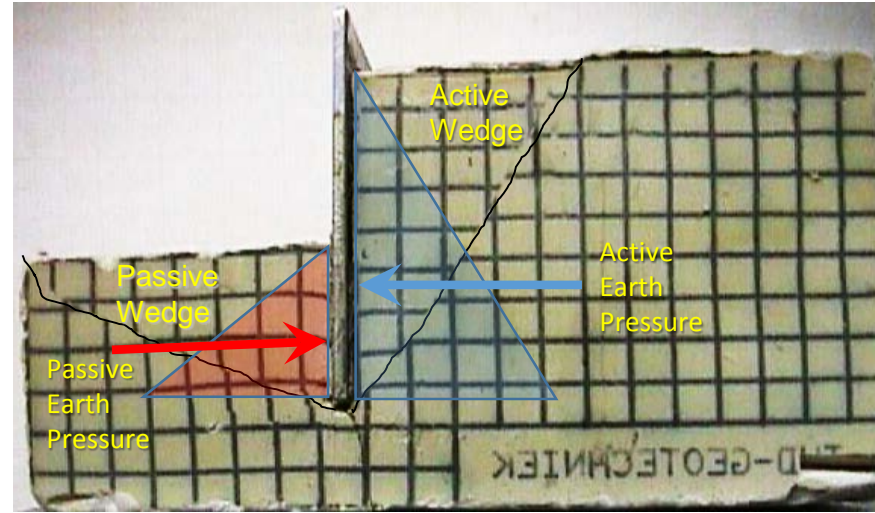
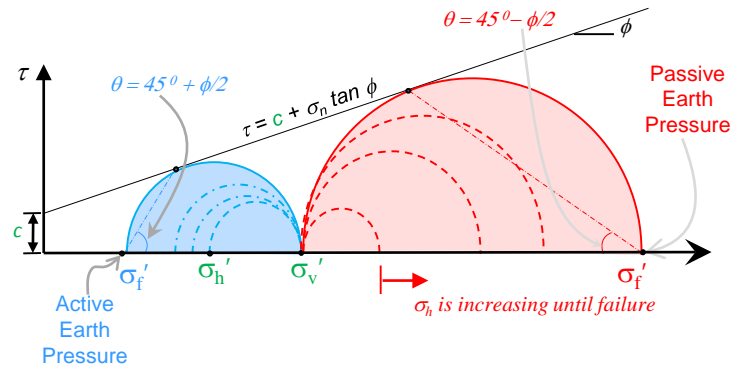
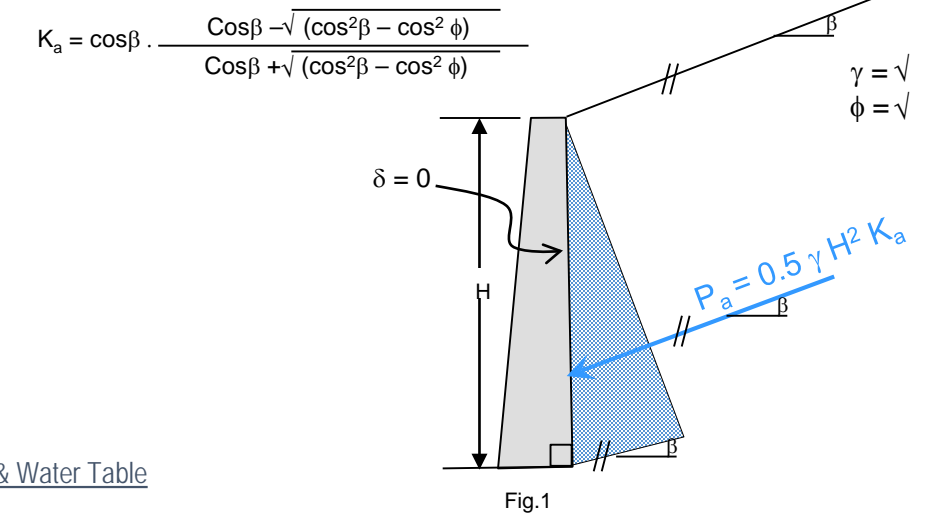


# Rankine's Earth Pressure Method for (c- φ) Soil

Rankine's Active and Passive Earth Pressure in (c- f) Soil



Rankine's Active Earth Pressure in (f) Soil with inclined backfill



## Active Earth Pressure

$$\sigma'_f = \sigma'_v \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) + 2c \tan \left( 45^\circ - \frac{\phi}{2} \right)$$

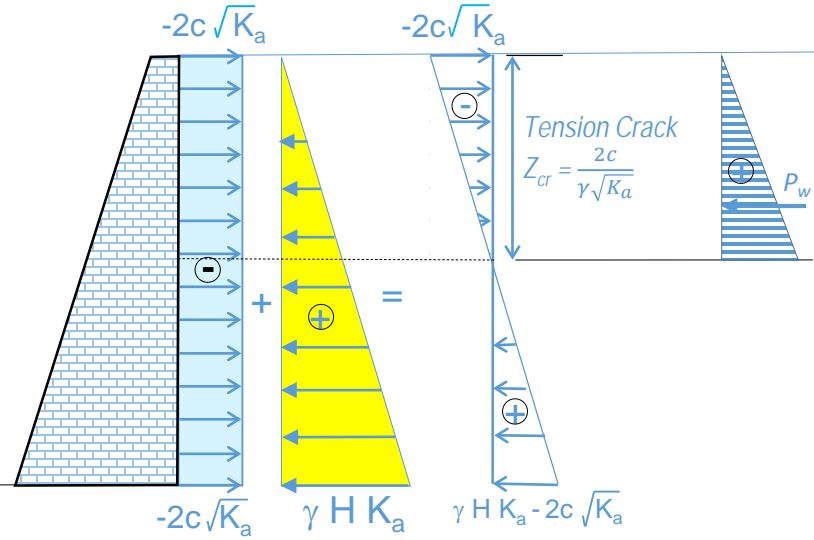
Or

$$\sigma'_f = \sigma'_v K_a - 2c \sqrt{K_a}$$

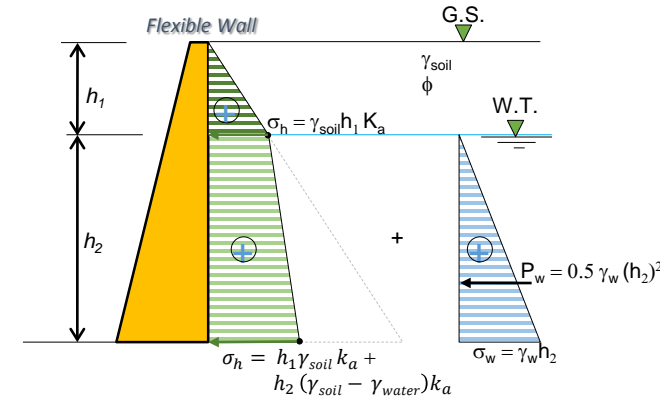
$$K_a = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) = \frac{1 - \sin \phi}{1 + \sin \phi}$$

Coefficient of active earth pressure

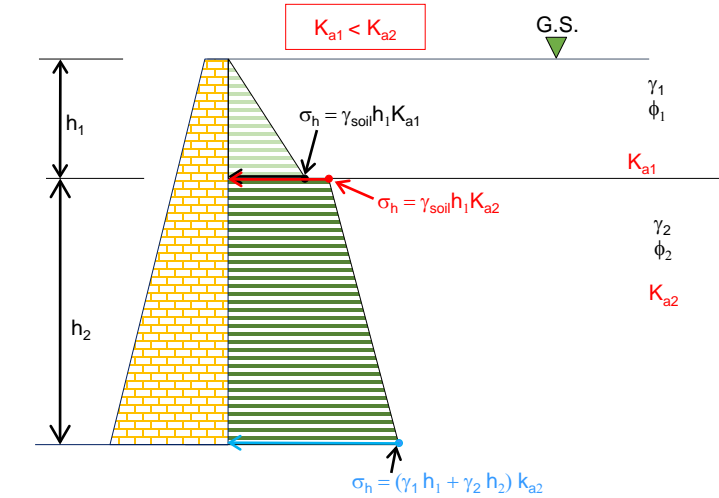
Effect of Cohesion of the Rankine's Active and Passive Earth Pressure



Rankine's Active Earth Pressure in f - Soil & Water Table



Effect of Two Soil Layers on Active Earth Pressure



## Passive Earth Pressure

$$\sigma'_p = \sigma'_v \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) + 2c \tan \left( 45^\circ + \frac{\phi}{2} \right)$$

