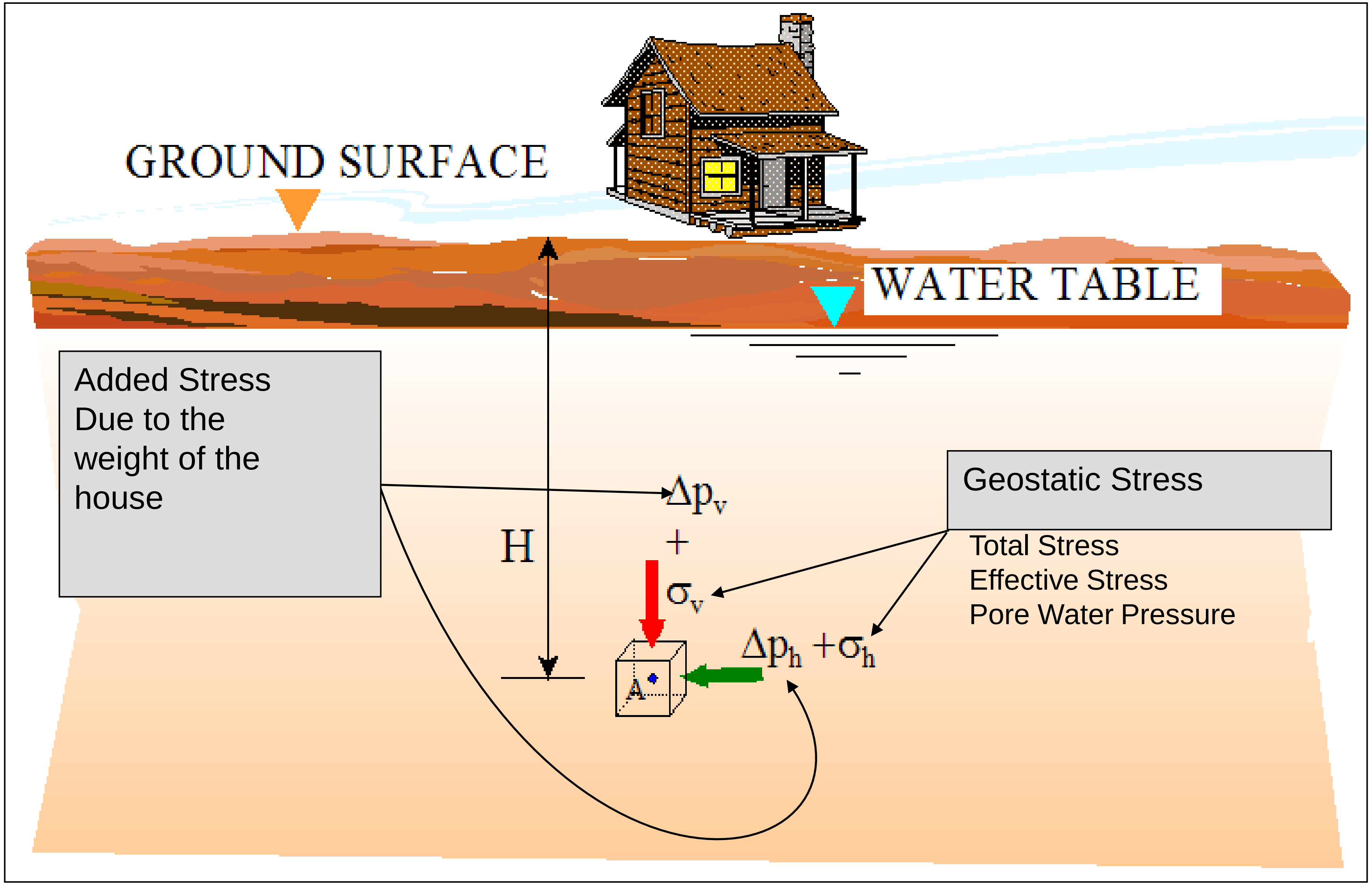
Stresses in Soil

By

Kamal Tawfiq, Ph.D., P.E.

Spring 2019

There are two types of lateral stresses in soil.



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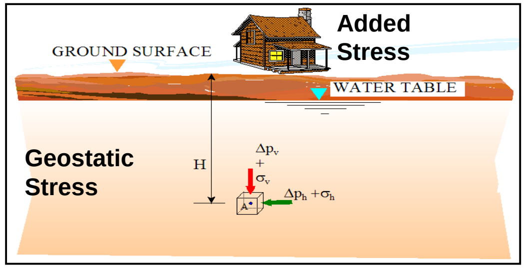


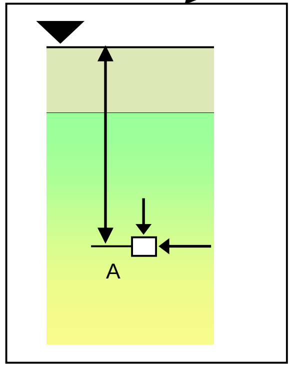
Stress Distribution in Soils

Geostatic Stresses

Total Stress
Effective Stress
Pore Water Pressure

Total Stress= Effective Stress+ Pore Water Pressure

$$\sigma_{total} = \sigma_{eff} + u$$

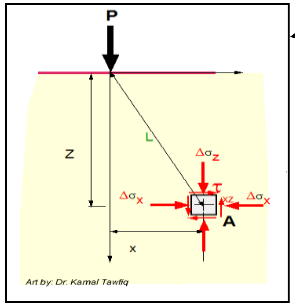


Added Stresses (Point, line, strip, triangular, circular, rectangular)

Bossinisque Equations

1. Point Load
2. Line Load
3. Strip Load
4. Triangular Load
5. Circular Load
6. Rectangular Load

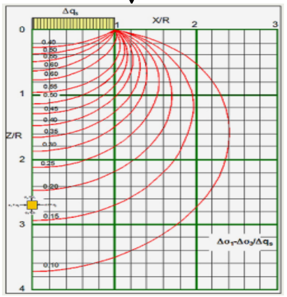
σ_y
σ_x
τ_{xy}



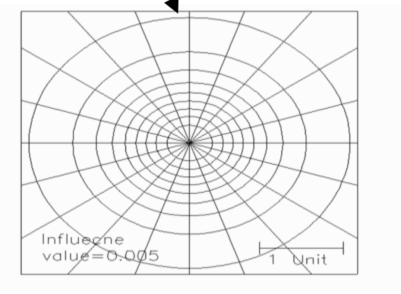
Influence Charts

$$\Delta\sigma_z = I_\sigma \cdot q$$

Stress Bulbs

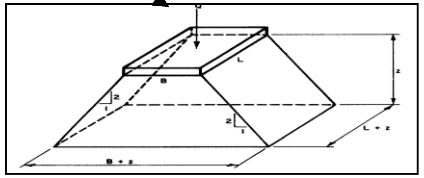


Newmark Charts



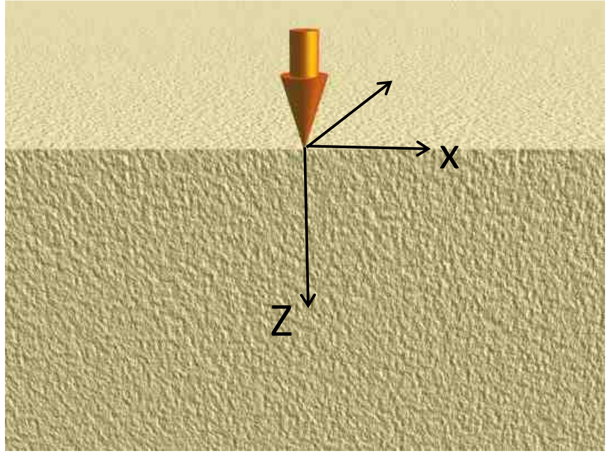
Westergaard's Method
(For Pavement)

