365 - 10,929.6 = 31,435.4 \text{ ft.lb/ft}$$

$$M_{net} = 31,435.4 = \sum F_v(X) = 7,990.(X)$$

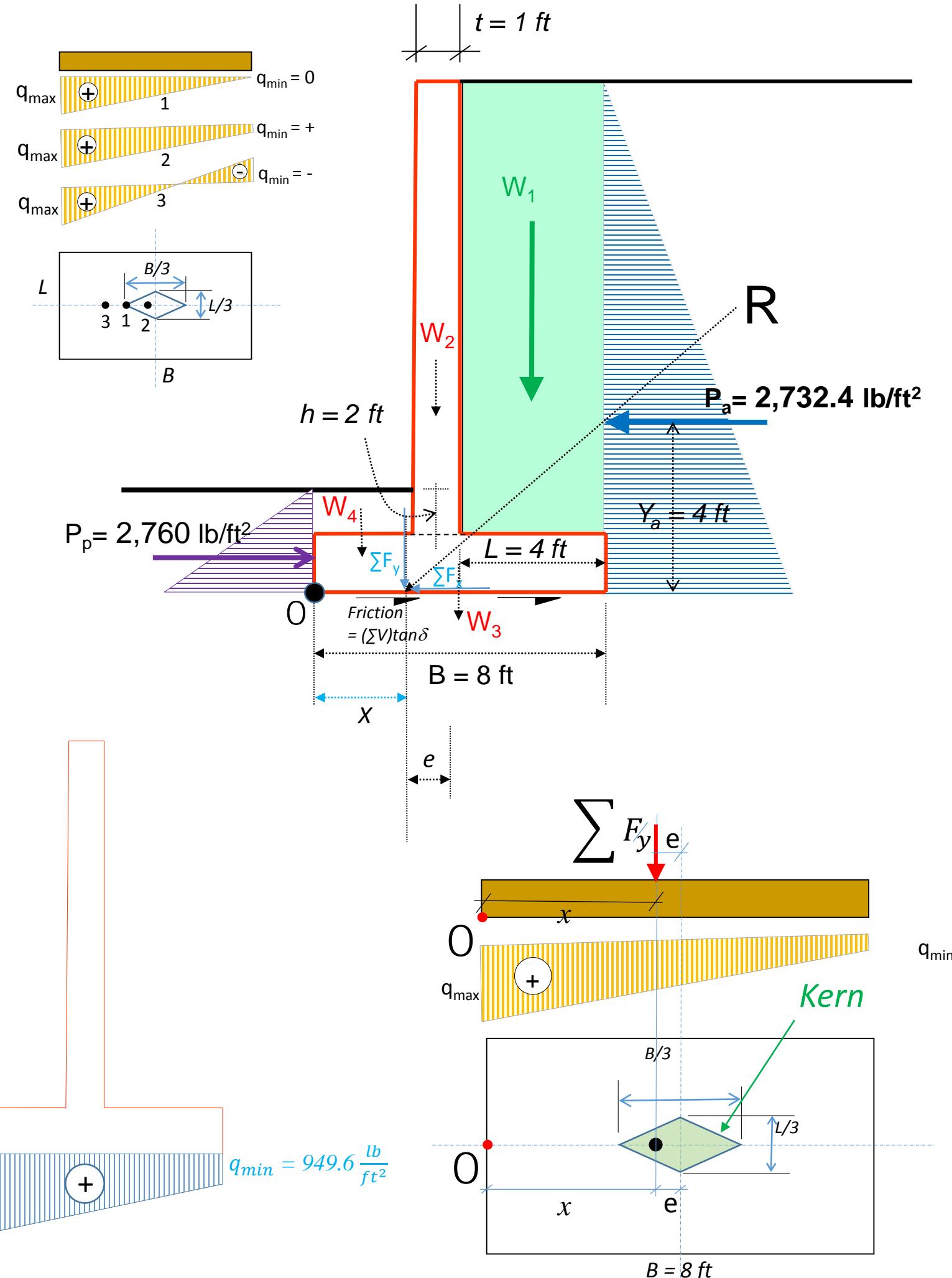
$$X = (M_{net} / \sum F_v) = 3.93 \text{ ft}$$