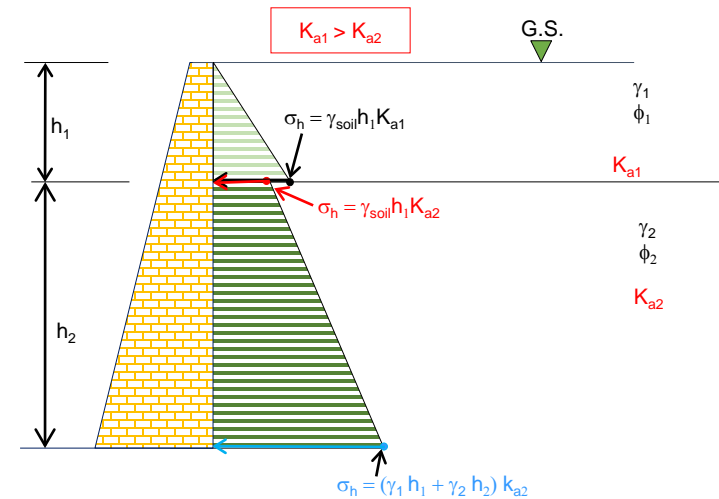
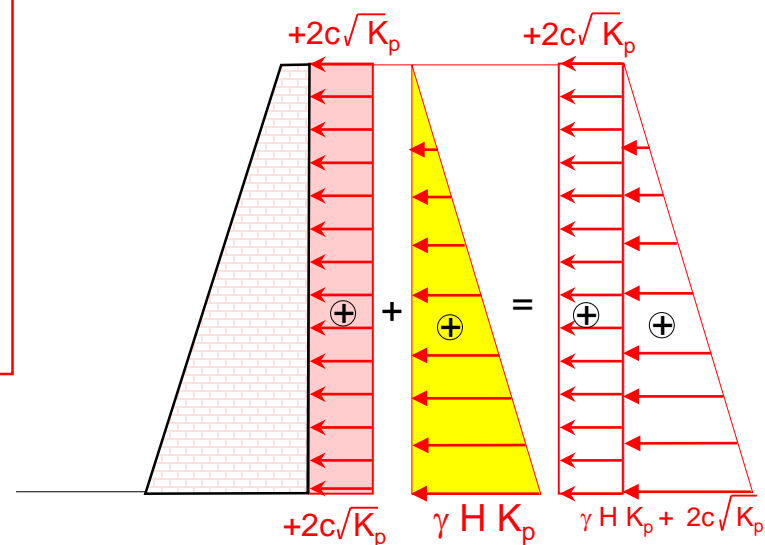
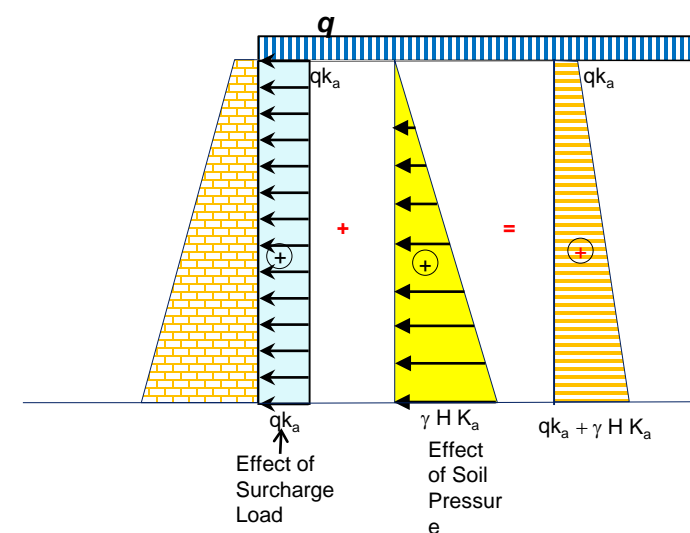
Or

$$\sigma'_p = \sigma'_v K_p + 2c \sqrt{K_p}$$

$$K_p = \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) = \frac{1 + \sin \phi}{1 - \sin \phi}$$

Coefficient of passive earth pressure

Effect of Surcharge (q) Load on Active Earth Pressure



# Coulomb Earth Pressure Method

Forces acting on the wall.

Wall Displacement =  $\Delta$



This active wedge will slide along the failure surface and will exert pressure on the wall. Also, the sliding wedge will push the wall away from the backfill.

**W**

Failure surface

Pressure from the sliding wedge

# Active Earth Pressure in $\phi$ – Soil (Using Rankine's Method)

Rankine's Method always assumes  $\delta = 0$

## Example - 1

### Given:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ( $\gamma$ ) = 115 pcf
- Angle of soil friction ( $\phi$ ) =  $30^\circ$
- Assume wall to be smooth ( $\delta=0$ )

### Find:

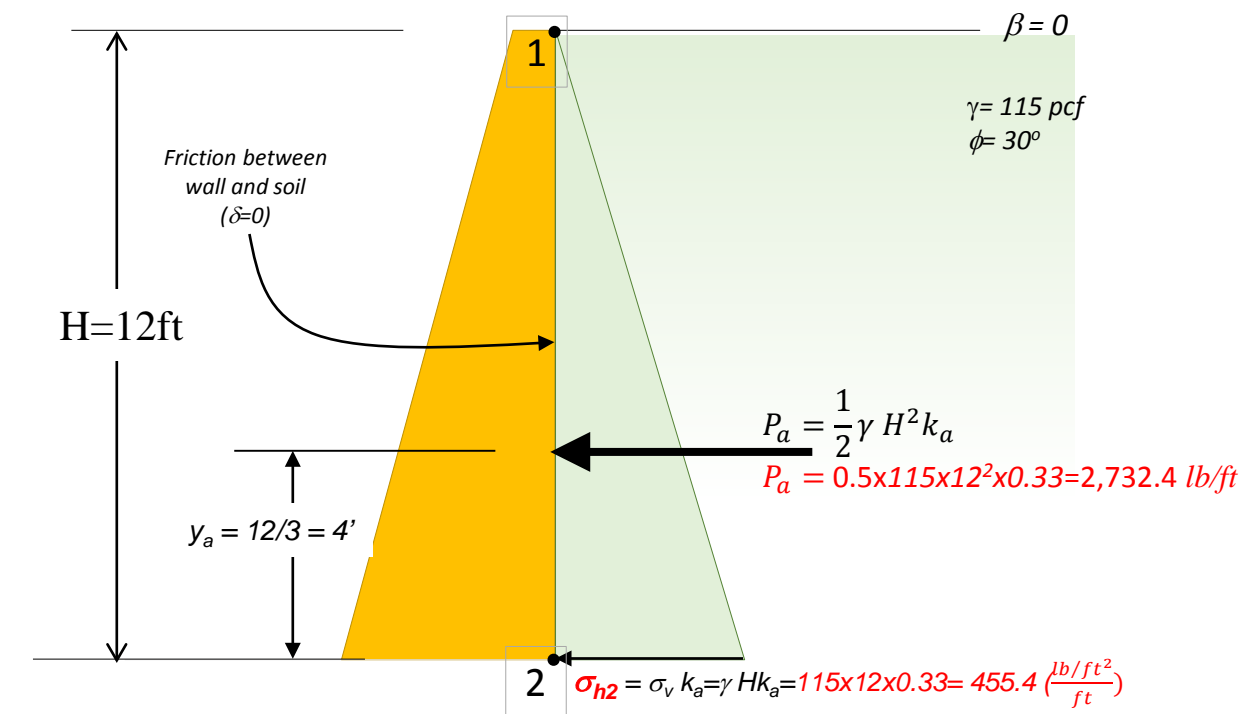
- Lateral force  $P_a$  acting on the wall

### Solution:

$$k_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30}{1 + \sin 30} = 0.33$$

Point	Vertical Stress $\sigma_v$ $\gamma H \left( \frac{\text{lb}/\text{ft}^2}{\text{ft}} \right)$	Horizontal Stress $\sigma_h$ $\gamma H k_a \left( \frac{\text{lb}/\text{ft}^2}{\text{ft}} \right)$	$P_a = \frac{1}{2} \gamma H^2 k_a$ (lb/ft)	$y_a$ (ft)
1	0	0	0.5x455.4x12=2,732.4	12/3=4
2	115x12=1,380	115x12x0.33= 455.4		

$$k_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$



# Active Earth Pressure in $\phi$ – Soil (Using Rankine's Method)

Rankine's Method always assumes  $\delta = 0$

## Example -2

### Given:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ( $\gamma$ ) = 115 pcf
- Angle of soil friction ( $\phi$ ) =  $30^\circ$
- Assume wall to be smooth ( $\delta=0$ )

### Find:

- Lateral force  $P_a$  acting on the wall

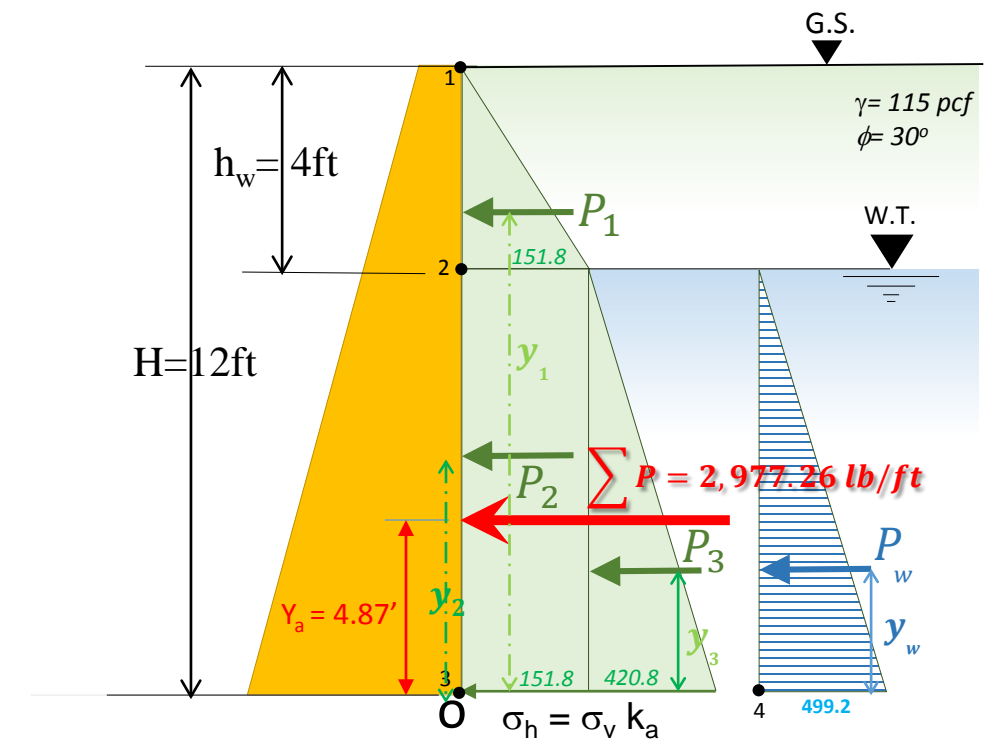
### Solution:

Because of the water table, the earth stress will be divided into soil and water pressure

$$k_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30}{1 + \sin 30} = 0.33$$

Point	Vertical Stress $\sigma_v$ $\gamma H \left(\frac{\text{lb}/\text{ft}^2}{\text{ft}}\right)$	Horizontal Stress $\sigma_h$ $\gamma H k_a \left(\frac{\text{lb}/\text{ft}^2}{\text{ft}}\right)$
1	0	0
2	$115 \times 4 = 460$	$460 \times 0.33 = 151.8$
3	$(115)(4) + (115 - 62.4)(8) = 460 + 420.8 = 880.8$	$[(115)(4) + (115 - 62.4)(8)] \times 0.33 = 290.66$
4	$62.4 \times 8 = 499.2$	$62.4 \times 8 = 499.2$ (Because $k_a$ for water = 1)

$$k_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$



To determine the resultant Force and its point of action

Lateral Force (lb/ft)	$y_n$ (ft)
$P_1 = 0.5 \times 151.8 \times 4 = 303.6$	$y_1 = (4/3) + 8 = 9.33$ (distance from $P_1$ to point O)
$P_2 = 151.8 \times 8 = 1214.4$	$y_2 = 8/2 = 4$ (distance from $P_2$ to point O)
$P_3 = 0.5 \times 138.86 \times 8 = 555.46$	$y_3 = 8/3 = 2.67$ (distance from $P_3$ to point O)
$P_w = 0.5 \times 62.4 \times 8^2 = 1996.8$	$y_w = 8/3 = 2.67$ (distance from $P_w$ to point O)
$\sum P = 2,977.26$	Take Moment about Point O = $\sum M_o = 303.6 \times 9.33 + 1214.4 \times 4 + 555.46 \times 2.67 + 1996.8 \times 2.67 = 14,504.72$ $y_a = \frac{\sum M_o}{\sum P} = \frac{14,504.72}{2,977.26} = 4.87 \text{ ft}$

# Active Earth Pressure in $\phi$ – Soil (Using Rankine's Method)

Rankine's Method always assumes  $\delta = 0$

## Example -3

### Given:

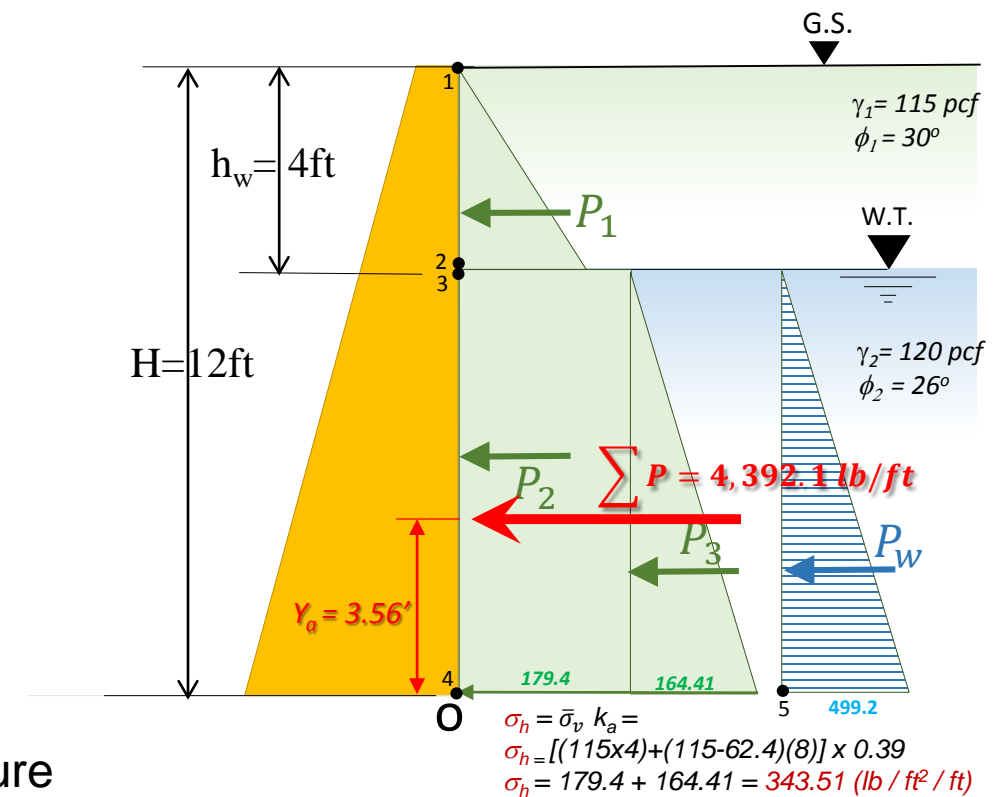
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ( $\gamma$ ) = 115 pcf
- Angle of soil friction ( $\phi$ ) =  $30^\circ$
- Assume wall to be smooth ( $\delta=0$ )

### Find:

- Lateral force  $P_a$  acting on the wall

### Solution:

Because of the water table, the earth stress will be divided into soil and water pressure



$$k_{a1} = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30}{1 + \sin 30} = 0.33$$

$$k_{a2} = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 26}{1 + \sin 26} = 0.39$$

Point	Vertical Stress $\sigma_v$ $\gamma H \left( \frac{\text{lb/ft}^2}{\text{ft}} \right)$	Horizontal Stress $\sigma_h$ $\gamma H k_a \left( \frac{\text{lb/ft}^2}{\text{ft}} \right)$
1	0	0
2	$115 \times 4 = 460$	$460 \times 0.33 = 151.8$
3	$115 \times 4 = 460$	$460 \times 0.39 = 179.4$
4	$(115)(4) + (115 - 62.4)(8) =$	$[(115 \times 4) + (115 - 62.4)(8)] \times 0.39 = 343.51$
5	$62.4 \times 8 = 499.2$	$62.4 \times 8 = 499.2$ (Because $k_a$ for water = 1)

Lateral Force (lb/ft)	$Y_n$ (ft)
$P_1 = 0.5 \times 151.8 \times 4 = 303.6$	$Y_1 = (4/3) + 8 = 9.33$
$P_2 = 179.4 \times 8 = 1435.2$	$8/2 = 4$
$P_3 = 0.5 \times 164.11 \times 8 = 656.45$	$8/3 = 2.67$
$P_w = 0.5 \times 499.2 \times 8 = 1996.8$	$8/3 = 2.67$
$\sum P = 4,392.1$	Take Moment about Point O = $\sum M_o = 303.6 \times 9.33 + 1435.2 \times 4 + 656.45 \times 2.67 + 1996.8 \times 2.67 = 15,657.57$ $Y_a = \frac{\sum M_o}{\sum P} = \frac{15,657.57}{4,392.1} = 3.56 \text{ ft}$

# Active Earth Pressure in $\phi$ – Soil (Using Rankine's Method)

Rankine's Method always assumes  $\delta = 0$

## Example - 4

### Given:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ( $\gamma$ ) = 115 pcf
- Angle of soil friction ( $\phi$ ) =  $30^\circ$
- Ground surface slope  $\alpha = 10^\circ$
- Assume wall to be smooth ( $\delta=0$ )