Approximate Method
1:2 Method

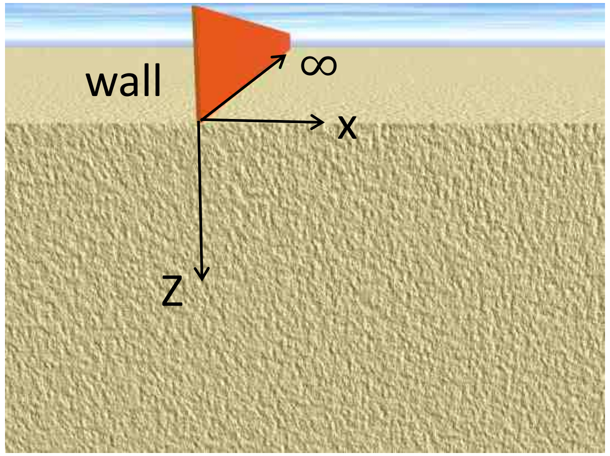


Types of Loads

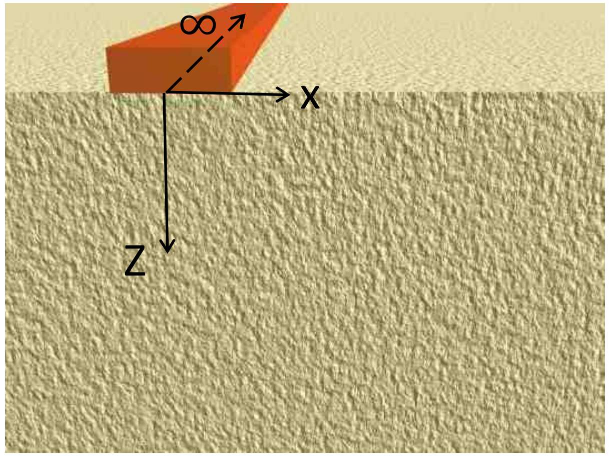
Point Load



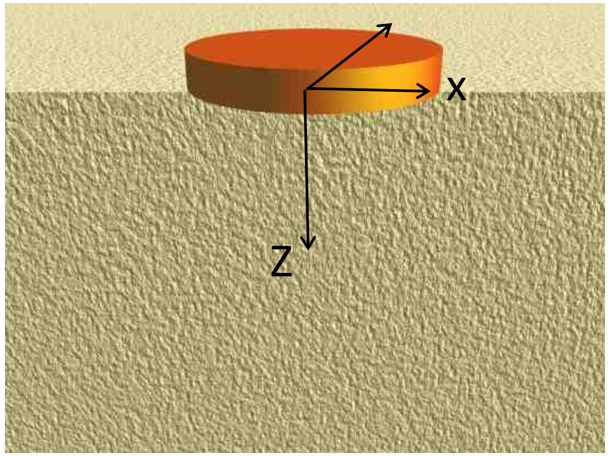
Line Load



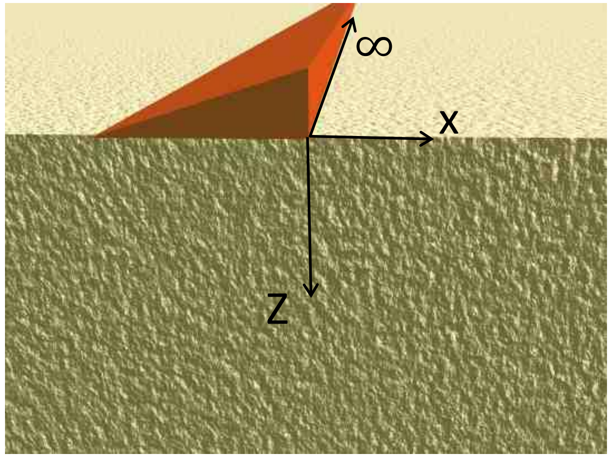
Strip Load



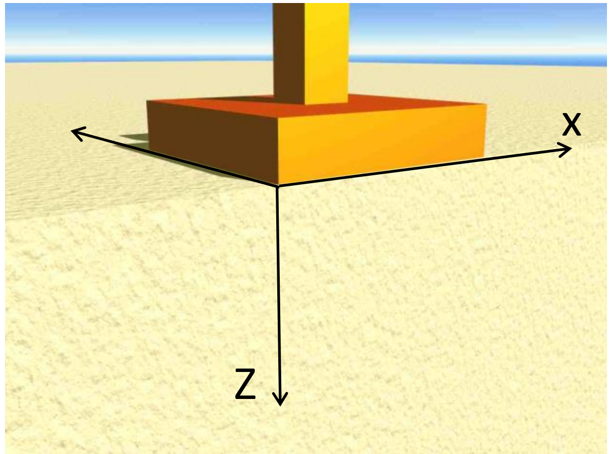
Circular Load



Triangular Load



Rectangular Load



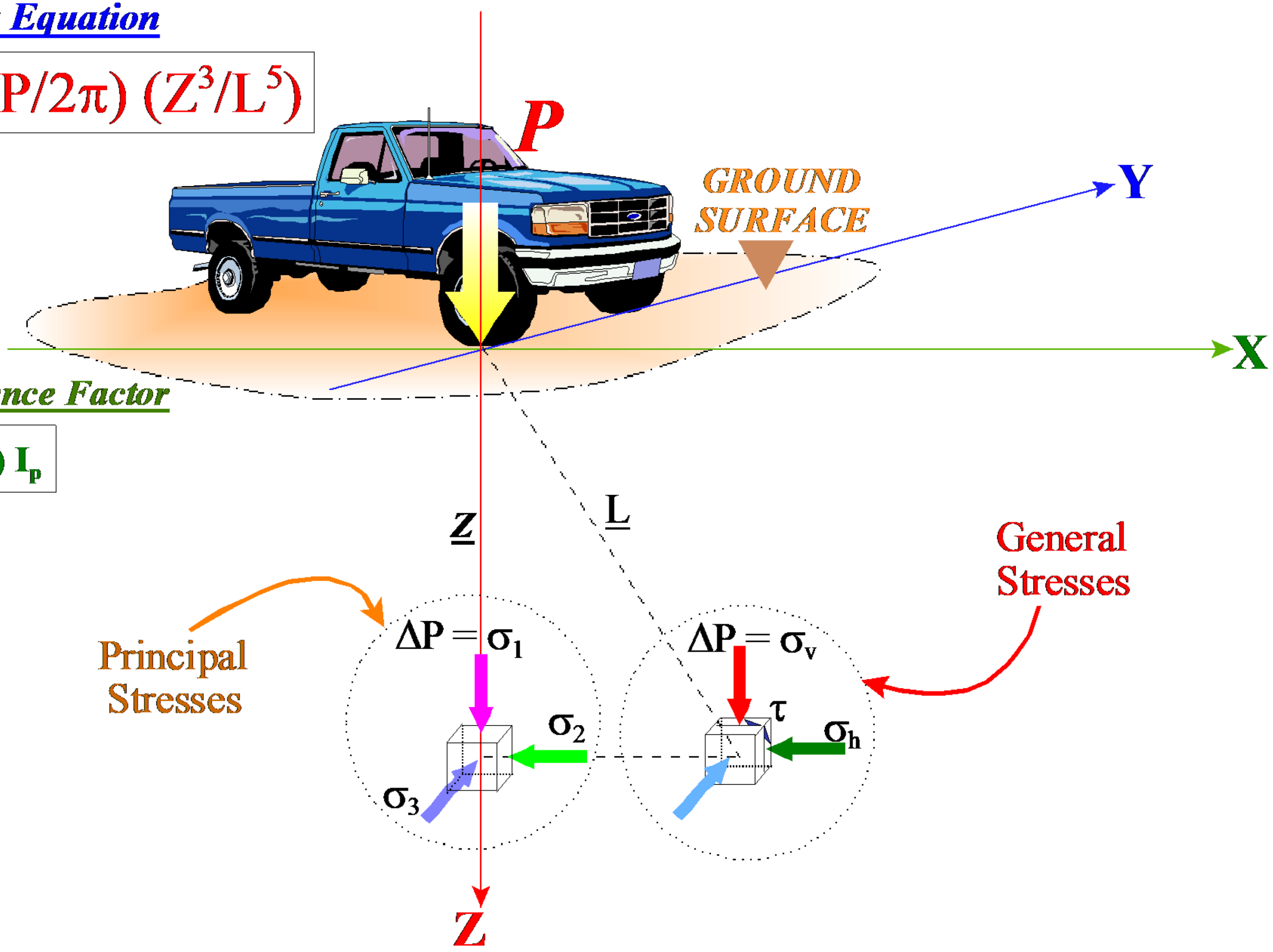
1. STRESSES CAUSED BY A POINT LOAD

Boussinesq's Equation

$$\Delta P_z = (3P/2\pi)\,(Z^3/L^5)$$

Using Influence Factor

$$\Delta P = (P/Z^2)\, I_p$$



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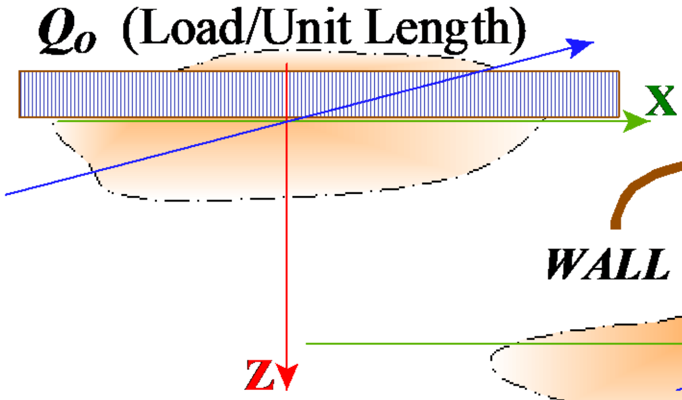
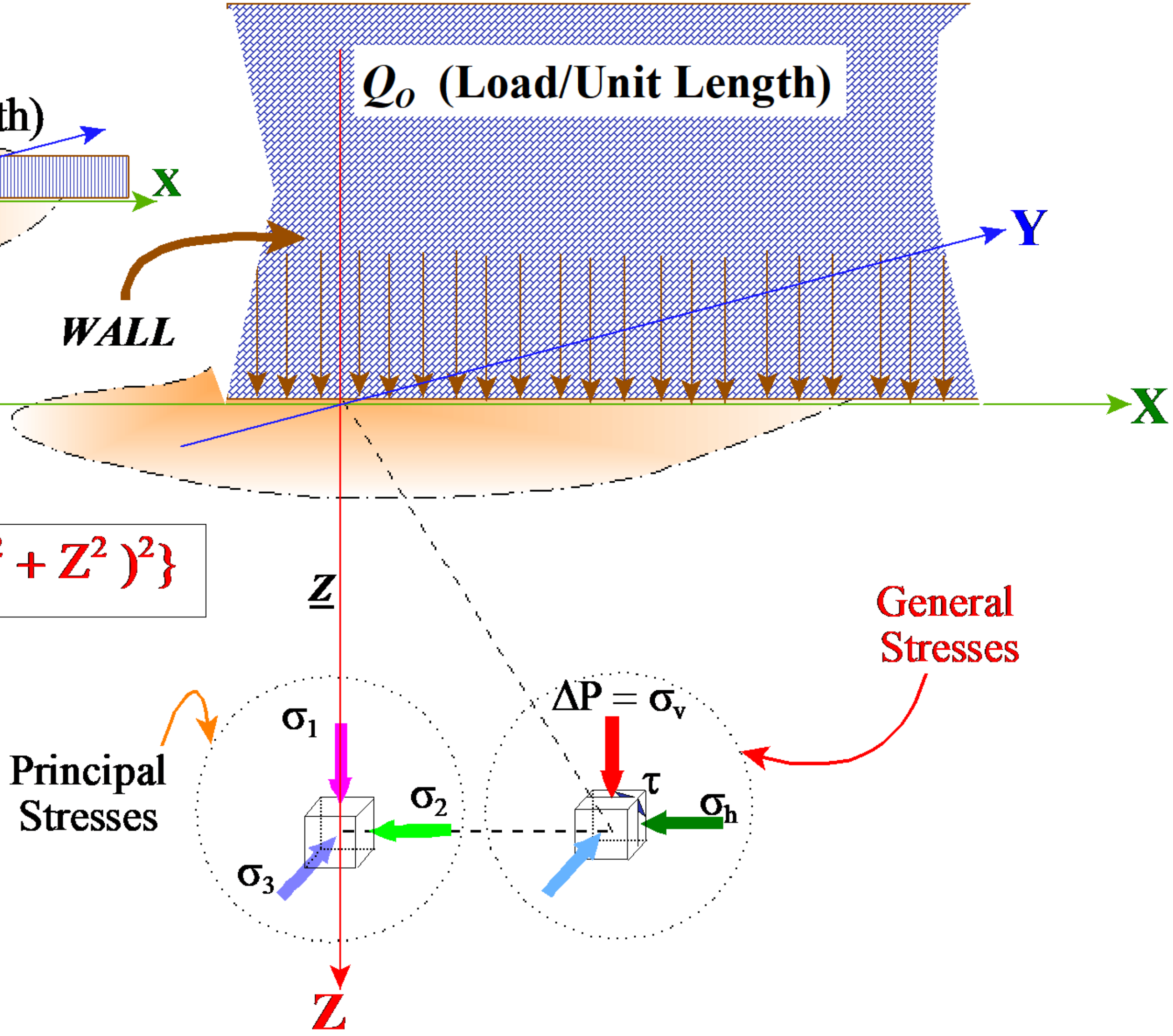
2. STRESSES CAUSED BY A LINE LOAD





Boussinesq's Equation

$$\Delta P_z = \{2\ Q_o\ Z^3/\pi\ (X^2 + Z^2)^2\}$$