$$e = (8/2) - 3.93 = 0.0656 \text{ ft} < B/6 \text{ or } 8/6 = 1.33 \text{ (Full contact)}$$

$$q_{max} = \frac{\Sigma F_y}{B} \left(1 + \frac{6e}{B}\right) = \frac{7,990}{8} \left(1 + \frac{(6)(0.0656)}{8}\right) = 1,047.8 \frac{lb}{ft^2} < q_{all} = 3,000 \frac{lb}{ft^2}$$

$$q_{min} = \frac{\sum F_y}{B} \left(1 - \frac{6e}{B}\right) = \frac{7,990}{8} \left(1 - \frac{(6)(0.0656)}{8}\right) = 949.6 \frac{lb}{ft^2}$$

$$q_{max} = 1,047 \frac{lb}{ft^2}$$



Example 1

Given

The cross section of a cantilever retaining wall is shown in Figure 1. Calculate the factors of safety with respect to overturning, sliding, and bearing capacity.

Solution

From the figure,

$$H^* = H_1 + H_2 + T_1 = 8 \tan 10^\circ + 19.5 + 2 = 22.91 \text{ ft}$$

The Rankine active force per unit length of wall =

$$K_a = \cos \alpha \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}$$

$$K_a = \cos 10^\circ \frac{\cos 10^\circ - \sqrt{\cos^2 10^\circ - \cos^2 30^\circ}}{\cos 10^\circ + \sqrt{\cos^2 10^\circ - \cos^2 30^\circ}} = 0.34$$