### Find:

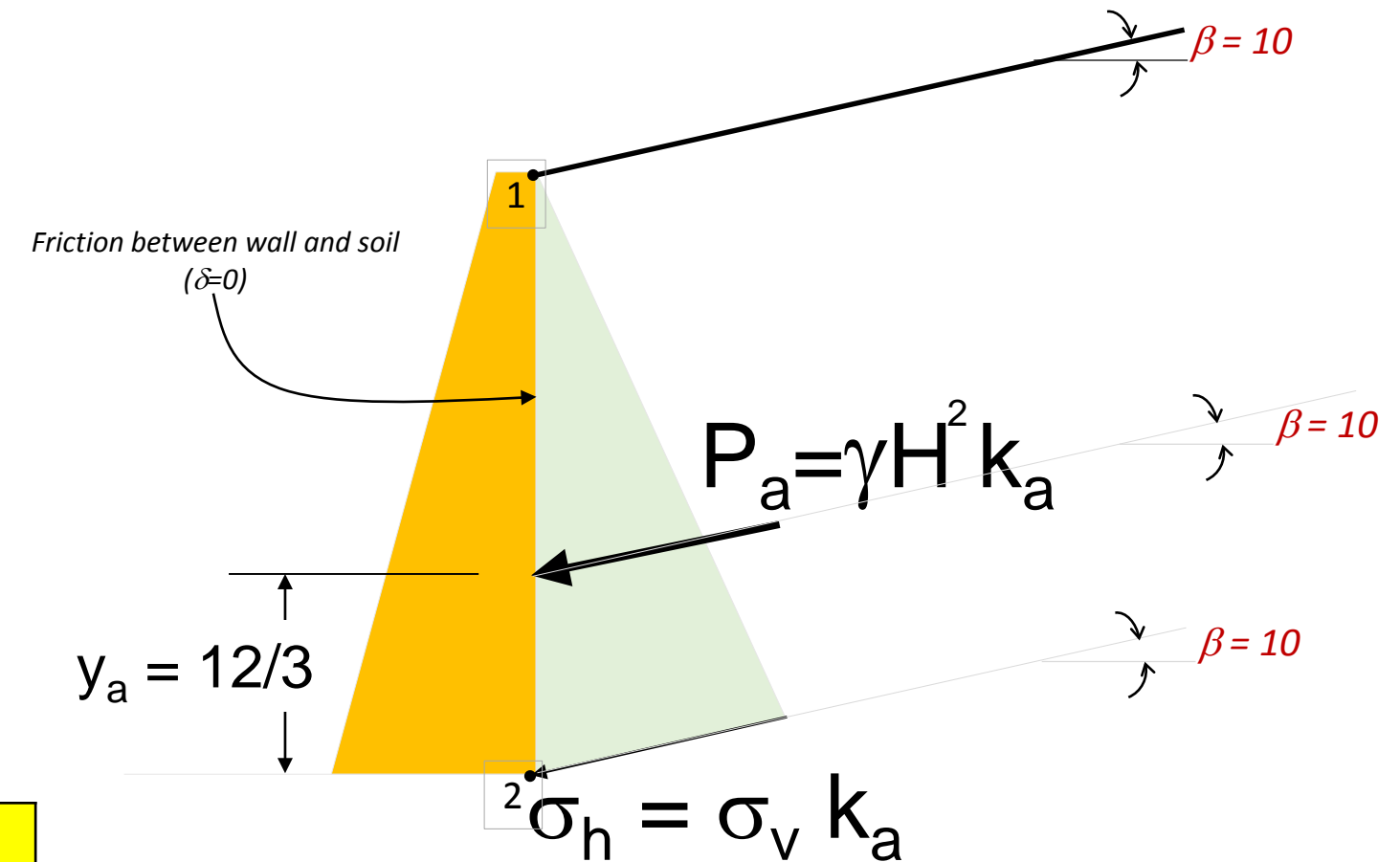
- Lateral force  $P_a$  acting on the wall

### Solution:

$$k_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} = \cos 10^\circ \frac{\cos 10^\circ - \sqrt{\cos^2 10^\circ - \cos^2 30^\circ}}{\cos 10^\circ + \sqrt{\cos^2 10^\circ - \cos^2 30^\circ}} = 0.35$$

Point	Vertical Stress $\sigma_v$ $\gamma H \left(\frac{\text{lb/ft}^2}{\text{ft}}\right)$	Horizontal Stress $\sigma_h$ $\gamma H k_a \left(\frac{\text{lb/ft}^2}{\text{ft}}\right)$	$P_a = \frac{1}{2} \gamma H^2 k_a$ (lb/ft)	$y_a$ (ft)
1	0	0	0.5x483x12 = 2,898	12/3=4
2	115x12=1,380	115x12x0.35= 483		

$$k_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$



# Active Earth Pressure in $\phi$ – Soil (Using Rankine's Method)

Rankine's Method always assumes  $\delta = 0$

## Example - 5

### Given:

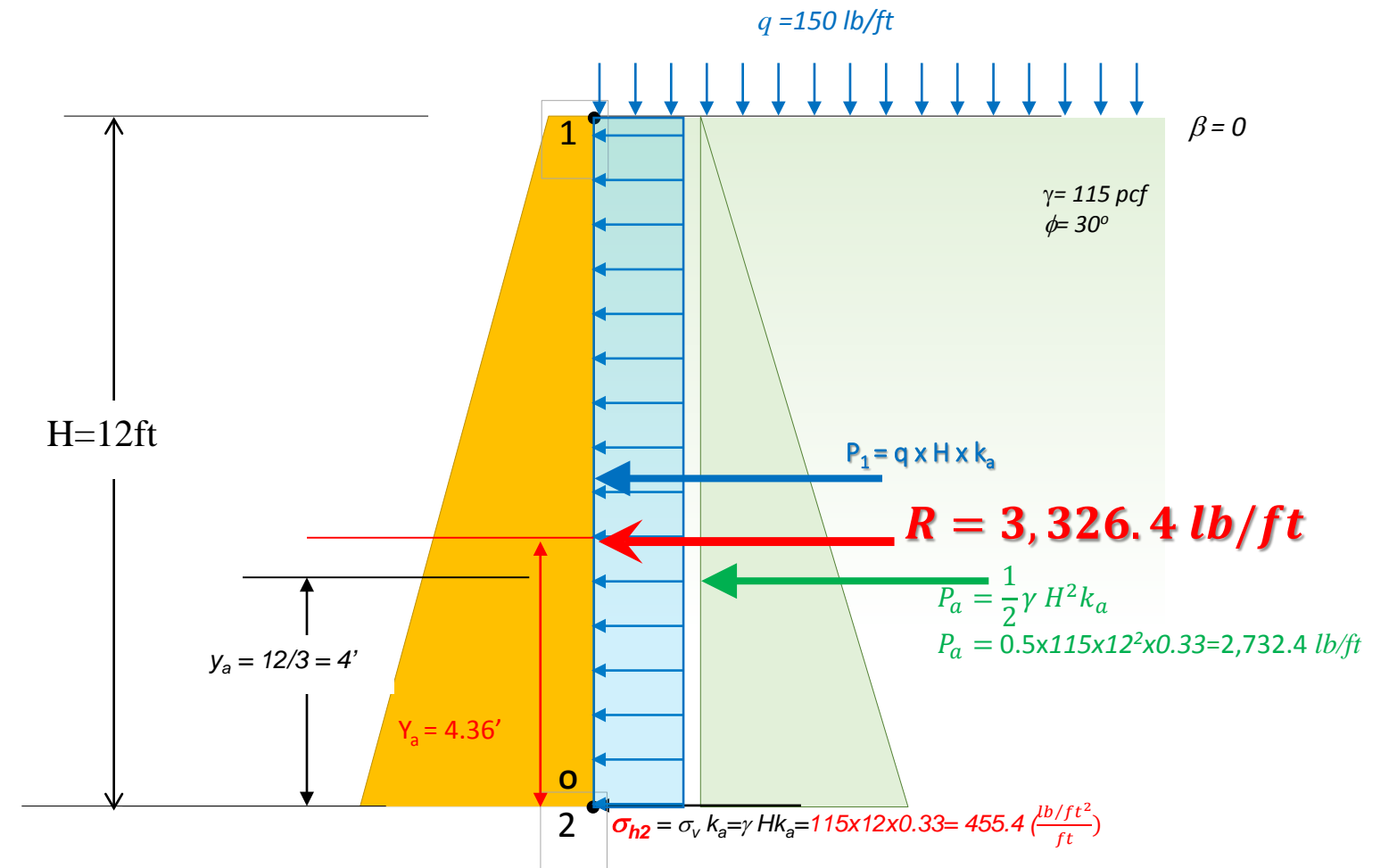
- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ( $\gamma$ ) = 115 pcf
- Angle of soil friction ( $\phi$ ) =  $30^\circ$
- Assume wall to be smooth ( $\delta=0$ )

### Find:

- Lateral force  $P_a$  acting on the wall

### Solution:

$$k_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 30}{1 + \sin 30} = 0.33$$