Using Influence Factor

$$\Delta P = (P/Z)\ I_L$$

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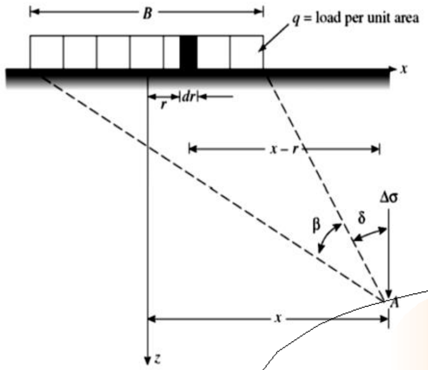
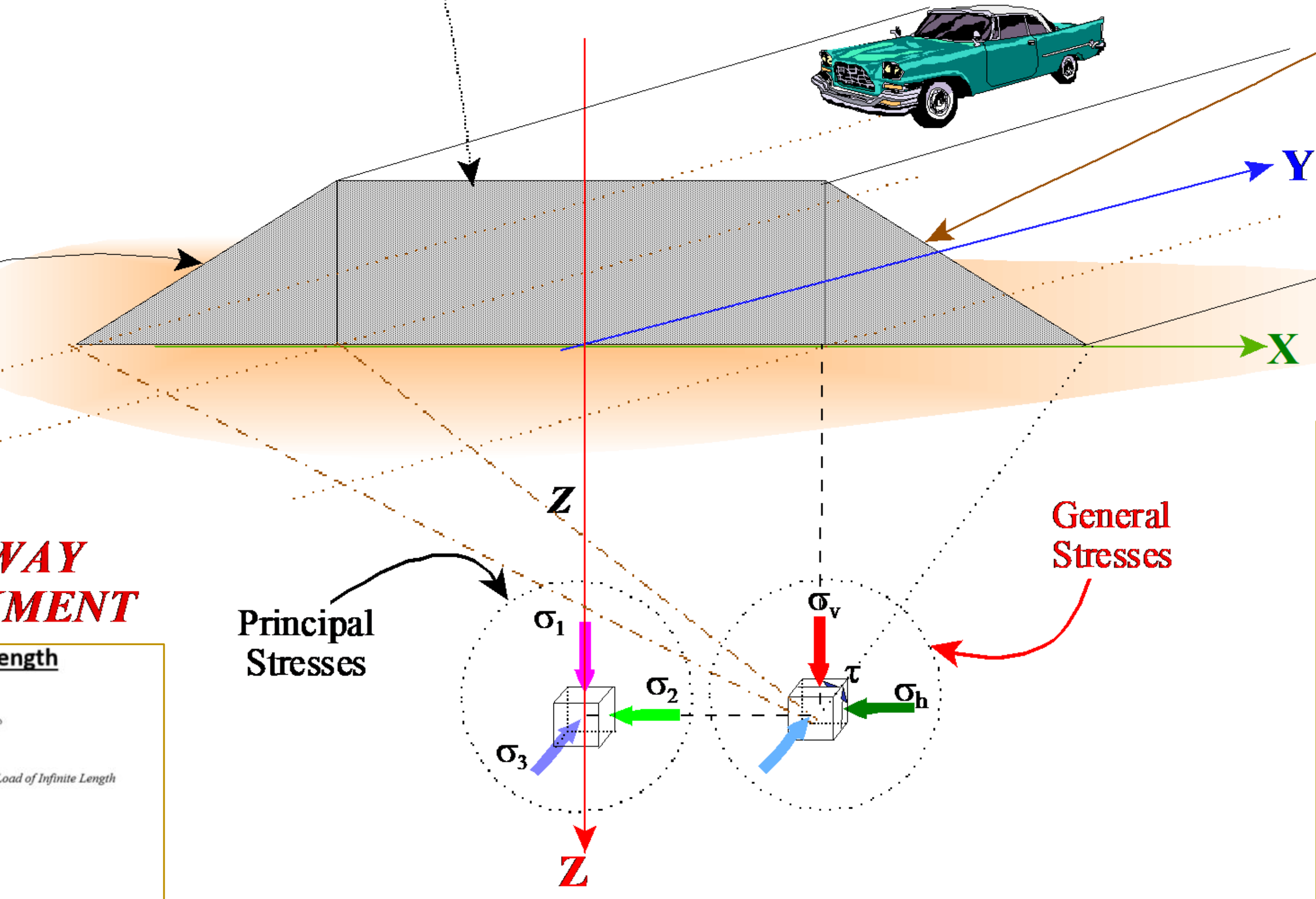
3. CONTINUOUS LOAD

$$\Delta P = q/\pi\ [\beta + \sin\beta \cos(\beta + 2\delta)]$$

4. TRIANGULAR LOAD

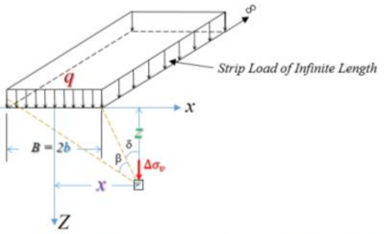
$$\Delta P = (q/\pi)\ \{(X/B)\,\alpha - (\sin 2\delta/2)\}$$





HIGHWAY EMBANKMENT

3- Strip Load of Infinite Length



$$\Delta\sigma_v = \left(\frac{q}{\pi}\right)\left[\tan^{-1}\left(\frac{z}{x-b}\right) - \tan^{-1}\left(\frac{z}{x+b}\right) - \frac{2bz\,(x^2-b^2-z^2)}{(x^2-b^2+z^2)^2 + 4\,b^2 z^2}\right]$$

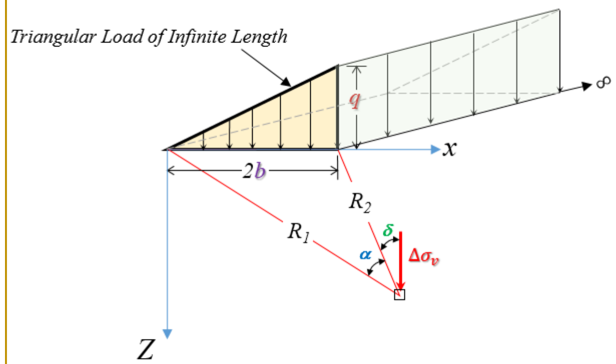
Or

$$\Delta\sigma_v = \left(\frac{q}{\pi}\right)[\beta + \sin\beta\,\cos(\beta + 2\delta)]$$ *where δ and β are angles*