P_a = Lateral Pressure from Surcharge + Lateral Pressure from Soil

$$P_{a1} = q H^* k_a$$

$$P_{a2} = \frac{1}{2} \gamma H^{*2} k_a$$

$$P_{a1} = 120 \times 22.91 \times 0.34 = 934.32 \text{ lb/ft}$$

$$P_{v1} = 934.32 \sin(10^\circ) = 162.24 \text{ lb/ft}$$

$$P_{h1} = 934.32 \cos(10^\circ) = 920.13 \text{ lb/ft}$$

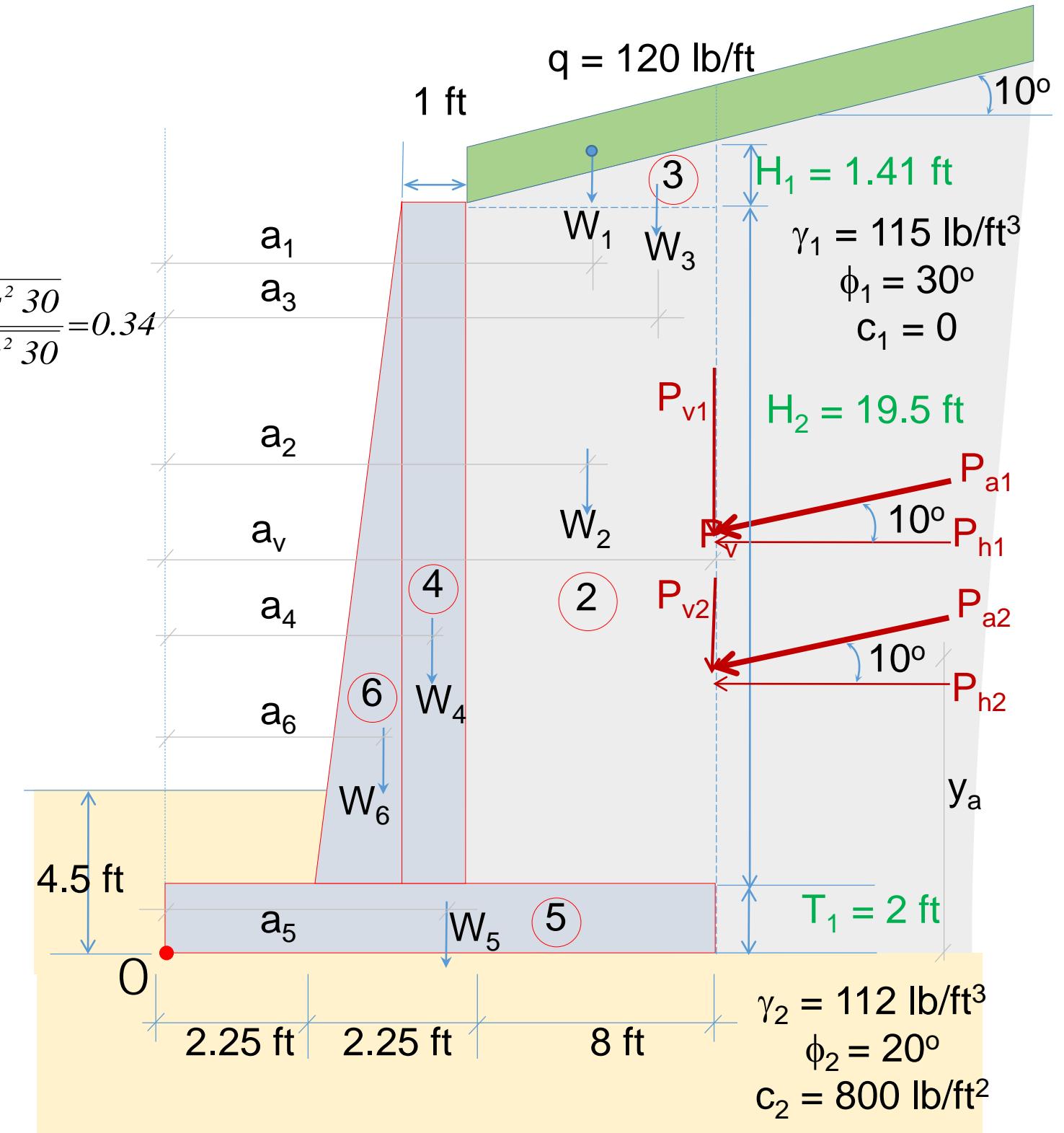
$$Y_{a1} = \frac{22.91}{2} = 11.45 \text{ ft}$$

$$P_{a2} = 0.5 \times 115 \times 22.91^2 \times 0.34 = 10252.22 = 11186.54 \text{ lb/ft}$$

$$P_{v2} = 10252.2 \sin(10^\circ) = 1780.28 \text{ lb/ft}$$

$$P_{h2} = 10252.2 \cos(10^\circ) = 10096.45 \text{ lb/ft}$$

$$Y_{a1} = \frac{22.91}{3} = 7.63 \text{ ft}$$



Driving Pressure (lb/ft ²)/ft	Resisting Pressure (lb/ft ²)/ft	Weight/Unit Length (lb/ft)	Moment Arm from Point O (ft)	Driving Moment (ft.lb/ft)	Resisting Moment (ft.lb/ft)
P _{h1} = 920.13			11.45	920.13x11.45 = 10535.49	
P _{h2} = 10096.45			7.64	10096.45x7.64 = 77136.88	
		W ₁ = 120 x 8 = 960	8.5		960x8.5 = 8,160
		W ₂ = 8X19.2X112 = 17,203.2	8.5		17,203.2x8.5 = 146,227.2
		W ₃ = 0.5X1.41X8X112 = 631.7	9.83		631.7x9.83 = 6,209.6
		W ₄ = 1X19.5X150 = 2,925	4.0		2925x4 = 11,700
		W ₅ = 12.5X2X150 = 3,750	6.25		3750x6.25 = 23,437.5
		W ₆ = 0.5X1.25X19.2X150 = 1,800	3.08		1800x3.08 = 5,544
		ΣP _v = 162.24+1780.28 = 1,942.52	12.5		1942.52x12.5 = 24,281.5
ΣP _h = 11,016.58	0	Σ F _y = 29,212.4		ΣM _D = 87,672.37	ΣM _R = 225,559.86