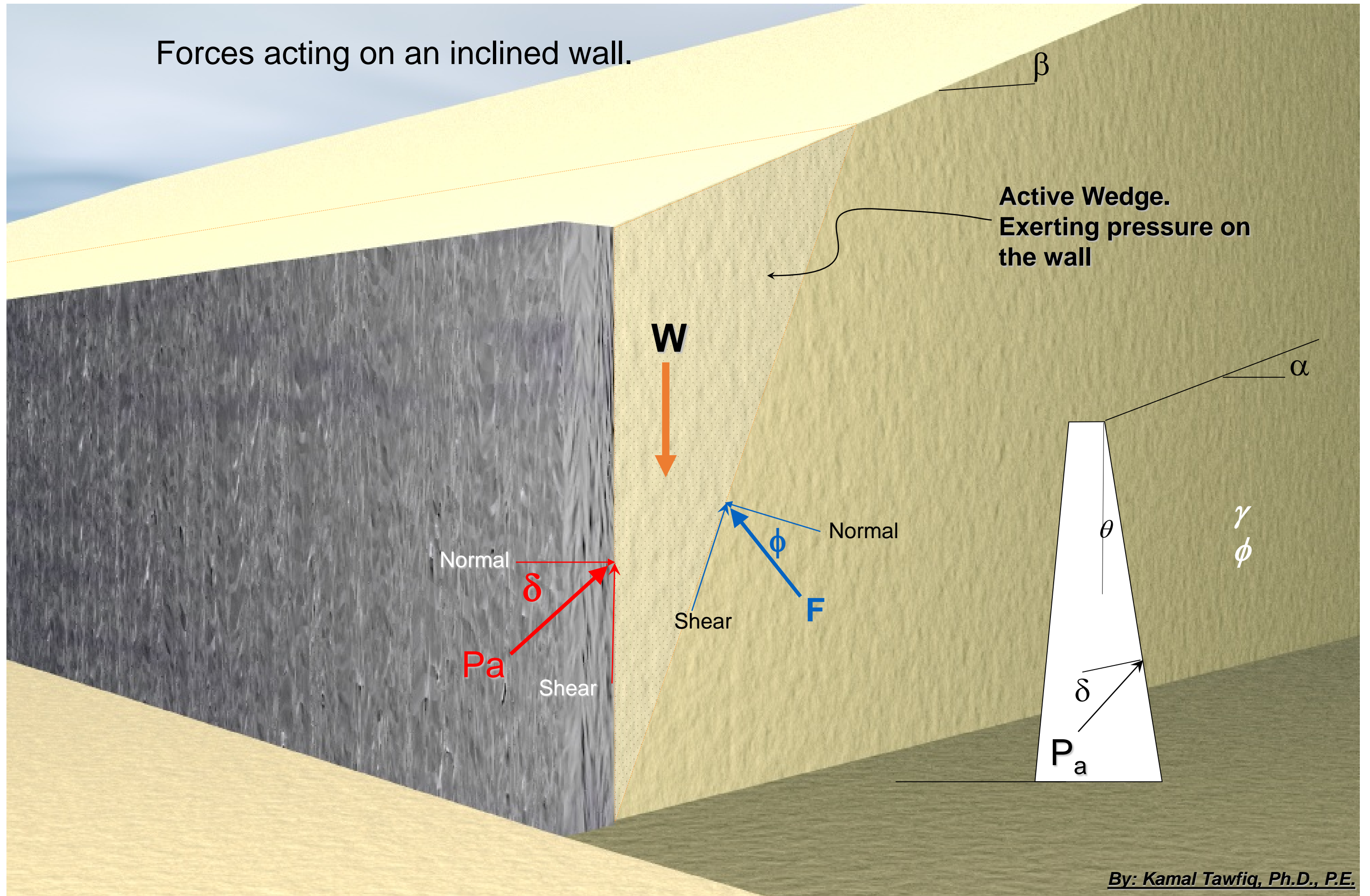


Point	Vertical Stress $\sigma_v$ $\gamma H \left(\frac{\text{lb/ft}^2}{\text{ft}}\right)$	Horizontal Stress $\sigma_h$ $\gamma H k_a \left(\frac{\text{lb/ft}^2}{\text{ft}}\right)$
1	150	$150 \times 0.33 = 49.5$
2	150	$150 \times 0.33 = 49.5$
3	$115 \times 12$	$115 \times 12 \times 0.33 = 455.4$

Lateral Force P (lb/ft)	Distance to O $y_a$ (ft)
$P_1 = 150 \times 0.33 \times 12 = 594$	$12/2 = 6$
$P_a = 0.5 \times 115 \times 12^2 \times 0.33 = 2,732.4$	$12/3 = 4$
$R = \sum P = 3,326.4$	Take Moment about Point O $= \sum M_o = 594 \times 6 + 2732.4 \times (12/3) = 14,493.6$ $y_a = \frac{\sum M_o}{\sum P} = \frac{14,493.6}{3,326.4} = 4.36\text{ ft}$

# Coulomb Earth Pressure Method

Forces acting on an inclined wall.





# COULOMB'S WEDGE THEORY

**W** = weight of the soil wedge  
**R** = resultant of the shear and normal forces on the failure surface BC  
**P<sub>a</sub>** = the active force per unit length of the wall. The direction of P<sub>a</sub> is inclined at an angle δ to the normal drawn and the face of the wall that supports the soil  
**δ** = the angle of friction between the soil and the wall

$$W = \gamma \cdot (\text{area of wedge } ABC)$$

From the triangles of forces,

$$\frac{P_a}{\sin(\theta - \phi)} = \frac{W}{\sin(180^\circ - \psi - \theta + \phi)}$$

$$P_a = \frac{W \sin(\theta - \phi)}{\sin(180^\circ - \psi - \theta + \phi)}$$

Substituting for W,

$$P_a = \frac{1}{2} \cdot \frac{\gamma H^2}{\sin^2 \alpha} \cdot \frac{\sin(\theta - \phi)}{\sin(180^\circ - \psi - \theta + \phi)} \cdot \frac{\sin(\theta + \alpha) \cdot \sin(\alpha + \beta)}{\sin(\theta - \beta)}$$

The maximum value of P<sub>a</sub> is obtained by equating the first derivative of P<sub>a</sub> with respect to θ to zero; or

(∂P<sub>a</sub>)/∂θ = 0, and substituting the corresponding value of θ.

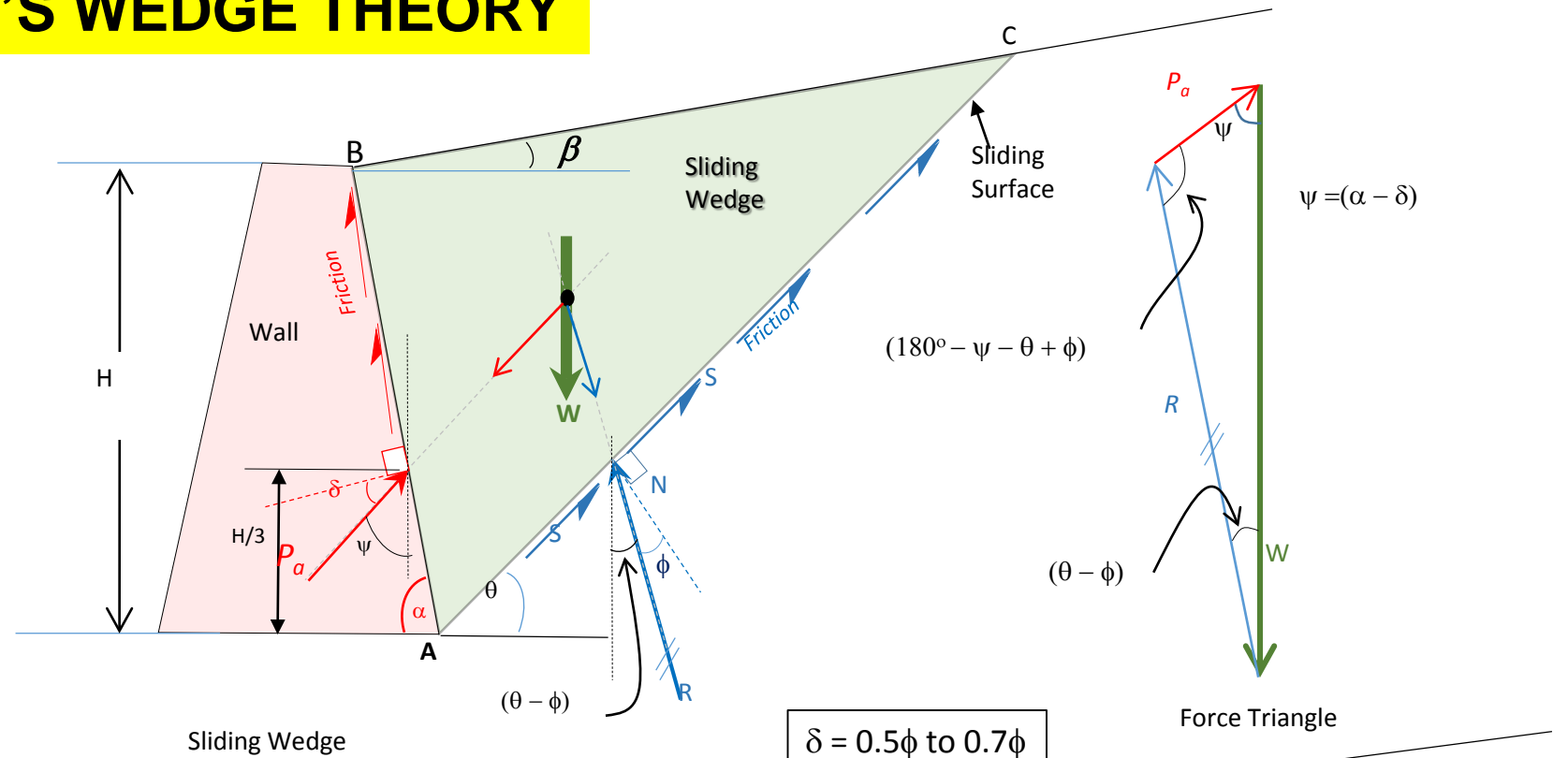
The value of P<sub>a</sub> so obtained is written as

$$P_a = \frac{1}{2} \cdot \gamma H^2 \cdot \frac{\sin^2(\alpha + \phi)}{\sin^2 \alpha \sin(\alpha - \delta) \left[ 1 + \sqrt{\frac{\sin(\phi + \delta) \sin(\phi - \beta)}{\sin(\alpha - \delta) \sin(\alpha + \beta)}} \right]^2}$$