Figure 3

See page 17

4- Triangular Load of Infinite Length



$$\Delta\sigma_v = \left(\frac{q}{2\pi}\right)\left(1 + \cos 2\delta - \frac{z}{b}\alpha\right)$$

Figure 4

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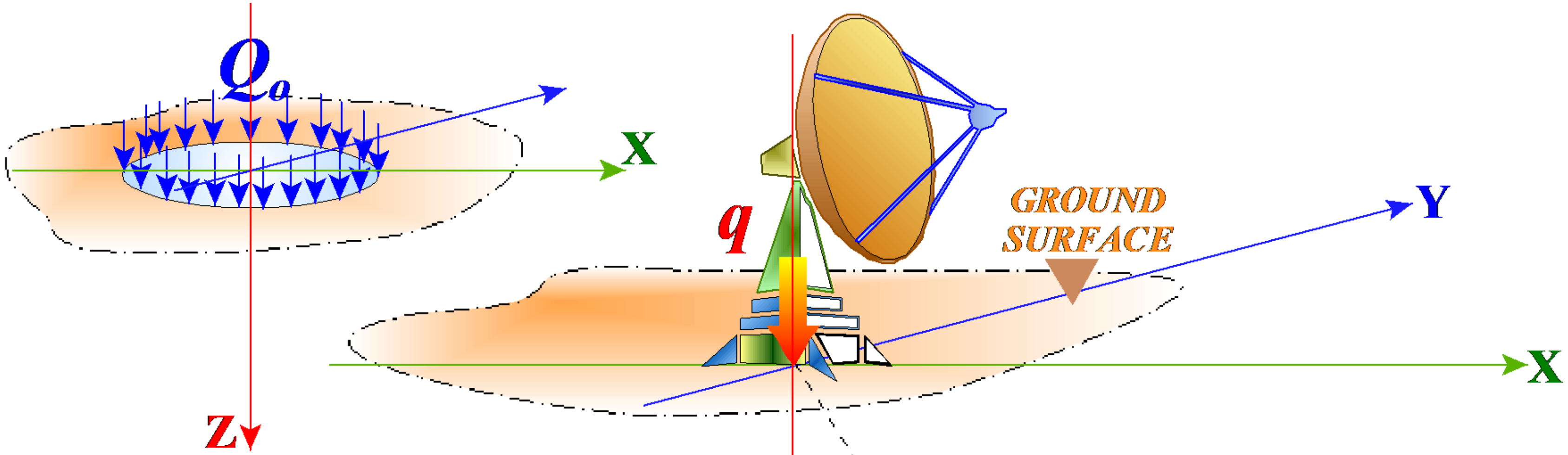
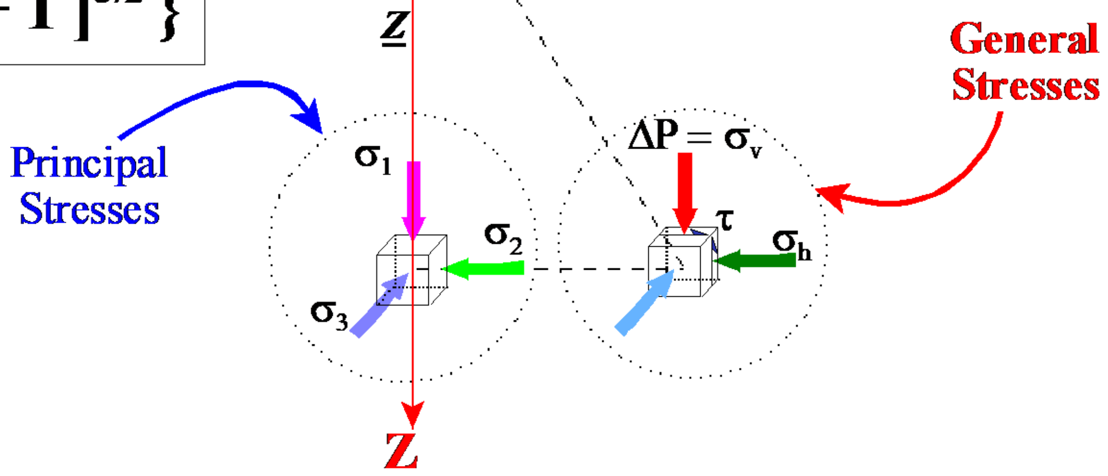
See page 17

5. VERTICAL STRESS DUE TO CIRCULAR LOAD



$$\Delta P = q \{ 1 - 1/[(R/Z)^2 + 1]^{3/2} \}$$



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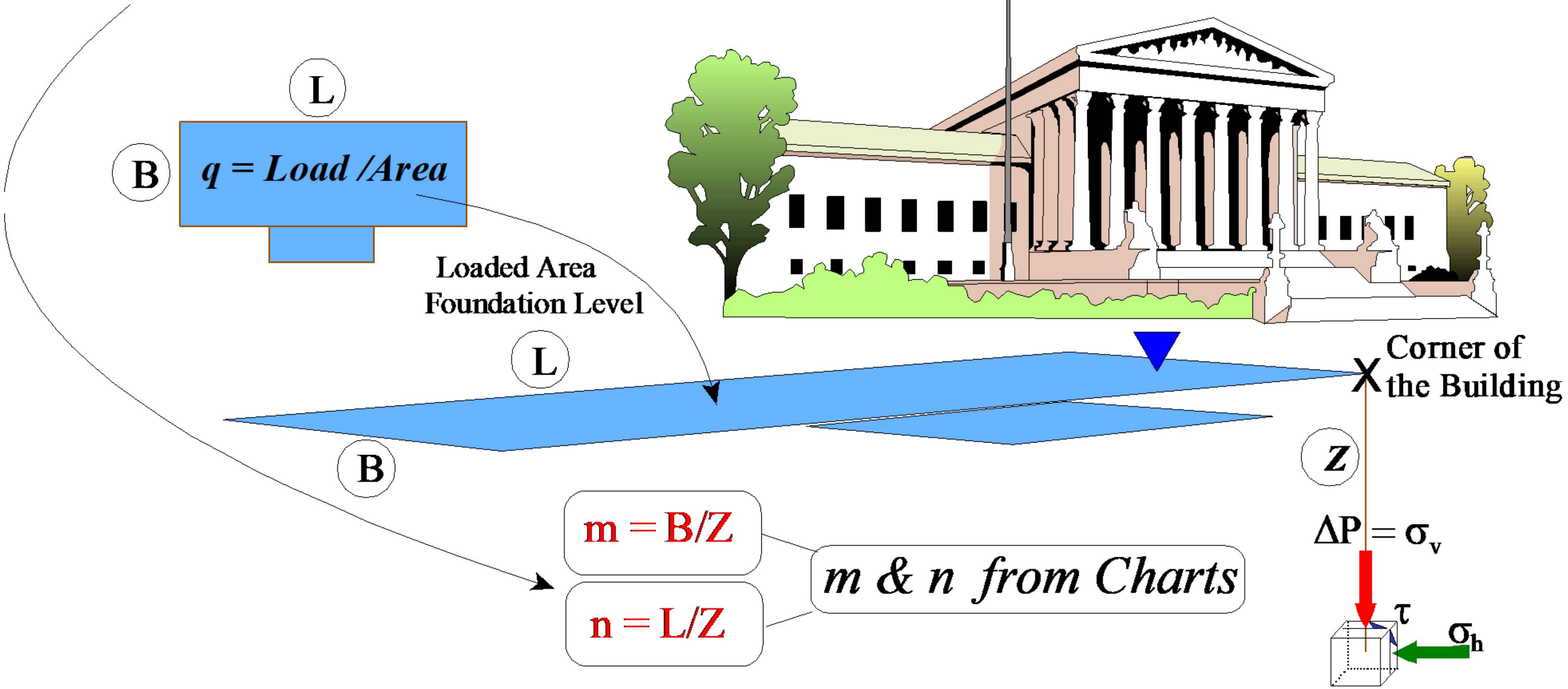
6. VERTICAL STRESS DUE TO RECTANGULAR LOAD

$$\Delta P = q\, I_R$$

$$I_R = 1/4\pi \{[(2.\, m.n\, (m^2 + n^2 + 1)^{1/2}) / (m^2 + n^2 + m^2.\, n^2 + 1)]\, [(m^2 + n^2 + 2)/(m^2 + n^2 + 1)] + \tan^{-1} (2.m\, .n\, (m^2 + n^2 + 1)^{1/2} / (m^2 + n^2 - m^2\, .\, n^2 + 1)\}$$