1- Factor of Safety Against Sliding = FS_(sliding) = $\frac{\sum F_y \tan(20) + P_p}{P_a} = \frac{[(29,212.4) \tan 20^\circ] + 0}{11,016.58} = 0.96 < 1.5 \text{ Not OK}$

Add Passive Resistance

P_p = 0.5 x 4.5² x 115 x 3 = 3493.13 lb/ft

2- Factor of Safety Against Sliding = FS_(sliding) = $\frac{\sum F_y \tan(20) + P_p}{P_a} = \frac{[(29,212.4) \tan 20^\circ] + 3493.13}{11,016.58} = 1.28 < 1.5 \text{ Not OK}$

3- Factor of Safety Against Overturning = FS_(overturning) = $\frac{\sum M_R}{\sum M_D} = \frac{225,559.86}{87,672.37} = 2.57 > 20 \text{ OK}$

4- Factor of Safety Bearing Capacity Failure = FS_(BC) =

$$M_{\text{net}} = \sum M_R - \sum M_D = 225,559.86 - 87,672.37 = 137,887.5 \text{ ft.lb/ft}$$

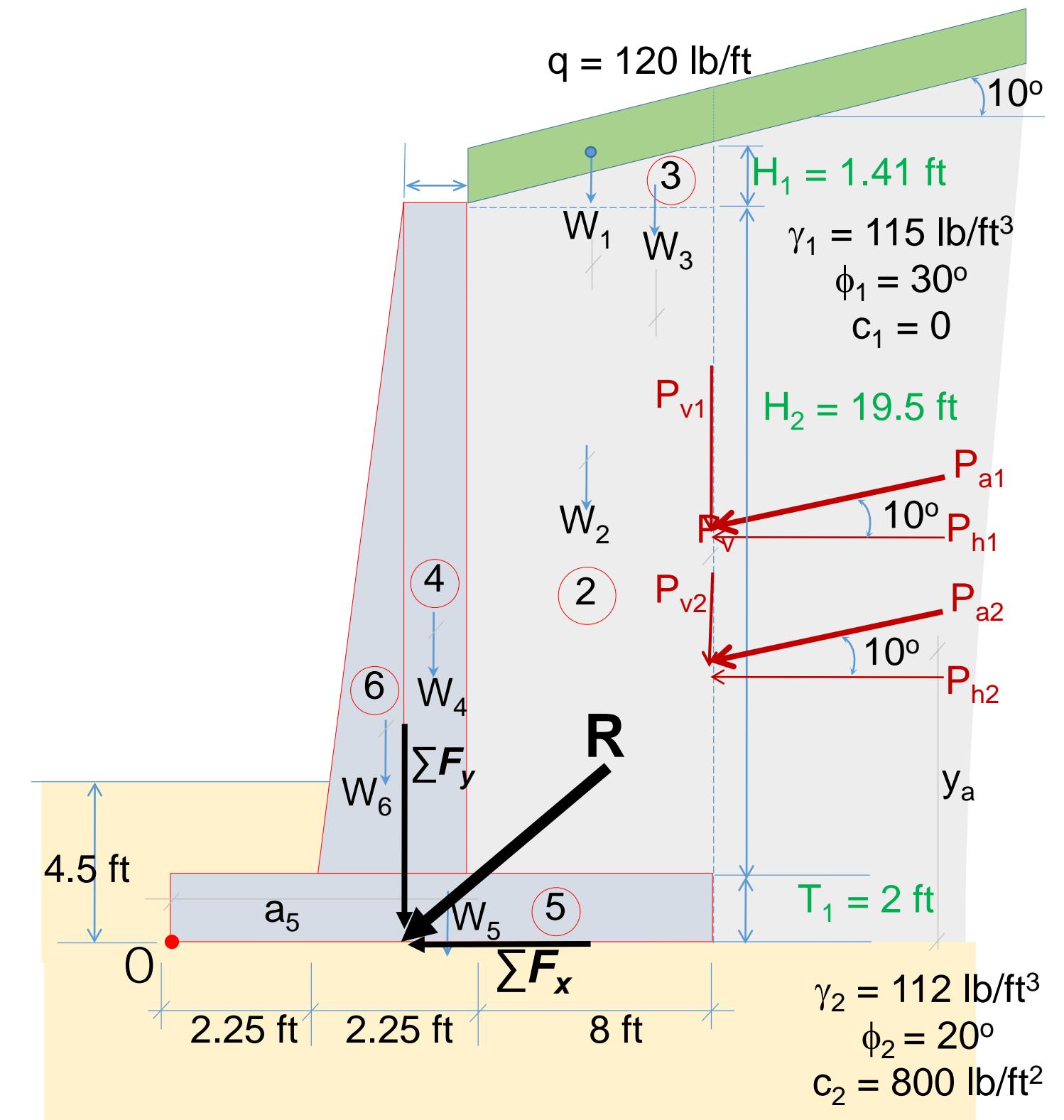
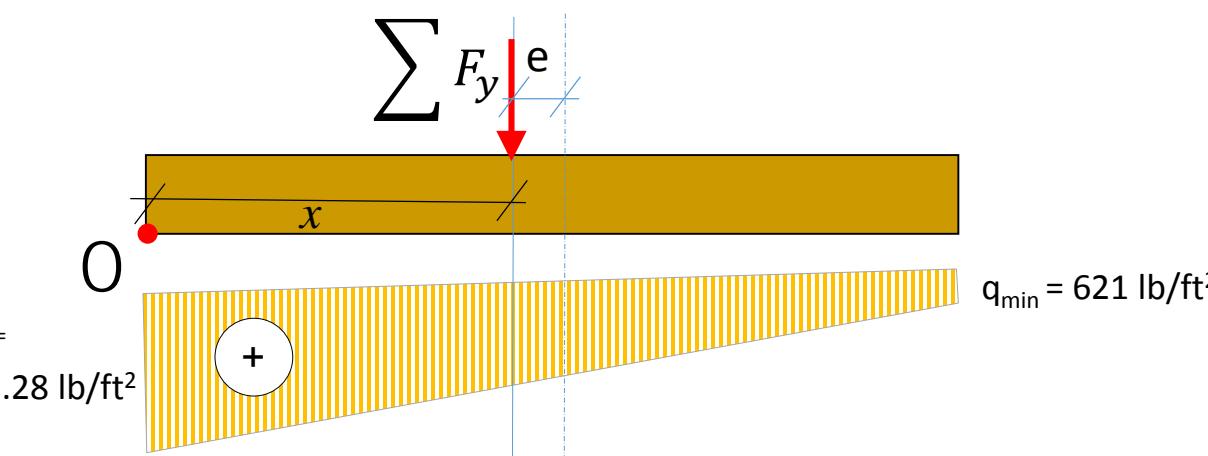
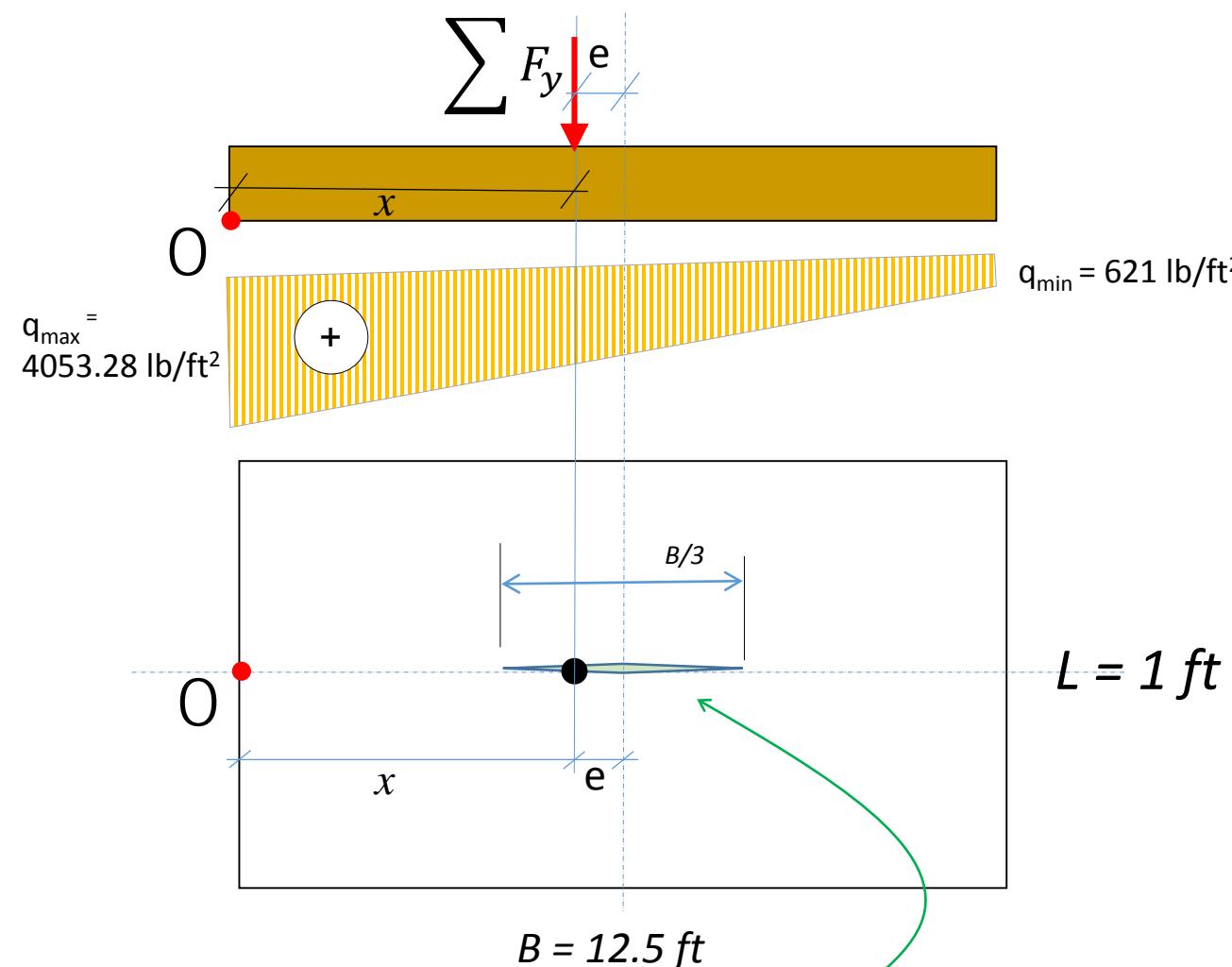
$$M_{\text{net}} = 137,887.5 = \sum F_y(X) = 29,212.4 \quad (X)$$

$$X = (M_{\text{net}} / \sum F_y) = 4.72 \text{ ft}$$

$$e = (12.5/2) - 4.72 = 1.53 \text{ ft} < B/6 \text{ or } 12.5/6 = 2.083 \text{ (Full contact)}$$

$$q_{\max} = \frac{\sum F_y}{B} \left(1 + \frac{6e}{B}\right) = \frac{29,212.4}{12.5} \left(1 + \frac{(6)(1.53)}{12.5}\right) = 4053.28 \frac{\text{lb}}{\text{ft}^2} > q_{\text{all}} = 3,000 \frac{\text{lb}}{\text{ft}^2} \quad \text{No good}$$

$$q_{\min} = \frac{\sum F_y}{B} \left(1 - \frac{6e}{B}\right) = \frac{29,212.4}{12.5} \left(1 - \frac{(6)(1.53)}{12.5}\right) = 621 \frac{\text{lb}}{\text{ft}^2}$$



Kern

Kamal Tawfiq