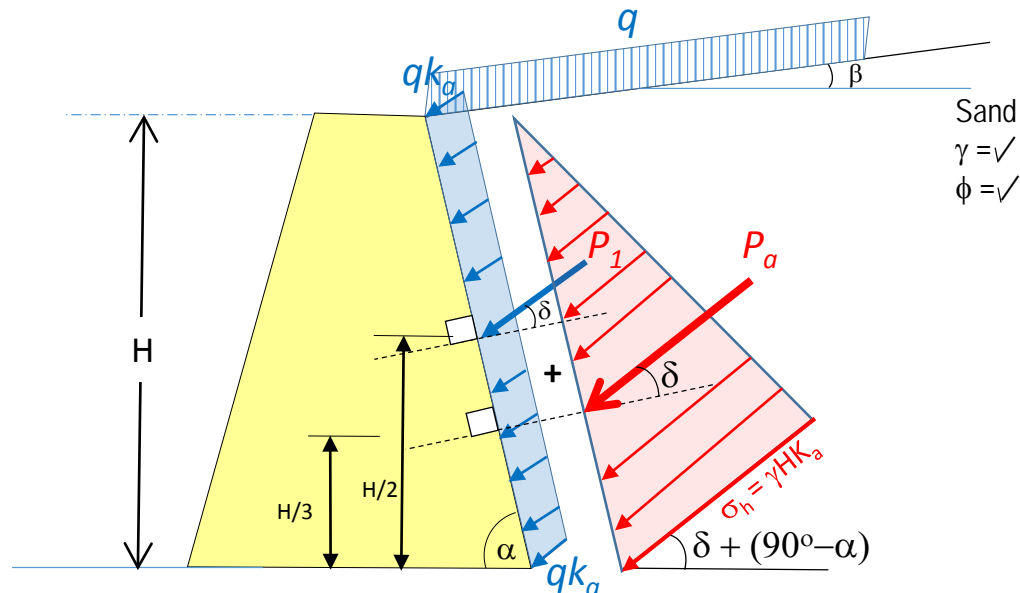
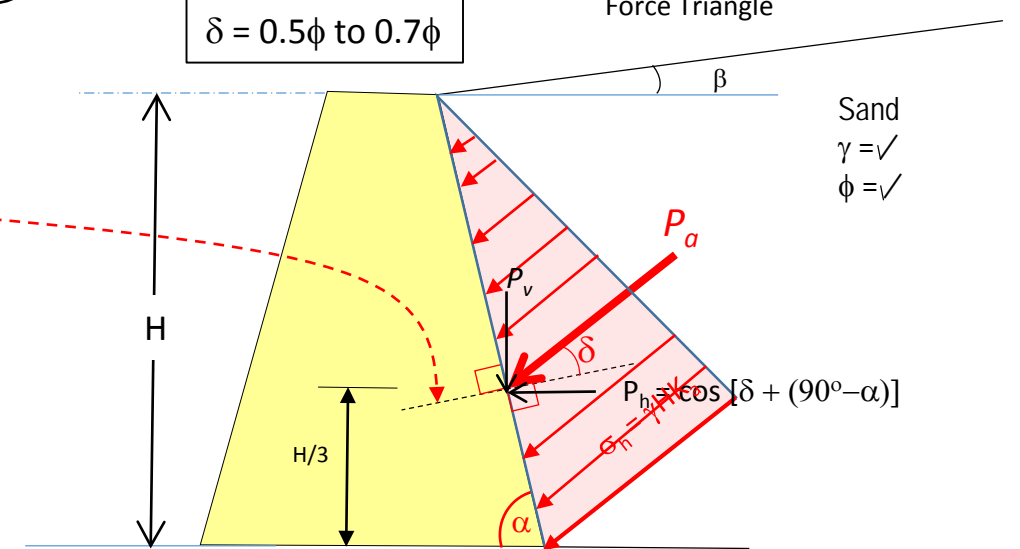
This is usually written as

$$P_a = \frac{1}{2} \cdot \gamma H^2 \cdot K_a$$

Where  $K_a$  being the coefficient of active earth pressure = 
$$\frac{\sin^2(\alpha + \phi)}{\sin^2 \alpha \sin(\alpha - \delta) \left[ 1 + \sqrt{\frac{\sin(\phi + \delta) \sin(\phi - \beta)}{\sin(\alpha - \delta) \sin(\alpha + \beta)}} \right]^2}$$