$$I_R = f\,(m,n)$$



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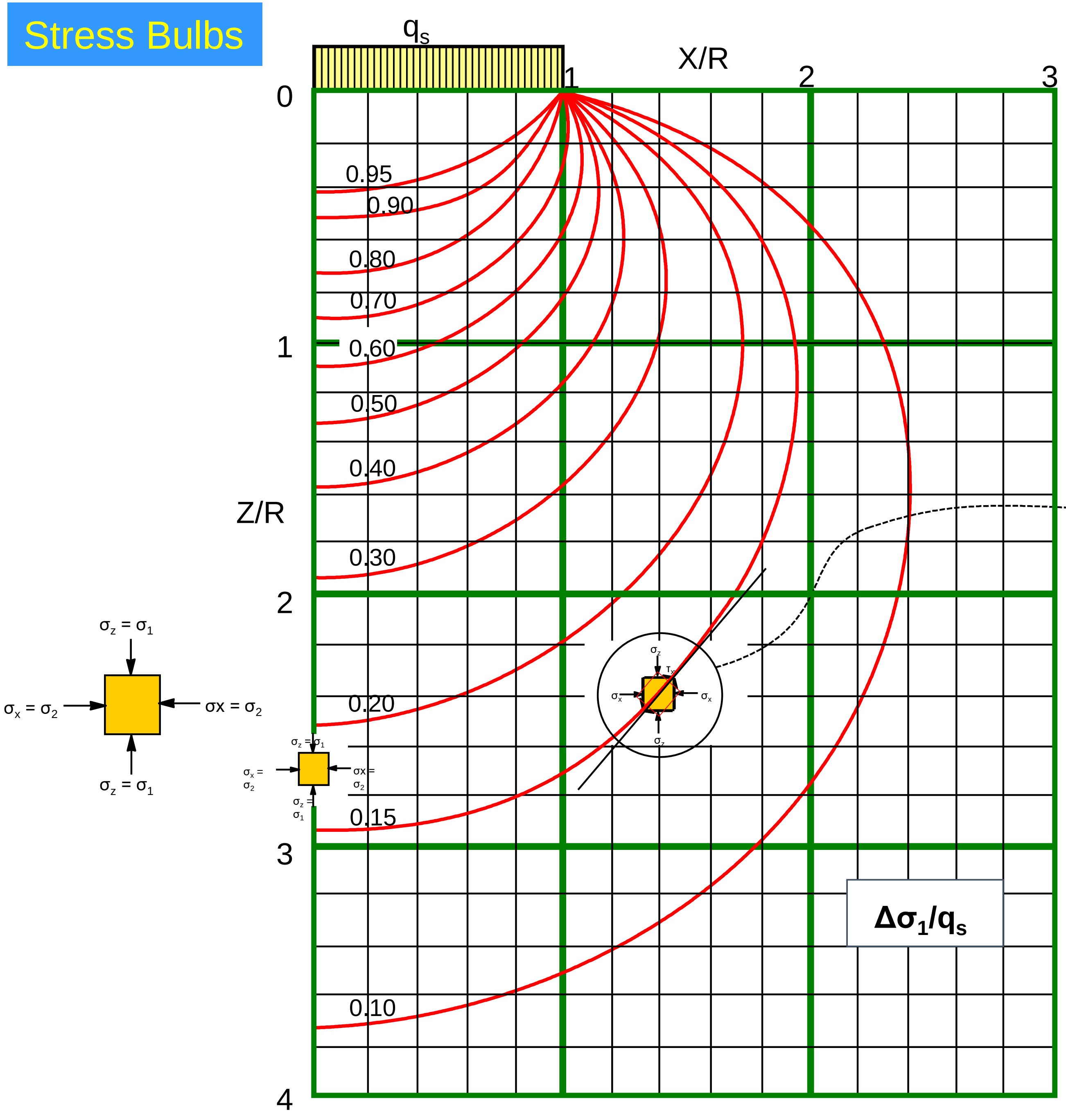
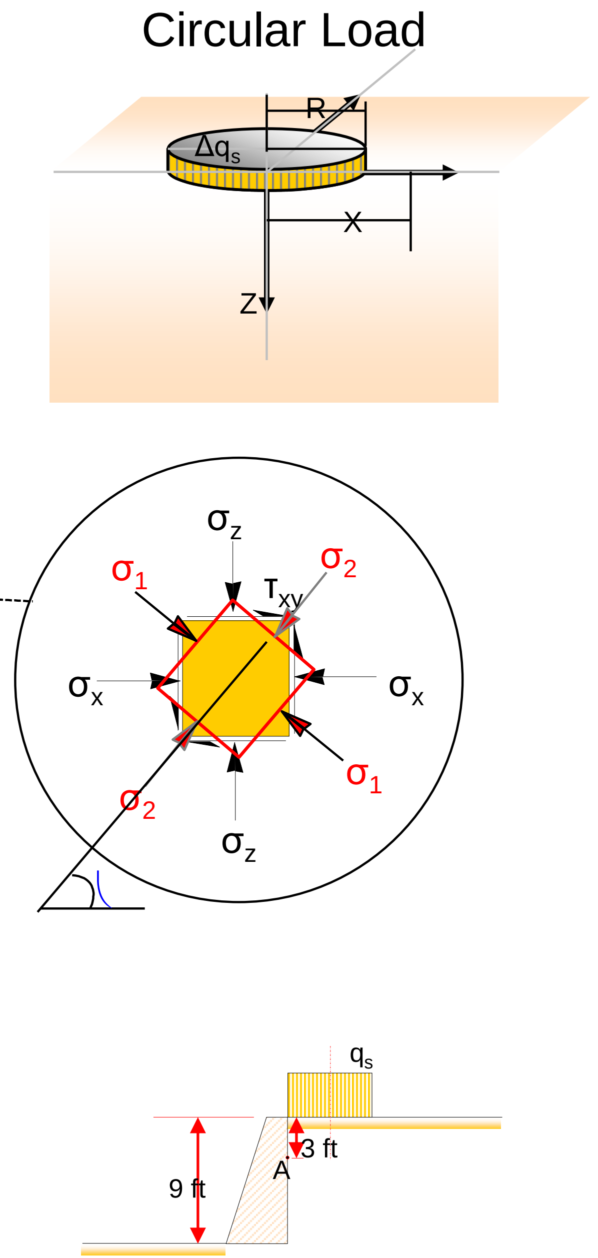
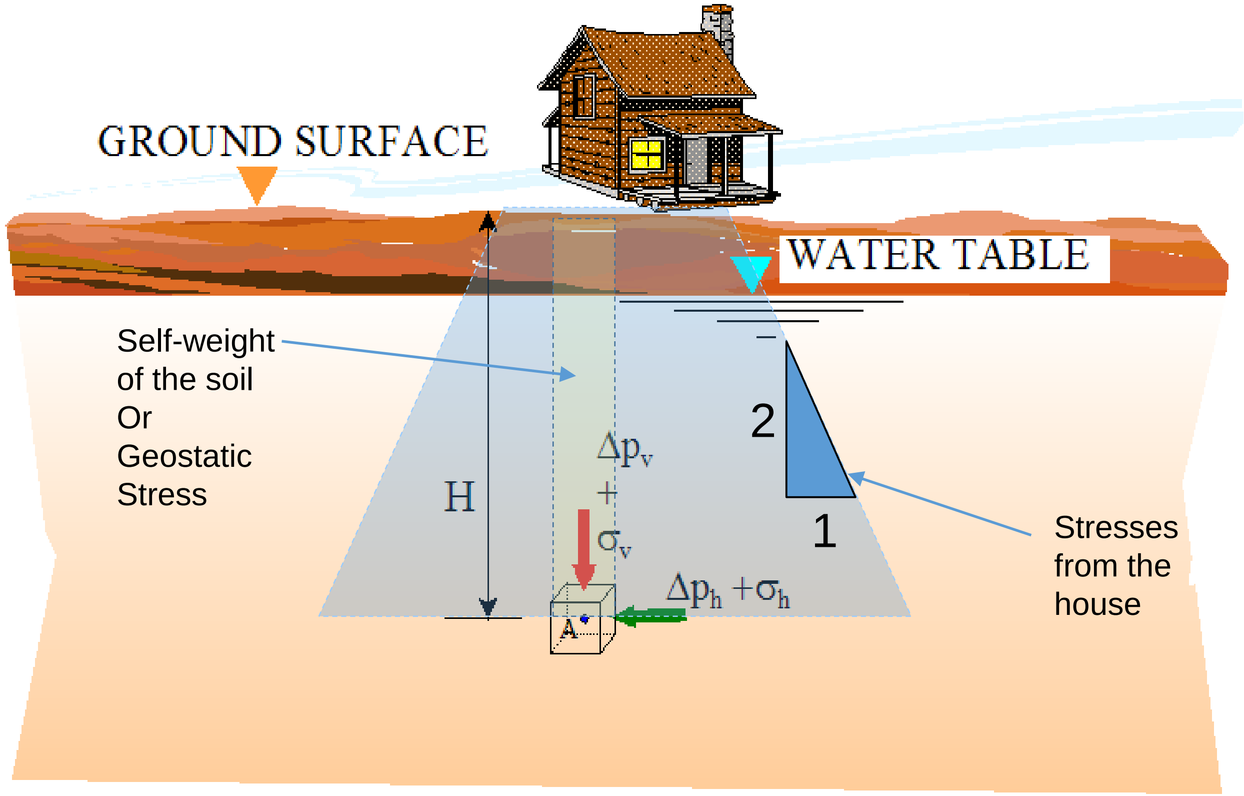
Stress Bulbs





Circular Load: (Major Principal Stress)/(Surface Stress)

Geostatic Stresses
&
Stress Distribution in Soils



2:1 Approximate Method

Study Guide for Homework # 2

Stress distribution in Soil Layers
Due to Geostatic Stresses and Added Loads

Refer to Figure 9.19. Calculate σ, u, and σ' at A, B, C, and D for the following cases and plot the variations with depth. (*Note:* e = void ratio, w = moisture content, G_s = specific gravity of soil solids, γ_d = dry unit weight, and γ_{sat} = saturated unit weight.)

Problem	Details of soil layer I	II	III
9.1	$H_1 = 5$ ft $\gamma_d = 112$ lb/ft^3	$H_2 = 6$ ft $\gamma_{sat} = 120$ lb/ft^3	$H_3 = 8$ ft $\gamma_{sat} = 125$ lb/ft^3
9.2	$H_1 = 5$ ft $\gamma_d = 100$ lb/ft^3	$H_2 = 10$ ft $\gamma_{sat} = 116$ lb/ft^3	$H_3 = 9$ ft $\gamma_{sat} = 122$ lb/ft^3
9.3	$H_1 = 3$ m $\gamma_d = 15$ kN/m^3	$H_2 = 4$ m $\gamma_{sat} = 16$ kN/m^3	$H_3 = 5$ m $\gamma_{sat} = 18$ kN/m^3
9.4	$H_1 = 4$ m $e = 0.4$ $G_s = 2.62$	$H_2 = 5$ m $e = 0.6$ $G_s = 2.68$	$H_3 = 3$ m $e = 0.81$ $G_s = 2.73$
9.5	$H_1 = 4$ m $e = 0.6$ $G_s = 2.65$	$H_2 = 3$ m $e = 0.52$ $G_s = 2.68$	$H_3 = 1.5$ m $w = 40\%$ $e = 1.1$

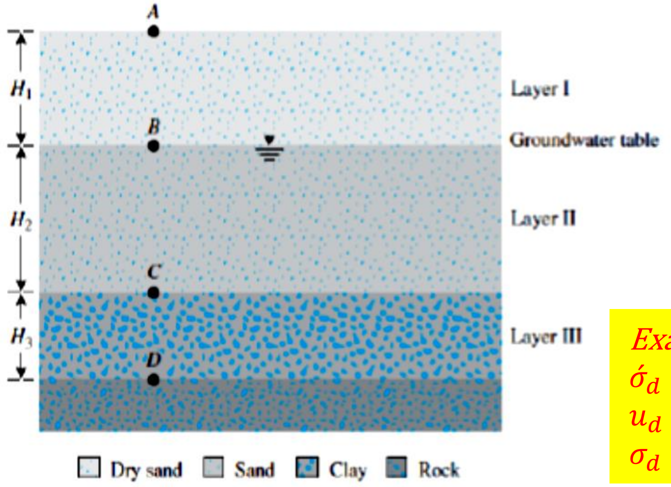


Figure 9.19

Example:

$$\acute{\sigma}_d = H_1 \cdot \gamma_{d1} + H_2 \cdot (\gamma_{sat2} - \gamma_w) + H_3 . (\gamma_{sat3} - \gamma_w)$$
$$u_d = (H_2 + H_3) . \gamma_w$$
$$\sigma_d = \acute{\sigma}_c + u_c$$

An earth embankment diagram is shown in Figure 10.39. Determine the stress increase at point A due to the embankment load.

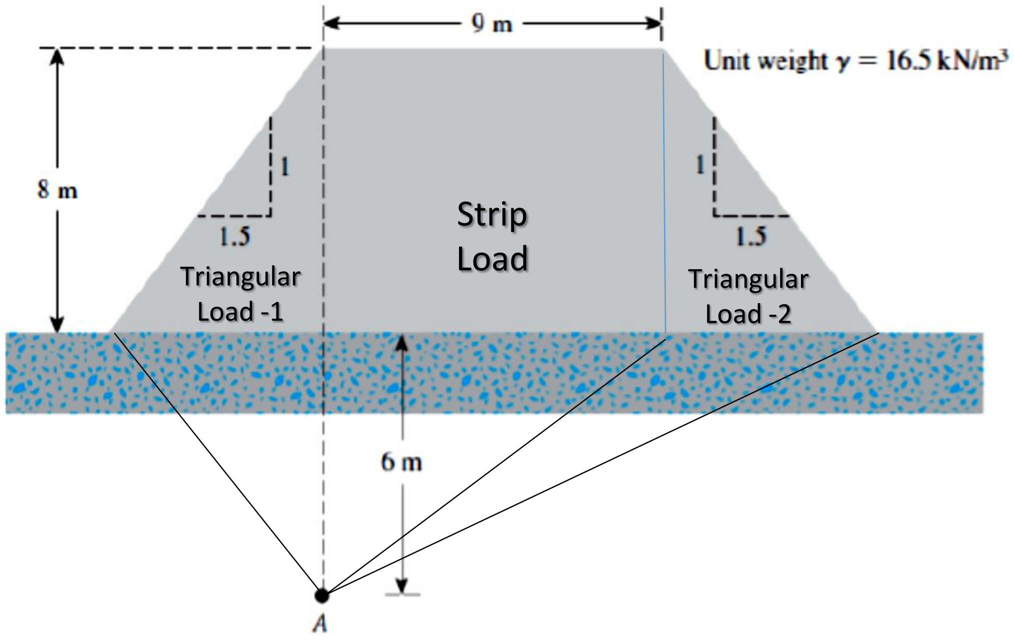


Figure 10.39

Stress Distribution in Soil Layers Due to Geoststic Stress & Added Loads

Let us consider a project where a high rise building will be constructed at a site

1. Before we start the construction of the building (Figure 1), the geostatic stress at point A is

$$\sigma_v = \overset{Z}{30} \; x \; \overset{\gamma_{soil}}{115} = 3{,}450 \; \frac{lb}{ft^2}$$

Note:

- *We used the weight of the soil particles only (i.e. the effective vertical stress)*
- *See Figure 6 (page 7) for other vertical geoststic stresses*

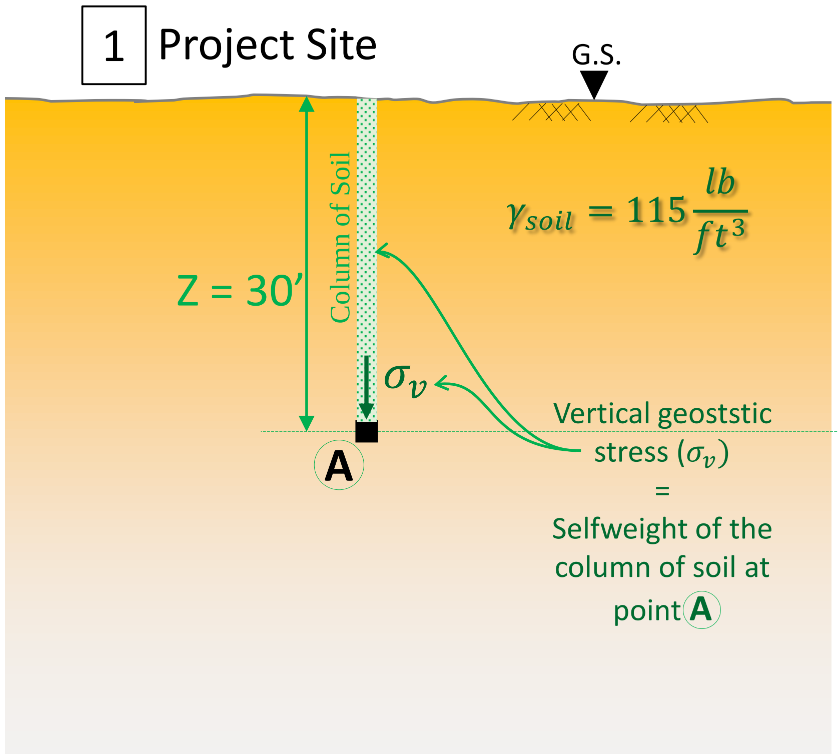


Figure 1

Before Construction and before adding the load of the building

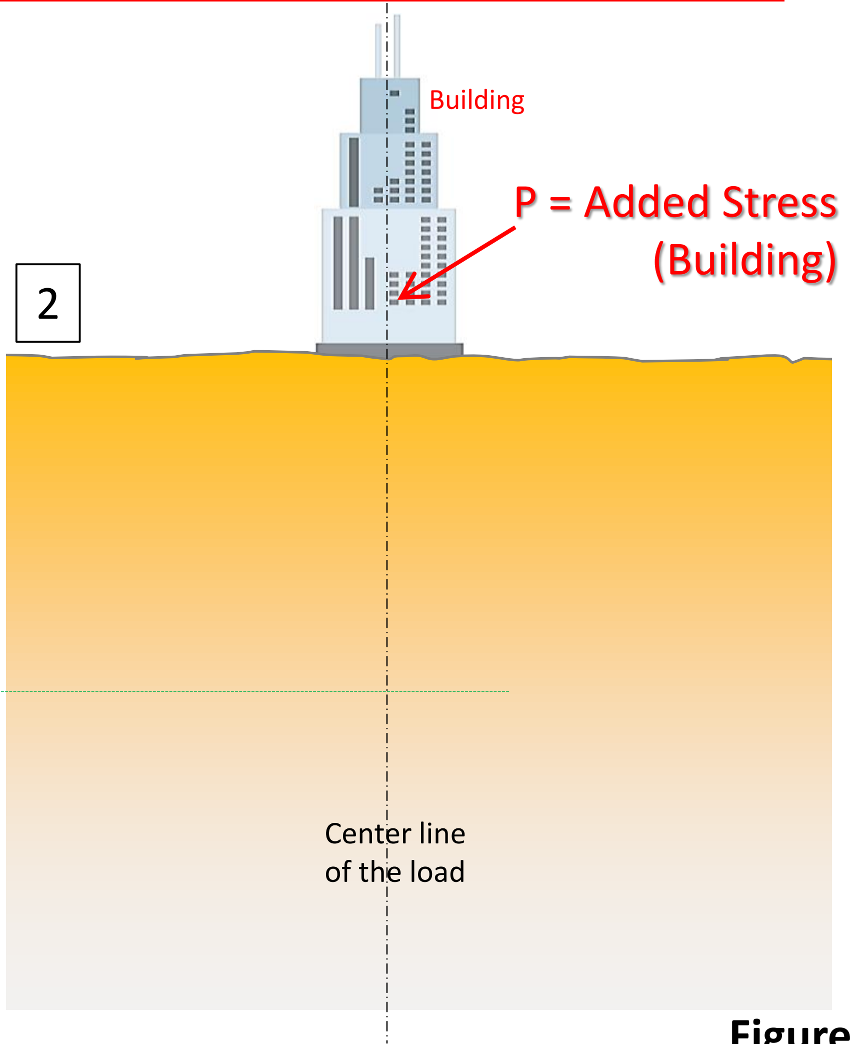


Figure 2

After the addition 0f the building load

2. After construction, the vertical stress at point **A** due to the addition of the building load can be determined using **Boussinesq's Equations.**