- Draw this perpendicular line first
- Then draw P<sub>a</sub> with an angle = α

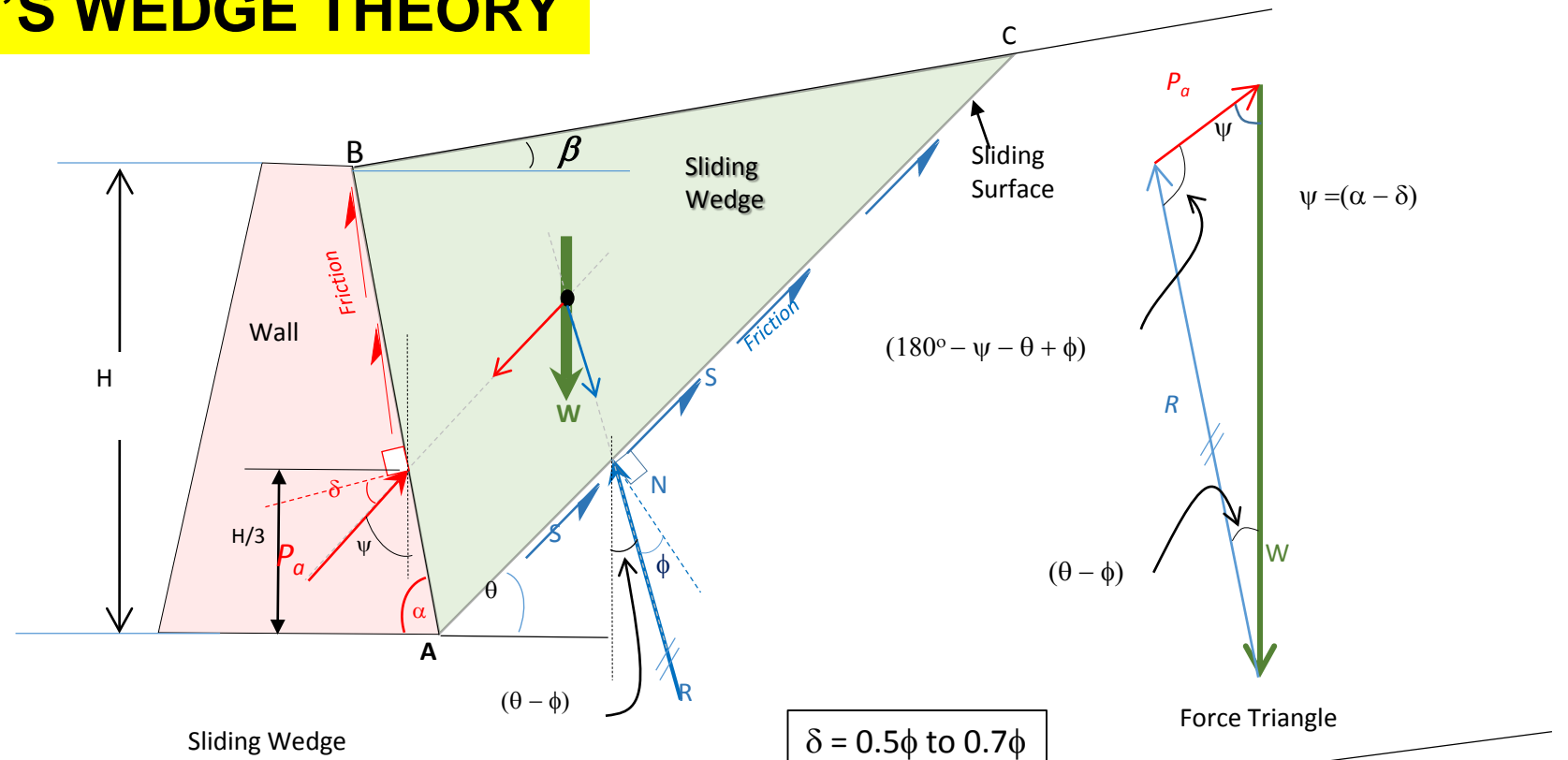


# COULOMB'S WEDGE THEORY

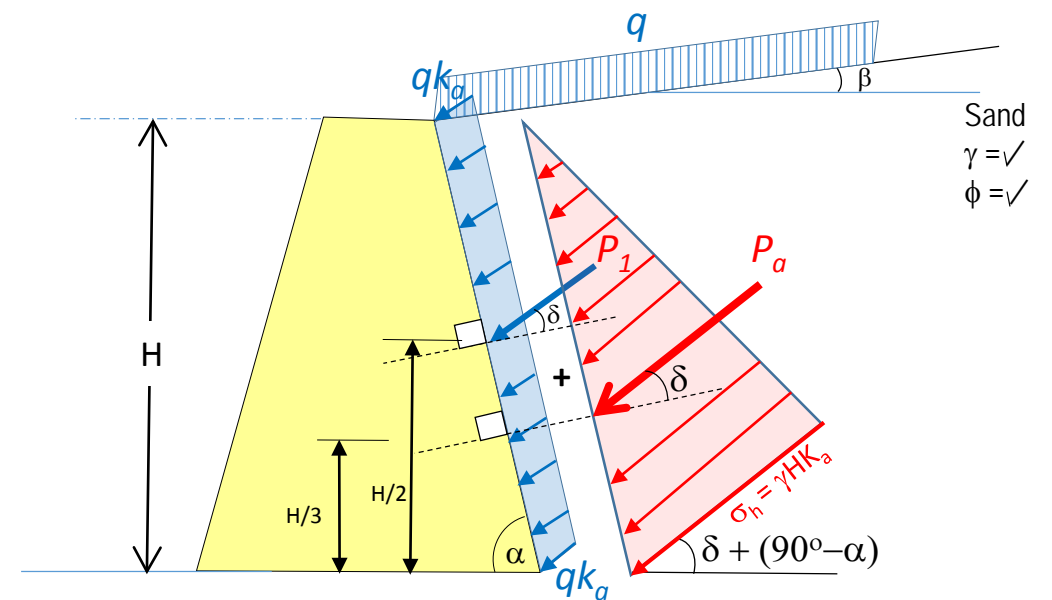
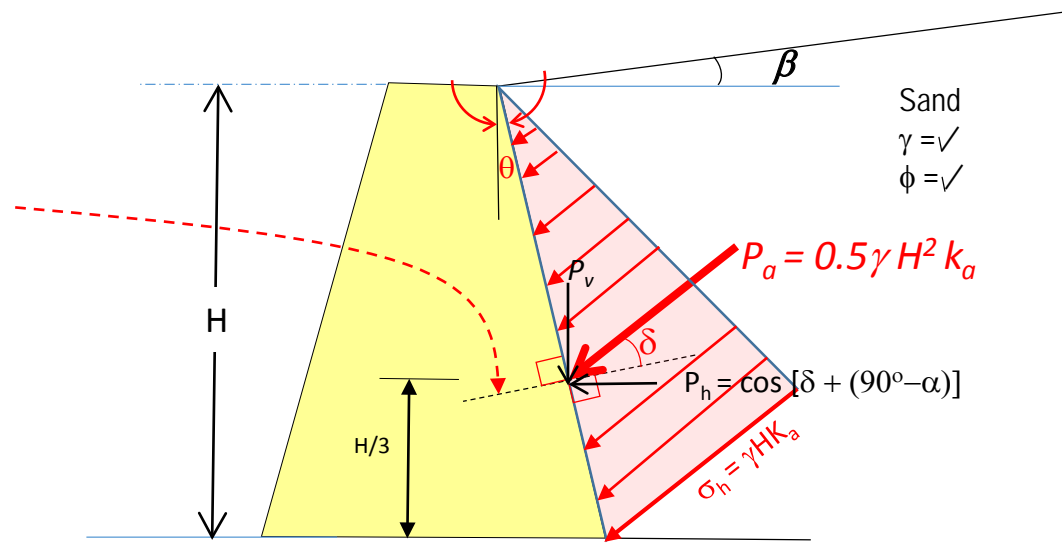
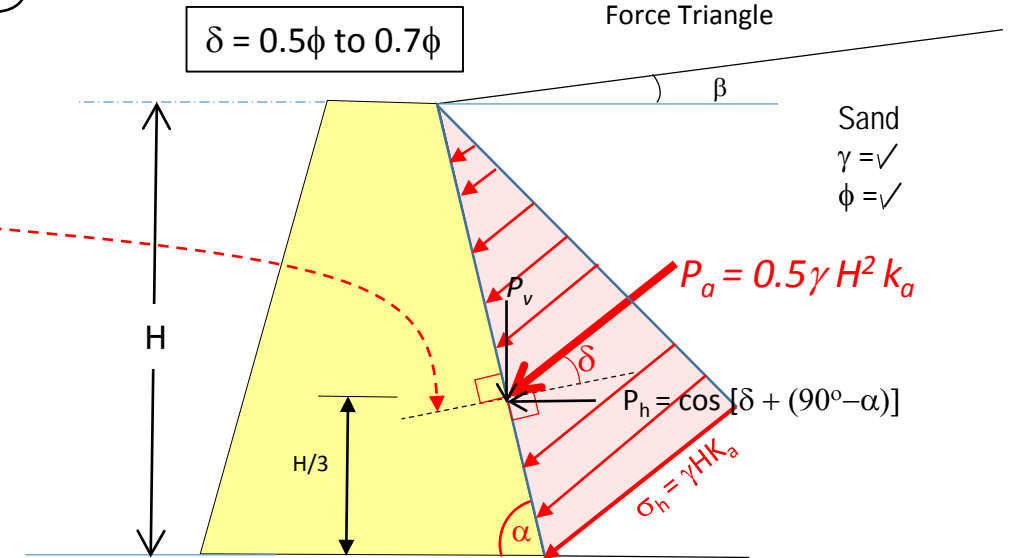
$$K_a = \frac{\sin^2(\alpha + \phi)}{\sin^2\alpha \sin(\alpha - \delta) \left[ 1 + \sqrt{\frac{\sin(\phi + \delta) \sin(\phi - \beta)}{\sin(\alpha - \delta) \sin(\alpha + \beta)}} \right]^2}$$

OR

$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \cos(\delta + \theta) \left[ 1 + \sqrt{\frac{\sin(\delta + \phi) \sin(\phi - \beta)}{\cos(\delta + \theta) \cos(\beta - \theta)}} \right]^2}$$



- Draw this perpendicular line first
- Then draw  $P_a$  with an angle =  $\alpha$



# Active Earth Pressure in $\phi$ – Soil (Using Coulomb's Method)

## Example - 6

### Given:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ( $\gamma$ ) = 115 pcf
- Angle of soil friction ( $\phi$ ) =  $30^\circ$
- Assume wall to be smooth ( $\delta=20$ )

### Find:

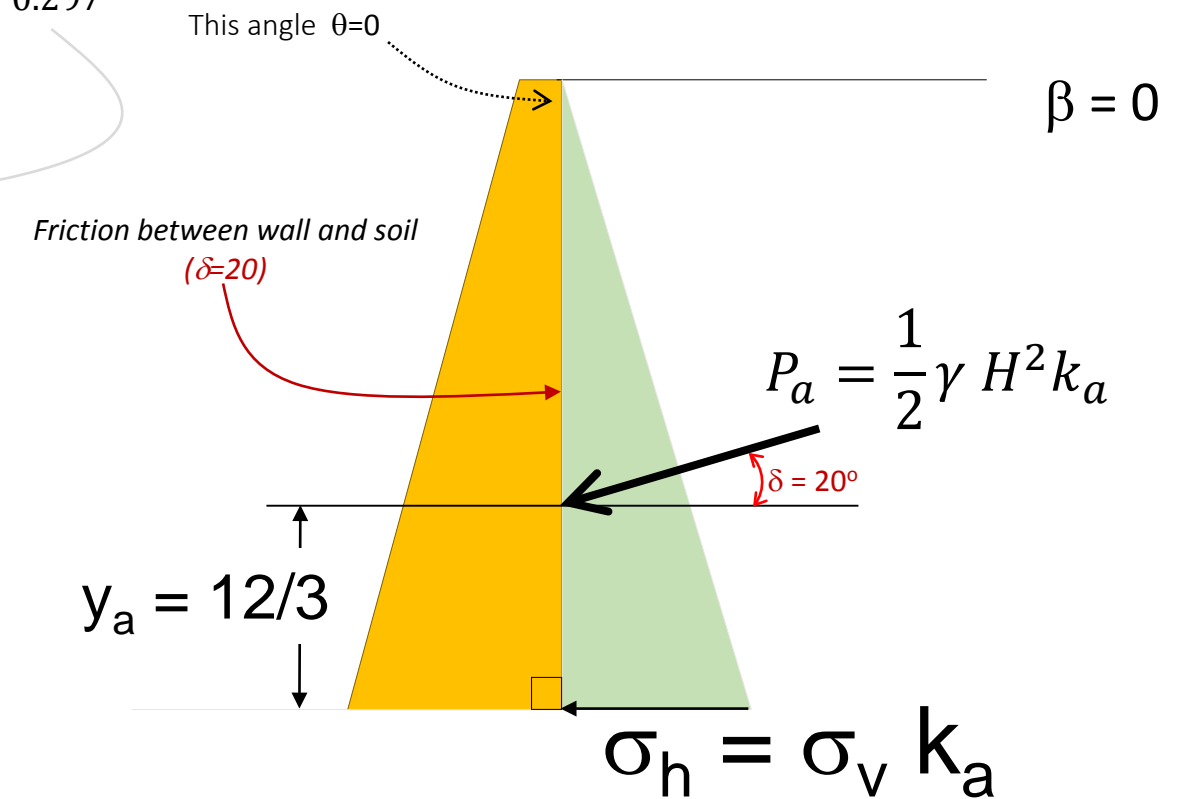
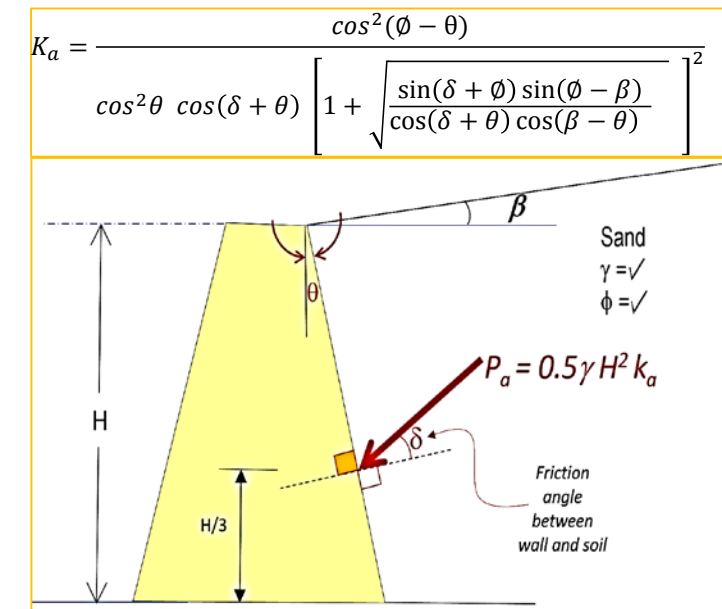
- Lateral force  $P_a$  acting on the wall