There are equations for each type of load. These loads are:

1. Point Load *(See Figure 3)*

2. Line Load

3. Strip Load *(See Figure 4)*

4. Triangular Load *(See Figure 5)*

5. Circular Load

6. Rectangular Load

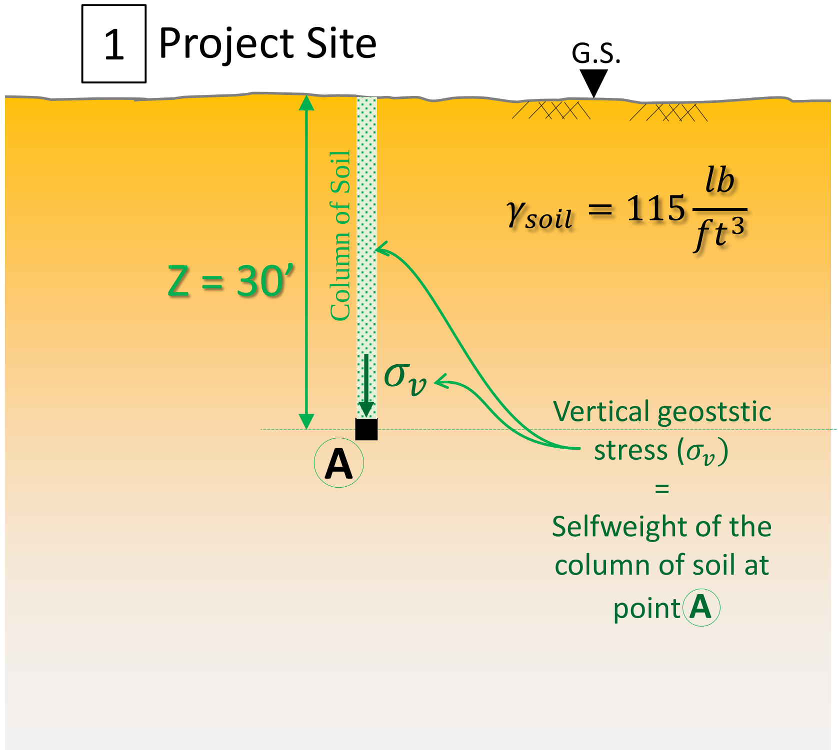


Figure 1

Before Construction and before adding the load of the building

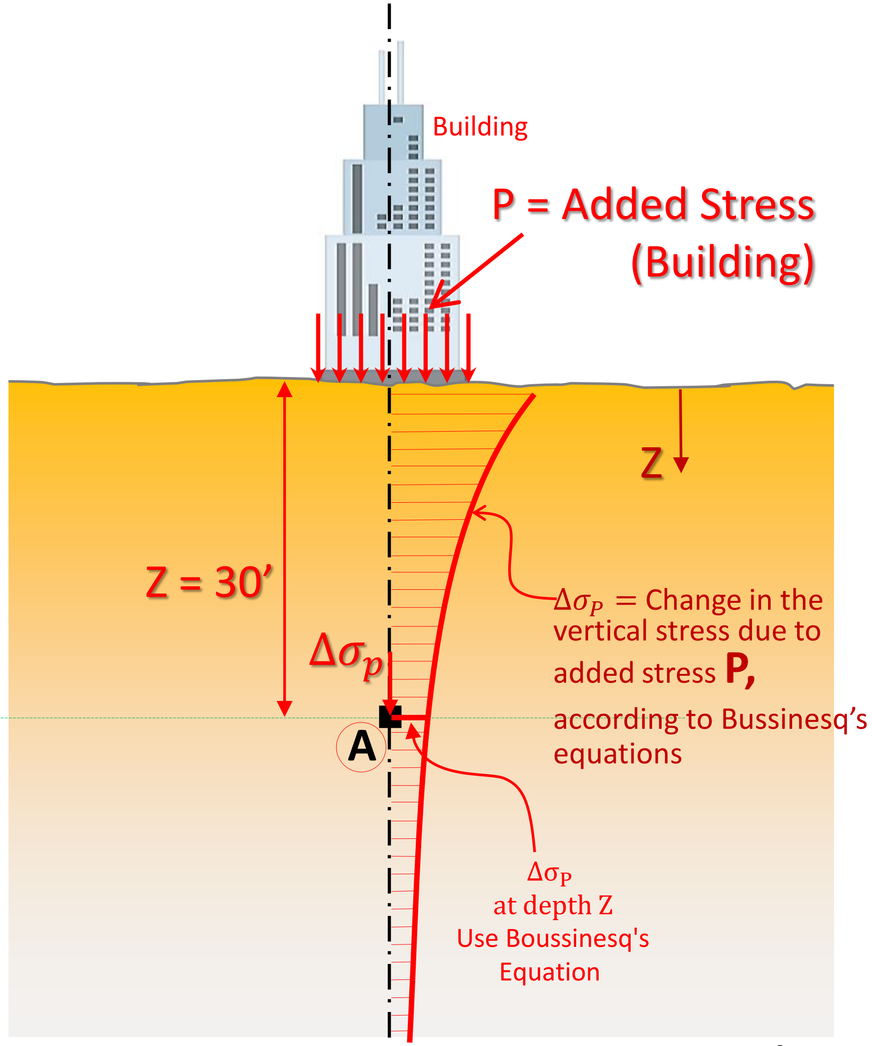


Figure 2

After the addition 0f the building load

Boussinesq's Equations.

1- Point Load

$$\textbf{Vertical Stress} = \Delta\sigma_v = \frac{3P}{2\pi^2 z^2 \left[1 + \left(\frac{x}{z}\right)^2\right]^{5/2}} = \frac{P}{z^2}\, I_p$$

At Point A along the center line

$$\textbf{Vertical Stress} = \Delta\sigma_v = \frac{3P}{2\pi^2 z^2} = \frac{P}{z^2}\, I_p$$

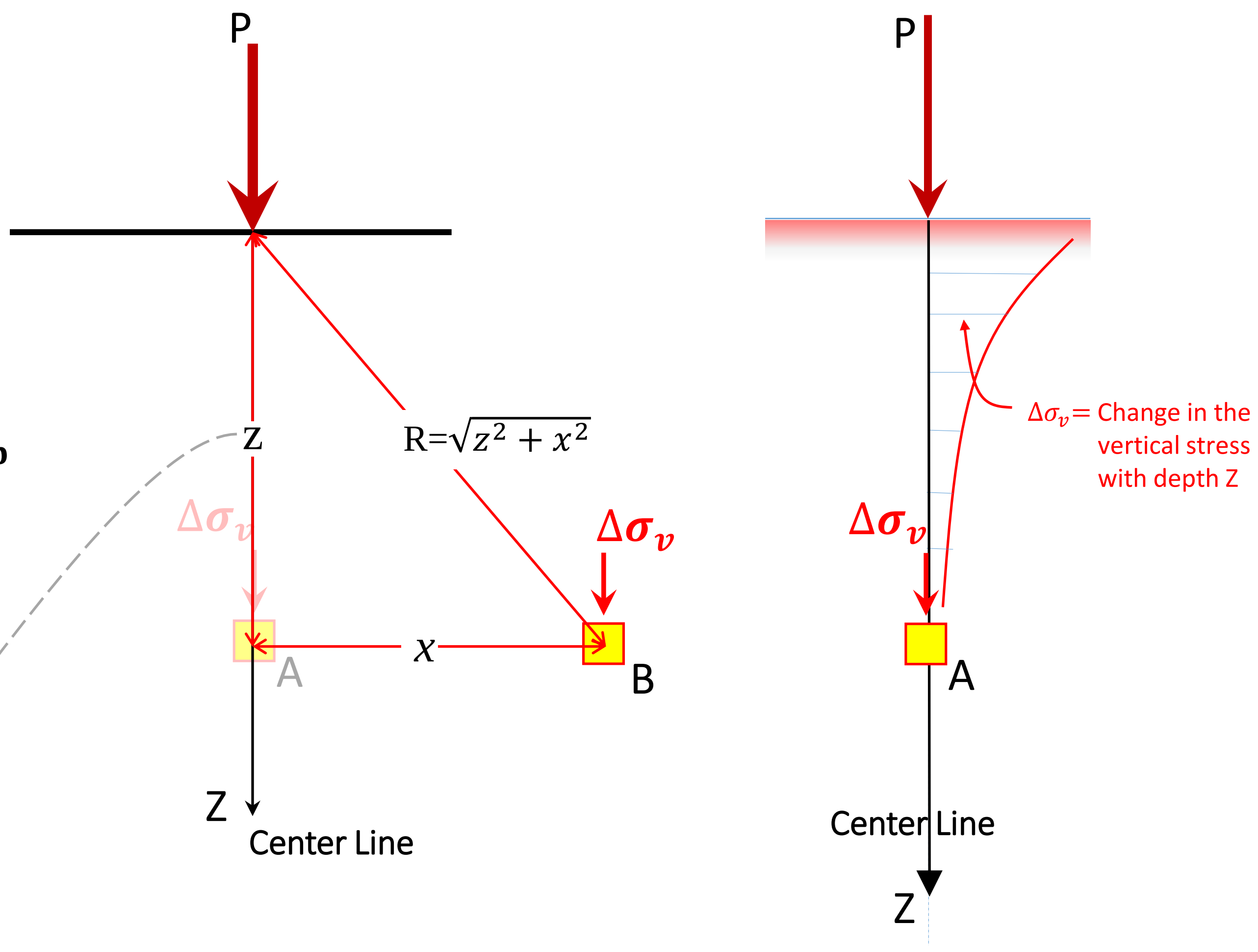
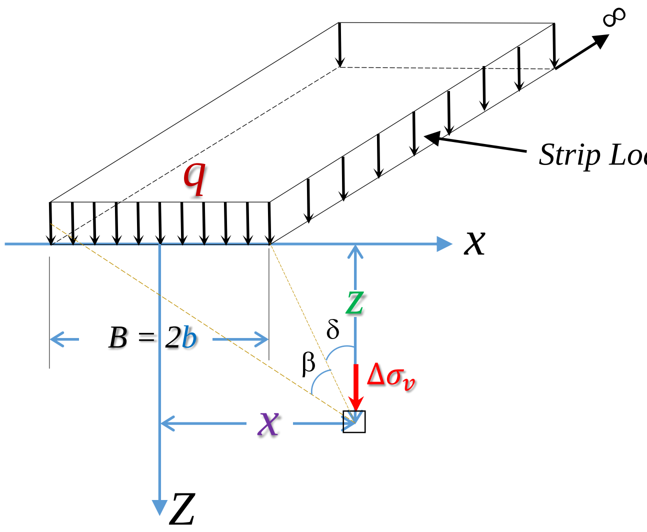


Figure 3

Boussinesq's Equations.

3- Strip Load of Infinite Length



$$\Delta\sigma_v = \left(\frac{q}{\pi}\right)\left[\tan^{-1}\left(\frac{z}{x-b}\right) - \tan^{-1}\left(\frac{z}{x+b}\right) - \frac{2bz\,(x^2-b^2-z^2)}{(x^2-b^2+z^2)^2 + 4\,b^2\,z^2}\right]$$

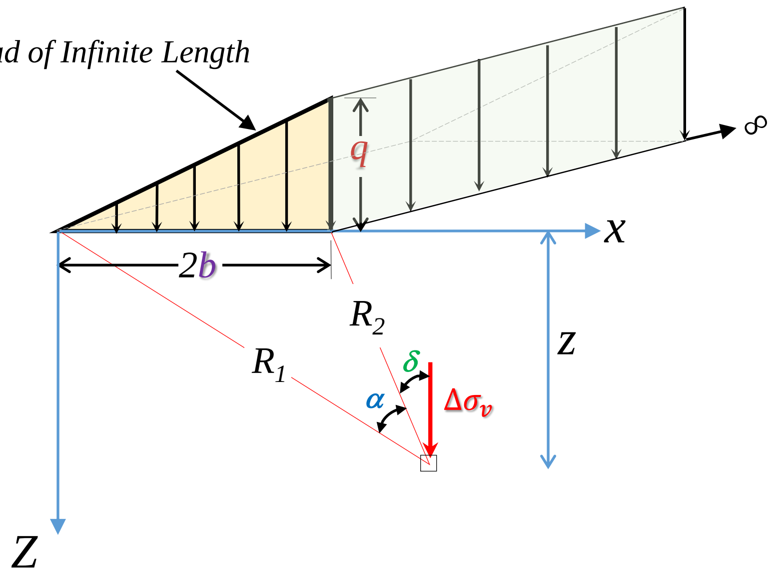
Or

$$\Delta\sigma_v = \left(\frac{q}{\pi}\right)\left[\beta + \sin\beta\,\cos(\beta + 2\delta)\right]$$

where δ and β are angles

Figure 3

4- Triangular Load of Infinite Length



$$\Delta\sigma_v = \left(\frac{q}{2\pi}\right)\left(1 + \cos 2\delta - \frac{z}{b}\alpha\right)$$

Figure 4

Geoststic Stresses

There are three stresses in acting at point **A.** These are

- Effective Stress = $\bar{\sigma}_v$
- Total Stress = σ_v
- Pore Water Pressure = u

Where

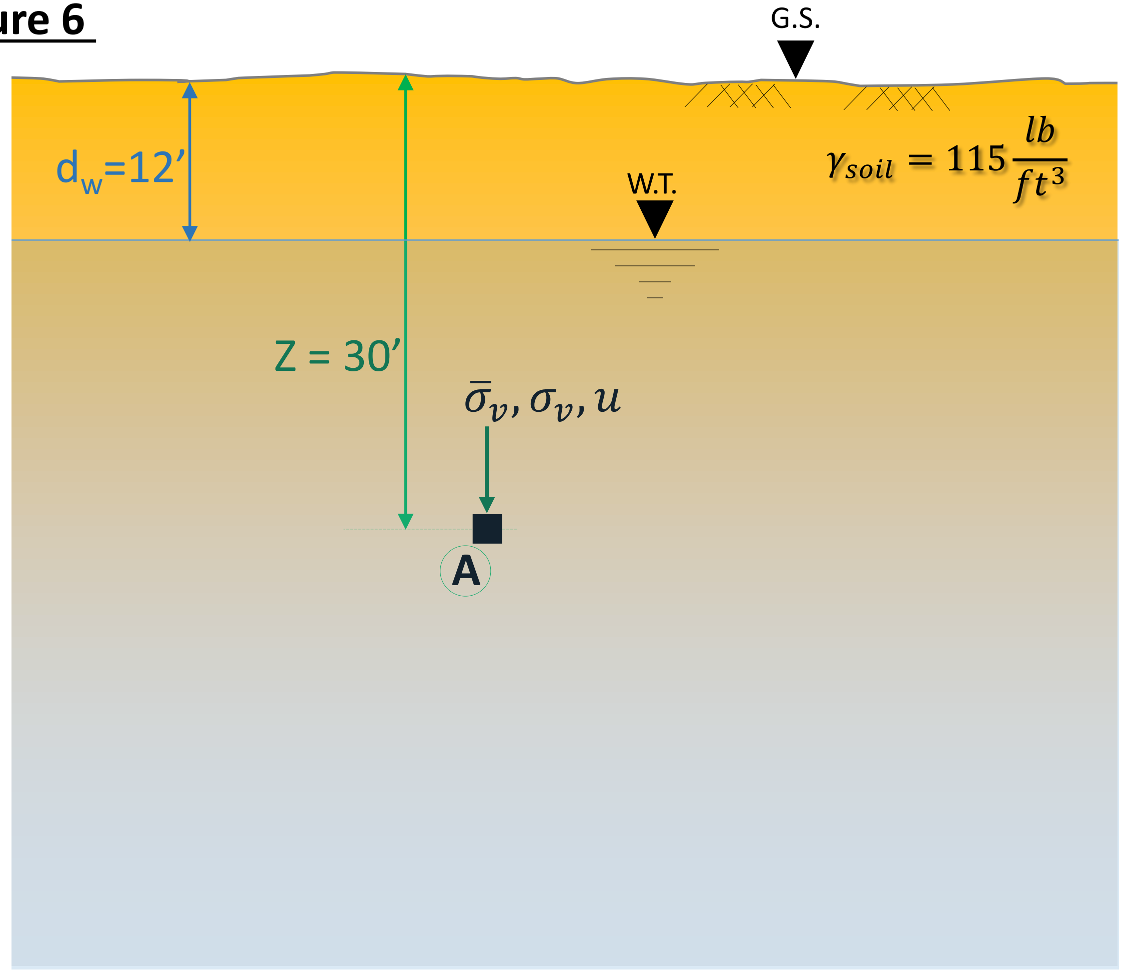
Total Stress = Effective Stress + PWP

$$\sigma_v = \bar{\sigma}_v + u$$

$$OR$$

$$\bar{\sigma}_v = \sigma_v - u$$

Figure 6



Question:

What is the total stress (σ_v)
effective stress ($\bar{\sigma}_v$)
pore water pressure (u)
at point **A**

Solution

effective stress ($\bar{\sigma}_{vA}$)$= 12' x\ 115 + (30-12)(115-62.4) = 2{,}326.8\ psf$

pore water pressure (u_A) = (30-12)(62.4) = 1,123.2 *psf*

total stress (σ_{vA}) = 30 x 115 = 3,450 *psf*