### Solution:

$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \cos(\delta + \theta) \left[ 1 + \frac{\sin(\delta + \phi) \sin(\phi - \beta)}{\cos(\delta + \theta) \cos(\beta - \theta)} \right]^2} = \frac{\cos^2(30 - 0)}{\cos^2 0 \cos(20 + 0) \left[ 1 + \frac{\sin(20 + 30) \sin(30 - 0)}{\cos(20 + 0) \cos(0 - 0)} \right]^2} = 0.297$$

Point	Vertical Stress $\sigma_v$ $\gamma H \left( \frac{\text{lb}/\text{ft}^2}{\text{ft}} \right)$	Horizontal Stress $\sigma_h$ $\gamma H k_a \left( \frac{\text{lb}/\text{ft}^2}{\text{ft}} \right)$	$P_a = \frac{1}{2} \gamma H^2 k_a$ (lb/ft)	$y_a$ (ft)
1	0	0	0.5x409.86x12=2,459.2	12/3=4
2	115x12=1,380	115x12x0.297= 409.86		

About 10% less than the Rankine's Earth Pressure



# Active Earth Pressure in $\phi$ – Soil (Using Coulomb's Method)

## Example - 7

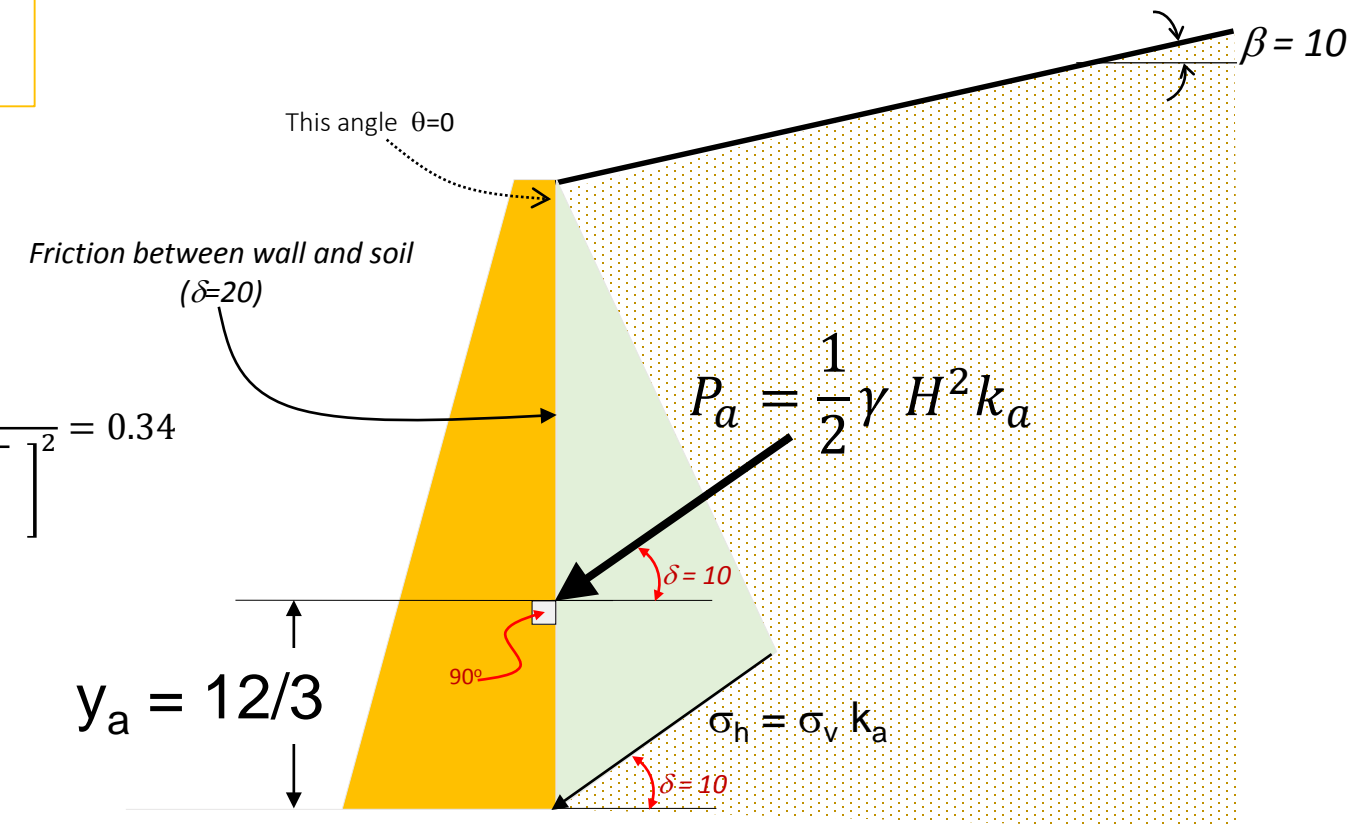
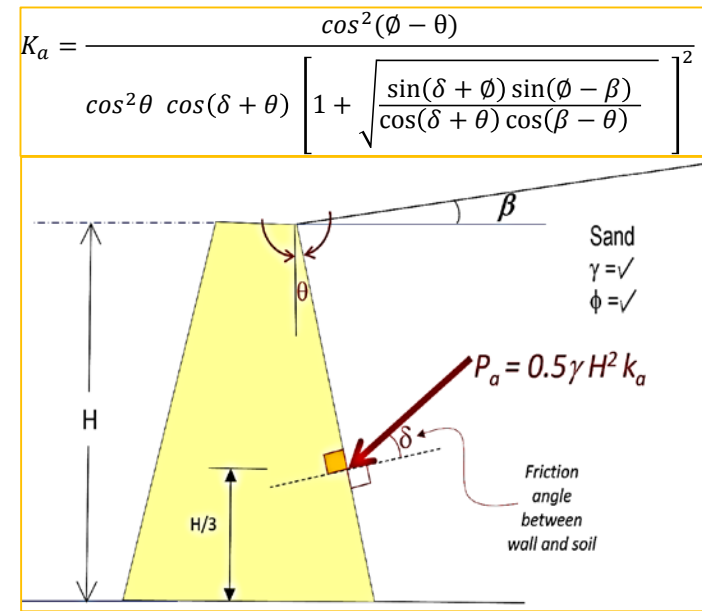
### Given:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ( $\gamma$ ) = 115 pcf
- Angle of soil friction ( $\phi$ ) =  $30^\circ$
- Ground surface slope  $\alpha = 10^\circ$
- Assume wall to be smooth ( $\delta=20^\circ$ )

### Find:

- Lateral force  $P_a$  acting on the wall

### Solution:



$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \cos(\delta + \theta) \left[ 1 + \frac{\sin(\delta + \phi) \sin(\phi - \beta)}{\cos(\delta + \theta) \cos(\beta - \theta)} \right]^2} = \frac{\cos^2(30 - 0)}{\cos^2 0 \cos(20 + 0) \left[ 1 + \frac{\sin(20 + 30) \sin(30 - 10)}{\cos(20 + 0) \cos(10 - 0)} \right]^2} = 0.34$$

Point	Vertical Stress $\sigma_v$ $\gamma H \left( \frac{\text{lb}/\text{ft}^2}{\text{ft}} \right)$	Horizontal Stress $\sigma_h$ $\gamma H k_a \left( \frac{\text{lb}/\text{ft}^2}{\text{ft}} \right)$	$P_a = \frac{1}{2} \gamma H^2 k_a$ (lb/ft)	$y_a$ (ft)
1	0	0	$0.5 \times 115 \times 12^2 \times 0.34 = 2,815.2$	$12/3 = 4$
2	$115 \times 12 = 1,380$	$115 \times 12 \times 0.34 = 469.2$		

About 3% less than the Rankine's Earth Pressure

# Active Earth Pressure in $\phi$ – Soil (Using Coulomb's Method)

## Example - 8

### Given:

- Vertical retaining wall (flexible)
- Wall height (H) = 12 ft
- Backfill unit weight ( $\gamma$ ) = 115 pcf
- Angle of soil friction ( $\phi$ ) =  $30^\circ$
- Ground surface slope  $\alpha = 10^\circ$
- Assume wall to be smooth ( $\delta=20^\circ$ )
- The wall batter angle  $\theta = 10^\circ$

### Find:

- Lateral force  $P_a$  acting on the wall

### Solution:

$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \cos(\delta + \theta) \left[ 1 + \frac{\sin(\delta + \phi) \sin(\phi - \beta)}{\cos(\delta + \theta) \cos(\beta - \theta)} \right]^2} = \frac{\cos^2(30 - 10)}{\cos^2 10 \cos(20 + 10) \left[ 1 + \frac{\sin(20 + 30) \sin(30 - 10)}{\cos(20 + 10) \cos(10 - 10)} \right]^2} = 0.475$$

Point	Vertical Stress $\sigma_v$ $\gamma H \left( \frac{\text{lb}/\text{ft}^2}{\text{ft}} \right)$	Horizontal Stress $\sigma_h$ $\gamma H k_a \left( \frac{\text{lb}/\text{ft}^2}{\text{ft}} \right)$	$P_a = \frac{1}{2} \gamma H^2 k_a$ (lb/ft)	$y_a$ (ft)
1	0	0	$0.5 \times 115 \times 12^2 \times 0.475 = 3,933$	$12/3 = 4$
2	$115 \times 12 = 1,380$	$115 \times 12 \times 0.475 = 655.5$		

About 28% more than Vertical wall ( $\theta = 0$ )

